

Performance Task #1 — Sunrise, Sunset

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- TF3.05 write the equation of a sinusoidal function, given its graph and given its properties
- TF4.01 determine, through investigation, the periodic properties of various models (e.g., the table of values, the graph, the equation) of sinusoidal functions drawn from various applications
- TF4.02 explain the relationship between the properties of a sinusoidal function and the parameters of its equation, within the context of an application, and over a restricted domain
- TF4.04 pose and solve problems related to models of sinusoidal functions drawn from various applications, and communicate the solutions with clarity and justification, using appropriate mathematical forms
- OC3.03 communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms
- OC3.05 use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve)

Text Page 555

Time Required Functions and Relations or Functions: 45–60 min

Materials and Equipment

- graphing calculator
- access to the Internet

Overview

This *Performance Task* allows students to demonstrate that they can

- demonstrate the relationships between the graphs and the equations of sinusoidal functions
- use radian measure in applications of trigonometric functions
- solve problems involving models of sinusoidal functions in various applications
- communicate mathematical reasoning clearly
- use graphing technology effectively
- solve multi-step problems

Description of Task

This *Performance Task* requires students to collect data about sunrise times and then fit a model to the data. They must test their models and use them to make predictions.

Management Suggestions

Have students work in pairs to gather data from the Internet or an almanac and then use this data for individual solutions.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*.

Assess Learning Skills as students work on this *Performance Task*, using the related generic rubrics. (See Assessment and Evaluation in the *Teacher Resource*.) Assess the ability of students to work independently and to organize their work. You might distribute the Learning Skills Self-Assessment Checklists (see Assessment and Evaluation) and ask students to fill out either the Teamwork section or the Organization section, but not both.

Guidelines for Solution

The solution will depend on the data collected by the students. Their reports should include the following:

- an accurate graph of the data
- an algebraic model of the data
- a justification of their choice of a model
- predictions based on the model
- a discussion of how the model was chosen and a description of how well the model fits the data

Performance Task #2 — Amusement Rides

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- TF3.05 write the equation of a sinusoidal function, given its graph and given its properties
- TF4.01 determine, through investigation, the periodic properties of various models (e.g., the table of values, the graph, the equation) of sinusoidal functions drawn from various applications
- TF4.02 explain the relationship between the properties of a sinusoidal function and the parameters of its equation, within the context of an application, and over a restricted domain
- TF4.03 predict the effects on the mathematical model of an application involving sinusoidal functions when the conditions in the application are varied
- TF4.04 pose and solve problems related to models of sinusoidal functions drawn from various applications, and communicate the solutions with clarity and justification, using appropriate mathematical forms
- OC3.03 communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions

Text Page 555

Time Required Functions and Relations or Functions: 45–60 min

Materials and Equipment

- graphing calculator or graphing software

Overview

This *Performance Task* allows students to demonstrate that they can

- demonstrate the relationships between the graphs and the equations of sinusoidal functions
- use radian measure in applications of trigonometric functions
- solve problems involving models of sinusoidal functions in various applications
- communicate mathematical reasoning clearly
- use graphing technology effectively
- solve multi-step problems

Description of Task

This *Performance Task* requires students to graph the height of someone riding on a Ferris wheel.

Management Suggestions

Students could work individually on this *Performance Task*.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*.

Assess Learning Skills as students work on this *Performance Task*. Assess the ability of students to work independently and to organize their work. You might distribute the Learning Skills Self-Assessment Checklists and ask students to fill out the Working Independently and the Organization sections. (See Assessment and Evaluation in the *Teacher Resource*.)

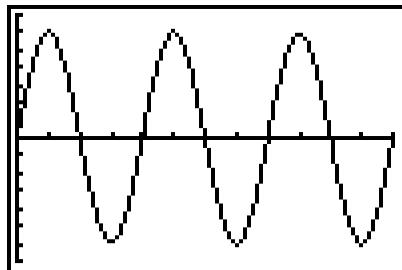
Guidelines for Solution

Part (a)

Create a table. The radius of the ring is 6 m.

