

Performance Task #1 — Building a Pyramid

Specific Expectations

Unless stated otherwise, the following expectations are common to the Functions and Relations course and the Functions course.

- FA1.01(U/C) write terms of a sequence, given the formula for the n th term
- FA1.02 determine a formula for the n th term of a given sequence (e.g., the n th term of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots$ is $\frac{n}{n+1}$)
- FA1.03 identify sequences as arithmetic or geometric, or neither
- FA1.04 determine the value of any term in an arithmetic or a geometric sequence, using the formula for the n th term of the sequence
- FA1.05 determine the sum of the terms of an arithmetic or a geometric series, using appropriate formulas and techniques
- FA3.05 communicate the solutions to problems and the findings of investigations with clarity and justification
- OC3.03 communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions

Text Page 207

Time Required Functions and Relations or Functions: 60–70 min

Materials and Equipment

- graphing calculator
- sugar cubes (or other cubes)

Overview

This *Performance Task* allows students to demonstrate that they can

- use sequences and series to model problem-solving situations
- solve real-world problems involving arithmetic and geometric sequences and arithmetic and geometric series
- communicate mathematics in various forms
- justify their reasoning

Description of Task

This *Performance Task* requires students to construct a physical model out of sugar cubes and then determine what type of sequence the model represents. Students will also be expected to write an algebraic model. They will need to look at the total number of cubes as a series and solve problems using the algebraic models for the sequence and the series. Students must explain and justify their models.

Management Suggestions

You might assign this *Performance Task* at the end of Chapter 1, since its focus is sequences and series, rather than financial applications. Students could work in pairs to build the pyramid and to start listing the terms of the sequence. Then they should develop the algebraic models on their own.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*. Use the rubric not only to assess the written responses but also to assess by observing.

Assess the Learning Skill Independence using the related generic rubric or by distributing the Independence Self-Reflection Sheet or the Learning Skills Self-Assessment Checklists to students. (See Assessment and Evaluation in the *Teacher Resource*.)

Guidelines for Solution

Create a table.

Layer	Number of Cubes in a Layer	Total Number of Cubes
1	1	1
2	4	5
3	9	14
4	16	30
5	25	55
6	36	91

Part (a)

In the 75th layer, there would be 75^2 or 5625 sugar cubes.

Part (b)

Create an algebraic model. The number of cubes, t_n , in each layer, n , is $t_n = n^2$. But the pattern of the total number of cubes must be cubic since the third differences* are constant. Students could use a graphing calculator (by entering lists and using cubic regression in the STAT CALC menu) to find the general term for the sequence of the total number of cubes,

$$t_n = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

So, for $n = 100$, you would need 338 350 sugar cubes to construct a model with 100 layers.

* The first differences of the sequence 1, 5, 14, 30, 55, 91, ... are 4, 9, 16, 25, 36, ... The second differences are 5, 7, 9, 11 and the third differences are 2, 2, 2.

Part (c)

Students should describe how they analyzed the problem and arrived at the algebraic models. They should further explain how to use the models to make predictions.

Performance Task #2 — Population Growth

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- FA1.03 identify sequences as arithmetic or geometric, or neither
- FA1.04 determine the value of any term in an arithmetic or a geometric sequence, using the formula for the n th term of the sequence
- FA1.05 determine the sum of the terms of an arithmetic or a geometric series, using appropriate formulas and techniques
- FA2.02 solve problems involving compound interest and present value
- FA2.05 demonstrate an understanding of the relationships between compound interest, geometric sequences, and exponential growth
- FA3.05 communicate the solutions to problems and the findings of investigations with clarity and justification
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions

Text Page 207

Time Required Functions and Relations or Functions: 45–60 min

Materials and Equipment

- scientific or graphing calculator

Overview

This *Performance Task* allows students to demonstrate that they can

- demonstrate an understanding between growth, compound interest, and geometric sequences and series
- use sequences and series to model solutions
- solve real-world problems about arithmetic and geometric sequences and series
- solve problems about compound interest and annuities
- communicate mathematics in various forms
- justify their reasoning

Description of Task

This *Performance Task* requires students to create a model for population growth and to make a prediction based on this model. The third part requires them to adjust their models for a decline in the population.

Management Suggestions

Students could work in pairs to brainstorm what needs to be done and then move apart to work on individual solutions.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*.

Assess the Learning Skill Initiative using the related generic rubric. (See Assessment and Evaluation in the *Teacher Resource*.)

Guidelines for Solution

Part (a)

China's population grows at an exponential rate. The relationship is similar to the growth of compound interest. The model is $P = 1.1 \times 10^9 (1.015)^n$, where P is the population and n is the number of years after 1990.