Angle of Revolution (radians)	Height Above Platform (m)
0	0
$\frac{\pi}{4}$	$\sqrt{18}$
$\frac{\pi}{2}$	6
$\frac{3\pi}{4}$	$\sqrt{18}$
π	0
$\frac{5\pi}{4}$	$-\sqrt{18}$
$\frac{3\pi}{2}$	-6
$\frac{7\pi}{4}$	$-\sqrt{18}$
2π	0

The graph of three revolutions is



Part (b)

The equation that models the motion of a person in this ride is $y = 6 \sin x$.

Part (c)

If the rider got on the ride in the pit, and the platform was in the pit, then translate the graph of the rider's height vertically up 6 units. If instead, the rider got on the ride in the pit but the platform was at the centre of the ride, then translate the graph horizontally

$\frac{\pi}{4}$ units right.

Performance Task #3 — Lawn Chairs

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- TF1.01 determine the sine, cosine, and tangent of angles greater than 90° , using a suitable technique (e.g., related angles, the unit circle), and determine two angles that correspond to a given single trigonometric function value
- TF1.02 solve problems in two dimensions and three dimensions involving right triangles and oblique triangles, using the primary trigonometric ratios, the cosine law, and the sine law (including the ambiguous case)
- OC3.01 explain mathematical processes, methods of solution, and concepts clearly to others

Text Page 555

Time Required Functions and Relations or Functions: 45–60 min

Materials and Equipment

- scientific calculator

Overview

This *Performance Task* allows students to demonstrate that they can

- solve problems involving the measures of sides and angles in oblique triangles
- communicate mathematical reasoning clearly
- solve multi-step problems

Description of Task

This *Performance Task* requires students to apply trigonometric ratios and laws to solve a multi-step problem involving several triangles.

Management Suggestions

Have pairs brainstorm what needs to be done and individuals work on solutions.

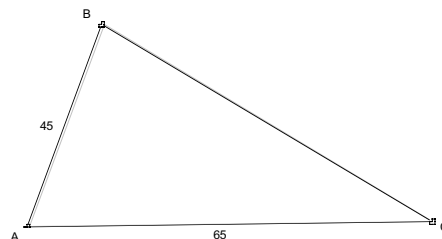
Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*.

Assess Learning Skills as students work, using the related generic rubrics. (See Assessment and Evaluation in the *Teacher Resource*.)

Guidelines for Solution

Students could approach this problem in various ways. Their solutions should include diagrams and proper mathematical conventions and integrate mathematical and narrative forms to explain their reasoning.



To find the position of the first notch, start with $\angle A$ and find the length of BC .

If the chair's angle is 95° , then $\angle A = 85^\circ$. To find the length of BC , use the cosine law.

$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos A \\a^2 &= 65^2 + 45^2 - 2(65)(45) \cos 85^\circ \\a &= 75.763\ 704\ 4\end{aligned}$$

Therefore, for the chair to be at a 95° angle, there will need to be a notch at about 75.8 cm from B .

Repeat these steps for the other given angles.

Alternatively, since $\angle C$ is fixed, use the sine law and the above information to find $\angle C$.

$$\frac{a}{\sin A} = \frac{c}{\sin C} \quad \text{Substitute to find } \angle C.$$

$$\frac{75.8}{\sin 85^\circ} = \frac{45}{\sin C} \quad \text{Solve for } \angle C.$$

$$\sin C = \frac{45 \sin 85^\circ}{75.8}$$

$$\angle C = \sin^{-1}(0.591\,408\,461\,9)$$

$$\angle C \approx 36.3^\circ$$

Then solve for a , substituting each value for $\angle A$ (which is not the angle of the chair).

$$a = \frac{c \sin A}{\sin C}$$

$$= \frac{45 \sin A}{\sin 36.3^\circ}$$

The other values are recorded in the table.

Angle of Chair	Length of BC
95°	75.8 cm
120°	65.9 cm
135°	53.8 cm
160°	26.0 cm
175°	6.63 cm

Performance Task #4 — Solving Systems of Trigonometric Equations

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- TF2.03 represent, in applications, radian measure in exact form as an expression involving π (e.g., $\frac{\pi}{3}$, 2π) and in approximate form as a real number (e.g., 1.05)
- TF2.04 determine the exact values of the sine, cosine, and tangent of the special angles 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$ and their multiples less than or equal to 2π
- TF2.05 prove simple identities, using the Pythagorean identity, $\sin^2 x + \cos^2 x = 1$, and the quotient relation, $\tan x = \frac{\sin x}{\cos x}$
- TF2.06 solve linear and quadratic trigonometric equations (e.g., $6 \cos^2 x - \sin x - 4 = 0$) on the interval $0 \leq x \leq 2\pi$
- TF2.07 demonstrate facility in the use of radian measure in solving equations and in graphing
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions
- OC3.05 use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve)

Text Page 556

Time Required Functions and Relations or Functions: 45–60 min

Materials and Equipment

- graphing calculator or graphing software

Overview

This *Performance Task* allows students to demonstrate that they can

- demonstrate the relationships between the graphs and the equations of sinusoidal functions
- use radian measure in applications of trigonometric functions
- communicate mathematical reasoning clearly
- use graphing technology effectively

Description of Task

This *Performance Task* requires students to solve a system of trigonometric equations both graphically and algebraically.

Management Suggestions

Students could work individually on this *Performance Task*.

Assessment Suggestions

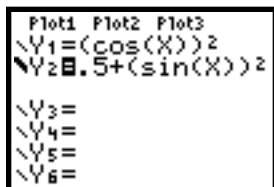
Assess Achievement using the rubric for this *Performance Task*.

Assess the Learning Skill Independence as students work on this *Performance Task* using the related generic rubric or distribute the Independence Self-Reflection Sheet to students. (See Assessment and Evaluation in the *Teacher Resource*.)

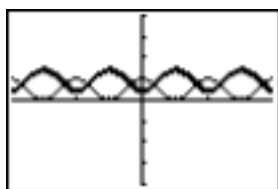
Guidelines for Solution

Graphic Solution

Encourage students to use graphing technology to graph each equation and then find the intersection of the curves. Students can verify the solution algebraically.

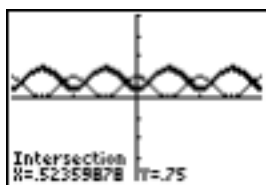


```
Plot1 Plot2 Plot3
Y1=(cos(X))^2
Y2=.5+(sin(X))^2
Y3=
Y4=
Y5=
Y6=
```



Use the intersect command of the CALCULATE menu to find one solution,

$$x = \frac{\pi}{6}, \text{ at } y = 0.75.$$



From the graph, it is clear that there are many other solutions such that x is a multiple of $\frac{\pi}{6}$.

Algebraic Solution

$$\cos^2 x = \frac{1}{2} + \sin^2 x \quad \text{Substitute } 1 - \sin^2 x \text{ for } \cos^2 x.$$

$$1 - \sin^2 x = \frac{1}{2} + \sin^2 x \quad \text{Simplify.}$$

$$\frac{1}{2} = 2 \sin^2 x$$

$$\frac{1}{4} = \sin^2 x \quad \text{Take square root of each side.}$$

$$\pm \frac{1}{2} = \sin x$$

$$\text{Therefore, } x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{-\pi}{6}, \frac{-5\pi}{6}, \frac{-7\pi}{6}, \frac{-11\pi}{6}.$$

Performance Task #5 — How High Is the Tower?

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- TF1.01 determine the sine, cosine, and tangent of angles greater than 90° , using a suitable technique (e.g., related angles, the unit circle), and determine two angles that correspond to a given single trigonometric function value
- TF1.02 solve problems in two dimensions and three dimensions involving right triangles and oblique triangles, using the primary trigonometric ratios, the cosine law, and the sine law (including the ambiguous case)
- OC3.01 explain mathematical processes, methods of solution, and concepts clearly to others
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions

Text Page 556

Time Required Functions and Relations or Functions: 60–70 min

Materials and Equipment

- scientific calculator

Overview

This *Performance Task* allows students to demonstrate that they can

- solve problems in two or three dimensions involving the measures of sides and angles in oblique and acute triangles
- communicate mathematical reasoning clearly
- solve multi-step problems

Description of Task

This *Performance Task* requires students to solve a complex trigonometry problem in three dimensions. Students should draw a diagram before solving.