Part (b)

$P = 2$ billion. Solve for n .

$$P = 1.1 \times 10^9 (1.015)^n$$

$$2 \times 10^9 = 1.1 \times 10^9 (1.015)^n$$

$$\frac{2 \times 10^9}{1.1 \times 10^9} = (1.015)^n$$

$$(1.015)^n = 1.8181$$

Students should estimate n by trial and error or using graphing technology.

$n \doteq 40$, which means that the population will reach two billion in 2030.

Part (c)

Create a table comparing population decline to growth.

Year	Population Decline	Population Growth
1991	$1.1 \times 10^9 (1.015) - 7 \times 10^6$	$1.1 \times 10^9 (1.015)^1 - 7 \times 10^6$
1992	$[1.1 \times 10^9 (1.015) - 7 \times 10^6] \times (1.015) - 7 \times 10^6$	$1.1 \times 10^9 (1.015)^2 - 7 \times 10^6 (1.015)^1 - 7 \times 10^6$
1993	$[1.1 \times 10^9 (1.015)^2 - 7 \times 10^6 (1.015) - 7 \times 10^6] \times (1.015) - 7 \times 10^6$	$1.1 \times 10^9 (1.015)^3 - 7 \times 10^6 (1.015)^2 - 7 \times 10^6 (1.015)^1 - 7 \times 10^6$
1994	$[1.1 \times 10^9 (1.015)^3 - 7 \times 10^6 (1.015)^2 - 7 \times 10^6 (1.015) - 7 \times 10^6] \times (1.015) - 7 \times 10^6$	$1.1 \times 10^9 (1.015)^4 - 7 \times 10^6 (1.015)^3 - 7 \times 10^6 (1.015)^2 - 7 \times 10^6 (1.015)^1 - 7 \times 10^6$

The population model has two components, the exponential model, which is similar to the model in (a) and (b), and a geometric series. Here is one model for the population:

$$P = 1.1 \times 10^9 (1.015)^n - \sum_{i=0}^{n-1} 7 \times 10^6 (1.015)^i, \text{ where } n \text{ is the number of years after 1990}$$

Performance Task #3 — A Picture Is Worth a Thousand Words

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- FA3.01 analyse the effects of changing the conditions in long-term savings plans (e.g., altering the frequency of deposits, the amount of deposit, the interest rate, the compounding period, or a combination of these) (Sample problem: Compare the results of making an annual deposit of \$1000 to an RRSP, beginning at age 20, with the results of making an annual deposit of \$3000, beginning at age 50)
- FA3.03 generate amortization tables for mortgages, using spreadsheets or other appropriate software
- FA3.04 analyse the effects of changing the conditions of a mortgage (e.g., the effect on the length of time needed to pay off the mortgage or changing the payment frequency or the interest rate)
- FA3.05 communicate the solutions to problems and the findings of investigations with clarity and justification
- OC3.01 explain mathematical processes, methods of solution, and concepts clearly to others
- OC3.02 present problems and their solutions to a group, and answer questions about the problems and the solutions
- OC3.03 communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions

Text Page 207

Time Required Functions and Relations or Functions: 60–70 min (preparing poster or presentation may need to be done outside of class time)

Materials and Equipment

- amortization tables, spreadsheet software, mortgage program
- graphing calculator
- large graph paper, chart paper

Overview

This *Performance Task* allows students to demonstrate that they can

- apply knowledge about compound interest and annuities to financial decision making
- use technology as an aid to financial decision-making
- demonstrate an understanding of terms and conditions of Canadian mortgages
- communicate mathematics in various forms
- justify their reasoning

Description of Task

This *Performance Task* is open ended and requires students to choose a current mortgage rate and then show the effects of changing the conditions of a mortgage. Students might use tables and graphs to show these effects.

Management Suggestions

Students could work in pair or small groups on this *Performance Task*. Students could write a report, present their findings as a group, or create a poster. If students give presentations, the time required may take more than one period.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*. Assess students' problem-solving abilities by observing groups as they work. Assess Communication skills by listening to groups' dialogues.

Assess the Learning Skill Teamwork if students work in groups. Because it might be difficult to observe and assess the abilities of all groups, assess the Teamwork of only a few groups by observing or distribute the Teamwork Self-Reflection Sheet. (See Assessment and Evaluation in the *Teacher Resource*.)

Guidelines for Solution

Solutions will vary, but students should fully explore the effects of varying the conditions of a mortgage.