Management Suggestions

Initially, students could work on this task individually, or in small groups or pairs to share ideas and begin drawing their diagrams. Students might want to make a three-dimensional model of the situation. Students could work in pairs to brainstorm what needs to be done and then work on their solutions individually.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task* and by observing students as they work and by examining the written solutions.

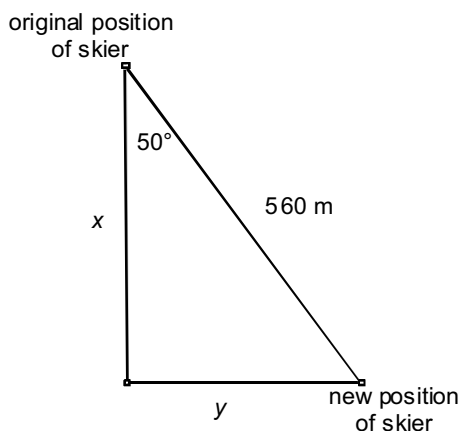
Assess the Learning Skills Teamwork or Independence using the related generic rubrics and by observing students as they work. Also assess students' written solutions. (See Assessment and Evaluation in the *Teacher Resource*.)

Guidelines for Solution

Students must visualize the problem in three dimensions. Drawing diagrams of the situation is sometimes difficult. Some students might draw several two-dimensional diagrams to represent the problem, while others might draw one diagram that shows three dimensions. To begin, students could build a three-dimensional model to understand and identify the different triangles. The following solution shows several diagrams, each representing a different two-dimensional component of the problem.

Part (a)

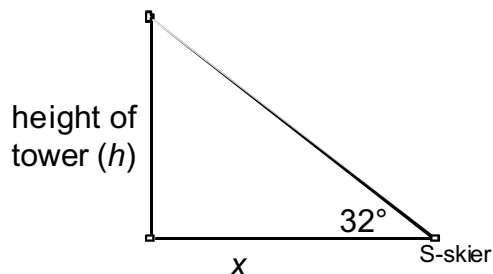
In the following diagrams, h represents the height of the tower, x represents the original distance of the skier from the tower, and y represents the distance of the skier from the tower after skiing on a bearing of 130° for 560 m.



Solve for x , using the above diagram.

$$\begin{aligned}\cos 50^\circ &= \frac{x}{560} \\ x &= 560 \cos 50^\circ \\ x &\approx 360\end{aligned}$$

Solve for h .



$$\begin{aligned}\frac{h}{x} &= \tan 32^\circ \\ \frac{h}{360} &= \tan 32^\circ \\ h &= 360 \tan 32^\circ \\ h &\approx 225\end{aligned}$$

Therefore, the height of the tower is about 225 m.

Part (b)

To find the distance from the tower to the skier, solve for y , using the first diagram.

$$\begin{aligned}\frac{y}{x} &= \sin 50^\circ \\ \frac{y}{360} &= \tan 50^\circ \\ y &= 360 \tan 50^\circ \\ y &\approx 429\end{aligned}$$

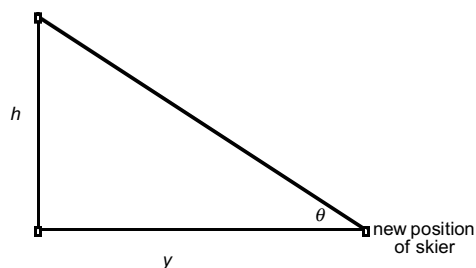
Therefore, the distance from the tower to the skier is about 429 m.

Part (c)

The following diagram is a side view of the skier (in his new position), viewing the top of the tower. Given $y = 429$ and $h = 225$,

$$\begin{aligned}\frac{h}{y} &= \tan \theta \\ \frac{225}{429} &= \tan \theta \\ \tan^{-1} \theta &= 0.524\ 475\ 524\ 5 \\ \theta &\approx 27.7^\circ\end{aligned}$$

The angle of elevation of the top of the tower from the new position is about 27.7° .



Performance Task #6 — Proving Identities

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- TF2.03 represent, in applications, radian measure in exact form as an expression involving π (e.g., $\frac{\pi}{3}$, 2π) and in approximate form as a real number (e.g., 1.05)
- TF2.04 determine the exact values of the sine, cosine, and tangent of the special angles 0 , $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$ and their multiples less than or equal to 2π
- TF2.05 prove simple identities, using the Pythagorean identity, $\sin^2 x + \cos^2 x = 1$, and the quotient relation, $\tan x = \frac{\sin x}{\cos x}$
- TF2.06 solve linear and quadratic trigonometric equations (e.g., $6 \cos^2 x - \sin x - 4 = 0$) on the interval $0 \leq x \leq 2\pi$
- TF2.07 demonstrate facility in the use of radian measure in solving equations and in graphing
- TF4.02 explain the relationship between the properties of a sinusoidal function and the parameters of its equation, within the context of an application, and over a restricted domain
- OC3.01 explain mathematical processes, methods of solution, and concepts clearly to others
- OC3.05 use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve)

Text Page 556

Time Required Functions and Relations or Functions: 60–70 min

Materials and Equipment

- graphing calculator or graphing software

Overview

This *Performance Task* allows students to demonstrate that they can

- demonstrate the relationships between the graphs and the equations of sinusoidal functions
- use radian measure in applications of trigonometric functions
- communicate mathematical reasoning clearly
- use graphing technology effectively
- solve multi-step problems

Description of Task

This *Performance Task* requires students to use graphing technology to visually determine whether each statement is an identity to be proven or an equation to be solved. In this way, students will see that an identity is true

for all values of the given variable, whereas an equation is true for selected values. The trigonometric functions in this *Performance Task* are more complex than those in chapters 5 or 6, but students can apply their graphing technology skills and their knowledge of periodic functions to find the solutions.

Management Suggestions

Individuals or groups can work on this *Performance Task*. Collect written solutions from individuals at the end of the period. Or, have each group fully explore graphing and proving or solving all four statements. Then each group can present the solution for one statement to the class. Allow students to use a TI-83 Plus calculator and the TI-83 view screen for their presentations.

Note: You might want to restrict the domains, for example, $-2\pi \leq x \leq 2\pi$, of the equations rather than have students describe the periodic nature of the solutions.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task* and by observing students as they work. Assess communication skills by examining written solutions or observing groups' presentations.

Assess the Learning Skills Teamwork and Organization using the related generic rubrics. Or distribute the Learning Skills Self-Assessment Checklists and ask students to fill out the Teamwork and Organization sections. (See Assessment and Evaluation in the *Teacher Resource*.)

EXTENSION

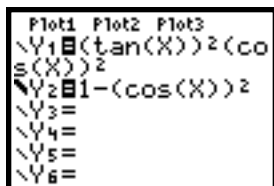
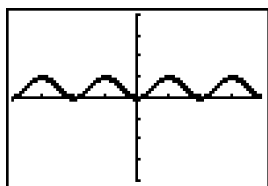
Have students work in groups to create and verify their own trigonometric equations and identities. Then groups could exchange the equations and identities and solve or prove them.

Guidelines for Solution

For each statement, students should use graphing technology to draw the graphs. Enter the left side of the statement into Y1 and the right side of the statement into Y2. The statement is an identity if both graphs are the same, and students then prove the identity. For those graphs that are not the same, students should find the intersection points to find the solutions.

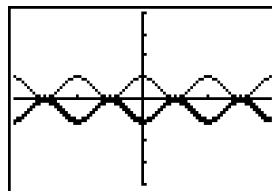
Part (a)

Graph each side of the statement. The graphs are the same, which means that the statement is an identity. Then prove the identity.

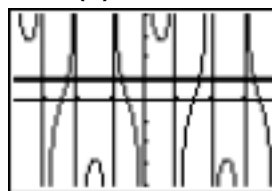
Part (b)

The statement is not an identity, but it is an equation with solutions that can be seen in the graph.



Students may decide that it is difficult to determine the intersection points by just looking at the graph. Then they will either zoom in to look at a specific region, or they will use the intersect command (CALCULATE menu) to find the intersection points. Solving algebraically yields $x = -\frac{\pi}{2}, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$.

Part (c)



The statement is not an identity, but it is an equation.