Performance Task #4 — Retirement Plans

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- FA1.05 determine the sum of the terms of an arithmetic or a geometric series, using appropriate formulas and techniques
- FA2.01 derive the formulas for compound interest and present value, the amount of an ordinary annuity, and the present value of an ordinary annuity, using the formulas for the n th term of a geometric sequence and the sum of the first n terms of a geometric series
- FA2.03 solve problems involving the amount and the present value of an ordinary annuity
- FA3.01 analyse the effects of changing the conditions in long-term savings plans (e.g., altering the frequency of deposits, the amount of deposit, the interest rate, the compounding period, or a combination of these) (Sample problem: Compare the results of making an annual deposit of \$1000 to an RRSP, beginning at age 20, with the results of making an annual deposit of \$3000, beginning at age 50)
- FA3.05 communicate the solutions to problems and the findings of investigations with clarity and justification
- OC3.02 present problems and their solutions to a group, and answer questions about the problems and the solutions
- OC3.03 communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms

Text Page 208

Time Required Functions and Relations or Functions: 45–60 min

Materials and Equipment

- spreadsheet software
- graphing calculator

Overview

This *Performance Task* allows students to demonstrate that they can

- solve problems involving compound interest and annuities
- solve real-world problems involving arithmetic and geometric sequences and series
- apply knowledge about compound interest and annuities to financial decision making
- use technology as an aid to financial decision making
- communicate mathematics in various forms
- justify their reasoning

Description of Task

Students are given information about two young women who are deciding when to start saving for their retirement. This *Performance Task* requires students to compare and contrast two methods and then write a convincing argument in favour of one woman's viewpoint.

Management Suggestions

Students could respond to this *Performance Task* individually or in pairs, with each partner taking a different position. Students then write a report, present their responses, or record a debate of two students who pretend to be Sara and Ritu.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*. Assess the problem-solving abilities of students by observing students if they work in pairs and assess Communication skills by listening to the pairs' conversations or presentations.

Assess the Learning Skill Organization by evaluating the organization of ideas in the written arguments or oral presentations.

Guidelines for Solution

Sara is 22, and we assume that she will make monthly deposits for 38 years. If Sara invests \$100/month at 9%/a, compounded monthly, she will have \$389 090.54 when she is 60 years old.

Ritu will start making deposits when she is 45, and we assume that she will make monthly deposits for 15 years. For Ritu to have \$389 090.54 by the time she is 60, she will need to deposit \$1028.24 each month.

This *Performance Task* requires students to choose the better plan and explain their choice. Some students may argue that Sara's monthly payment is much lower than Ritu's. However, some may argue that a person may better afford a higher payment when he or she is older. Also, some may support Ritu's position that a young person does not have the money to set aside in savings.

Performance Task #5 — Buying a Car

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- FA1.03 identify sequences as arithmetic or geometric, or neither
- FA1.04 determine the value of any term in an arithmetic or a geometric sequence, using the formula for the n th term of the sequence
- FA1.05 determine the sum of the terms of an arithmetic or a geometric series, using appropriate formulas and techniques
- FA2.01 derive the formulas for compound interest and present value, the amount of an ordinary annuity, and the present value of an ordinary annuity, using the formulas for the n th term of a geometric sequence and the sum of the first n terms of a geometric series
- FA2.02 solve problems involving compound interest and present value
- FA2.03 solve problems involving the amount and the present value of an ordinary annuity
- FA2.05 demonstrate an understanding of the relationships between compound interest, geometric sequences, and exponential growth
- FA3.01 analyse the effects of changing the conditions in long-term savings plans (e.g., altering the frequency of deposits, the amount of deposit, the interest rate, the compounding period, or a combination of these) (Sample problem: Compare the results of making an annual deposit of \$1000 to an RRSP, beginning at age 20, with the results of making an annual deposit of \$3000, beginning at age 50)
- FA3.05 communicate the solutions to problems and the findings of investigations with clarity and justification
- OC3.01 explain mathematical processes, methods of solution, and concepts clearly to others
- OC3.03 communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions

Text Page 208

Time Required Functions and Relations or Functions: 70–100 min (students who create a poster will need more time outside of class)

Materials and Equipment

- spreadsheet software
- graphing calculator
- automobile magazines, print advertisements, or consumer reports, showing current interest and depreciation rates

Overview

This *Performance Task* allows students to demonstrate that they can

- model solutions using sequences and series
- demonstrate an understanding of the relationships between growth, compound

interest, and geometric sequences and series

- solve real-world problems involving arithmetic and geometric sequences and series
- solve problems involving compound interest and annuities
- apply knowledge about compound interest and annuities to financial decision making
- use technology as an aid to investigating patterns and financial decision making
- communicate mathematics in various forms
- justify their reasoning

Description of Task

For this *Performance Task*, students will explore both the cost of buying a car on credit and the depreciation of the car's value.

Management Suggestions

Pairs could brainstorm what needs to be done and then work on solutions individually. Students could write a report or create a poster to show the cost of purchasing a car and the depreciation of the car's value.

EXTENSION

Invite students to obtain current information on a particular car as well as current interest and depreciation rates to make the problem more relevant. Students should look at other options such as leasing and to compare leasing to buying a car on credit.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*. Also assess by observing students at work and examining the written responses.

Assess the Learning Skill Teamwork if students work in pairs.

Guidelines for Solution

Solutions should include graphs and tables and show

- the value of the car over time
- the monthly payment, remaining principal, interest paid, and the outstanding balance after each payment, for the life of the loan

You should find this information in the solutions:

- Harpreet needs to borrow \$14 000.
- The monthly payment is \$348.39.
- The total of the monthly payments is \$16 722.75, and she pays \$2722.75 in interest.
- The value of her car after four years is \$7912.13 for a depreciation rate of 18% per year.

Performance Task #6 — Prescribing Medicine

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- FA1.04 determine the value of any term in an arithmetic or a geometric sequence, using the formula for the n th term of the sequence
- FA2.01 derive the formulas for compound interest and present value, the amount of an ordinary annuity, and the present value of an ordinary annuity, using the formulas for the n th term of a geometric sequence and the sum of the first n terms of a geometric series
- FA2.02 solve problems involving compound interest and present value
- FA2.05 demonstrate an understanding of the relationships between compound interest, geometric sequences, and exponential growth
- FA3.05 communicate the solutions to problems and the findings of investigations with clarity and justification
- OC3.01 explain mathematical processes, methods of solution, and concepts clearly to others
- OC3.03 communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms

Text Page 208

Time Required Functions and Relations or Functions: 70–100 min (Students could explore the *Performance Task* in class and write solutions outside of class time.)

Materials and Equipment

- spreadsheet software
- graphing calculator

Overview

This *Performance Task* allows students to demonstrate that they can

- use sequences and series to model problem-solving situations
- demonstrate an understanding of the relationships between growth, compound interest, and geometric sequences and series
- solve problems involving exponential growth
- communicate mathematics in various forms
- justify their reasoning

Description of Task

This *Performance Task* requires students to analyze the metabolism of a medication over ten days. The half-life is 2 h and the medication is taken every 4 h. Students should use a table, a graph, and an algebraic model to explore and

explain the problem.

Management Suggestions

Students should do this *Performance Task* in groups. Some students will wonder why traces of a substance can be found in someone's system long after he or she has taken the substance. Distribute an article about drug testing of athletes to stimulate whole-class discussion.

EXTENSION

Students could research the half-life of different medications or stimulants, for example, caffeine or antihistamines, and research and compare different rates of metabolism, for example, for smokers and nonsmokers, or for children and adults.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*. Also assess by observing students at work and examining written solutions.

The solutions to this *Performance Task* will provide an excellent opportunity to assess the Learning Skills Teamwork and Initiative.

Guidelines for Solution

Part (a)

Solutions should include a chart, a graph, and an algebraic model that show the amount of medicine in the patient's system. Here is one example of a table.

Time (h)	Amount of First Dose	Amount of Second Dose	Amount of Third Dose
0	30		
2	15		
4	7.5	30	
6	3.75	15	
8	1.875	7.5	30
10	0.9375	3.75	15
12	0.46875	1.875	7.5
14	0.234375	0.9375	3.75
...	...	...	...

Part (b)

The numbers in each column form a geometric sequence. After ten days, or 240 h, the amount of the first dose that will be in the patient's system is $30(0.5)^{120}$. The amount of the second dose is $30(0.5)^{118}$ and the amount of the third dose is $30(0.5)^{116}$.

Therefore, the total dose in the patient's system will be $30(0.5)^{120} + 30(0.5)^{118} + 30(0.5)^{116} + \dots$

The general term of this geometric series is $30(0.5)^t$, where t is the time in hours.

For the variables, $a = 30(0.5)^{120}$, $r = 0.5^{-2} = 2^2 = 4$. Since 60 doses are given in 240 h, $n = 60$. The sum of the terms of the geometric sequence is then

$$S_{60} = \frac{30(0.5)^{120}(4^{60} - 1)}{4 - 1}$$

$$= 10$$

Therefore, 10 mg of this medication will be in the patient's system after ten days.

Part (c)

After another ten days, $10(0.5)^{120}$ mg, or about 7.523×10^{-36} mg, will be in the patient's system.

Note: Alternatively, a student may argue that there are 61 doses, the last dose is taken at the end of the tenth day. In this case, there would be 40 mg of the medication in the patient's system at the end of the tenth day and 3.01×10^{-35} mg at the end of the 20th day.