This equation cannot easily be solved algebraically. Instead, solve this equation by analyzing the graph and referring to the periodic nature of the trigonometric relations. Students should focus on using the graphing calculator and on the nature and domain of the variables. Students might use the intersect command (CALCULATE menu) and the Table features or TRACE points on the graph. For $-2\pi \leq x \leq 2\pi$, $x \approx 2.653, 5.201, -1.083, -3.63, -7.366$.

The last value is outside the stated domain, but it may help students see a pattern. Students may include the observation that differences between pairs of consecutive values alternate between 3.736 and 2.547.

Part (d)

The statement is an identity. Graph $y = 1$ for each side of the statement.

Students should also include the restriction on the left side, $\sin x \neq 0$, and prove the identity.

Performance Task #7 — The Tide Is High

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- TF2.03 represent, in applications, radian measure in exact form as an expression involving π (e.g., $\frac{\pi}{3}$, 2π) and in approximate form as a real number (e.g., 1.05)
- TF3.03 determine the amplitude, period, phase shift, domain, and range of sinusoidal functions whose equations are given in the form $y = a \sin(kx + d) + c$ or $y = a \cos(kx + d) + c$
- TF3.04 sketch the graphs of simple sinusoidal functions [e.g., $y = a \sin x$, $y = \cos kx$, $y = \sin(x + d)$, $y = a \cos kx + c$]
- TF3.05 write the equation of a sinusoidal function, given its graph and given its properties
- TF4.01 determine, through investigation, the periodic properties of various models (e.g., the table of values, the graph, the equation) of sinusoidal functions drawn from various applications
- TF4.02 explain the relationship between the properties of a sinusoidal function and the parameters of its equation, within the context of an application, and over a restricted domain
- TF4.04 pose and solve problems related to models of sinusoidal functions drawn from various applications, and communicate the solutions with clarity and justification, using appropriate mathematical forms
- OC3.03 communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions
- OC3.05 use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve)

Text Page 557

Time Required Functions and Relations or Functions: 60–120 min

Materials and Equipment

- graphing calculator
- large graph paper

Overview

This *Performance Task* allows students to demonstrate that they can

- demonstrate the relationships between the graphs and the equations of sinusoidal functions
- use radian measure in applications of trigonometric functions
- solve problems involving models of sinusoidal functions in various applications
- communicate mathematical reasoning clearly
- use graphing technology effectively
- solve multi-step problems

Description of Task

This *Performance Task* requires students to graph experimental data about the changing height of a tide, to find a trigonometric model that fits the data, and to make predictions based on their models.

Management Suggestions

Pairs or small groups could work on this *Performance Task*. Some group members could graph the data on large graph paper while another member of the group graphs the data using a graphing calculator. Students should recognize that the values for amplitude and period, for example, are estimates, because the data will not perfectly fit the sinusoidal model. Once each group graphs the data and explores possible equations, each student should then work individually to develop a model and to justify its components.

Alternatively, small groups could answer all parts of this *Performance Task* and present their solutions to the class.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*. Use the rubric not only to assess the written responses but also to assess by observing.

Assess the Learning Skill Teamwork by distributing the Teamwork Self-Reflection Sheet to students. (See Assessment and Evaluation in the *Teacher Resource*.)

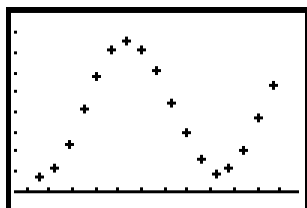
EXTENSION

Ask students to find other data about the changing height of a tide in Canada or elsewhere in the world and to determine whether the data fits the same model. Encourage students to consider the factors that would explain the differences between groups of data.

Guidelines for Solution

Part (a)

For L1, enter 13, 14, 15, and so on for 1 P.M. to 8 P.M.



Part (b)

High tide is about 7.6 m and low tide is about 0.7 m.

Part (c)

The amplitude is half the difference between high tide and low tide, so the value of a is about 3.45.

From the graph, the period appears to be about 7.5 units or 8 units. You may see in the WINDOW settings that the x -scale (Xscl) is about $1.57 \left(\frac{\pi}{2} \right)$. Therefore, the period could

be any time from 11.7 h to 12.6 h. A realistic period is about 12 h (or half a day), due to the

nature of tides. Also, if each unit is $\frac{\pi}{2}$ and

there are about 8 units, the period is 4π , which is twice the period for the base curve $y = \sin x$.

Part (d)

Using sinusoidal regression (STAT CALC menu) yields $y = 3.4 \sin [0.5(x + 5.3)] + 4.2$.

Solutions may include other models. In all cases, students should test their models using data values and justify the components by referring to period, amplitude, phase shift, and horizontal stretch or compression.

Part (e)

To determine the heights, extrapolate from the graph or substitute in the equation.

For 2:00 A.M. on August 15, substitute $x = 2$ in the equation and solve for y . The height of the tide is about 2.5 m. For 11:00 P.M. on August 15, substitute $x = 23$; the height is about 7.6 m. For 3:00 A.M. on August 16, substitute $x = 27$; the height is about 2.7 m.

Performance Task #8 — A Trigonometric Trip

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- TF1.01 determine the sine, cosine, and tangent of angles greater than 90° , using a suitable technique (e.g., related angles, the unit circle), and determine two angles that correspond to a given single trigonometric function value
- TF1.02 solve problems in two dimensions and three dimensions involving right triangles and oblique triangles, using the primary trigonometric ratios, the cosine law, and the sine law (including the ambiguous case)
- OC3.02 present problems and their solutions to a group, and answer questions about the problems and the solutions
- OC3.03 communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions

Text Page 557

Time Required Functions and Relations or Functions: 60–120 min

Materials and Equipment

- Students may need various materials, depending on their trip design.

Overview

This *Performance Task* allows students to demonstrate that they can

- solve problems involving the measures of sides and angles in acute, oblique, and right triangles
- communicate mathematical reasoning clearly
- use graphing technology effectively
- solve multi-step problems

Description of Task

This *Performance Task* requires students to create a trigonometric problem in three dimensions and its solution. The problem must involve finding an inaccessible height or distance in the surrounding area. It is suggested that students solve one another's problems by going outside of the classroom to take measurements.

Management Suggestions

Pairs could do this *Performance Task*. Once students have created problems and verified solutions, plan the “trigonometric walk.”

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*. Use the rubric to assess both the problems and their solutions as well as the solutions of other students' problems.

Assess the Learning Skills Teamwork and Initiative using the related generic rubrics. Or distribute the Learning Skills Self-Assessment Checklists and ask students to fill out the Teamwork and Initiative sections. (See Assessment and Evaluation in the *Teacher Resource*.)

Guidelines for Solution

Solutions will vary, but they should describe the following:

- finding an inaccessible height
- working in three dimensions
- a right triangle
- using the sine law or the cosine law

Performance Task #9 — Math Music

Specific Expectations

The following expectations are common to the Functions and Relations course and the Functions course.

- TF2.04 determine the exact values of the sine, cosine, and tangent of the special angles $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples less than or equal to 2π
- TF3.01 sketch the graphs of $y = \sin x$ and $y = \cos x$, and describe their periodic properties
- TF3.02 determine, through investigation, using graphing calculators or graphing software, the effect of simple transformations (e.g., translations, reflections, stretches) on the graphs and equations of $y = \sin x$ and $y = \cos x$
- TF3.04 sketch the graphs of simple sinusoidal functions [e.g., $y = a \sin x$, $y = \cos kx$, $y = \sin(x + d)$, $y = a \cos kx + c$]
- TF4.01 determine, through investigation, the periodic properties of various models (e.g., the table of values, the graph, the equation) of sinusoidal functions drawn from various applications
- TF4.02 explain the relationship between the properties of a sinusoidal function and the parameters of its equation, within the context of an application, and over a restricted domain
- TF4.04 pose and solve problems related to models of sinusoidal functions drawn from various applications, and communicate the solutions with clarity and justification, using appropriate mathematical forms
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions
- OC3.05 use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve)

Text Page 558

Time Required Functions and Relations or Functions: 70–120 min

Materials and Equipment

- graphing technology
- oscilloscope and synthesizer for Extension

Overview

This *Performance Task* allows students to demonstrate that they can

- demonstrate the relationships between the graphs and the equations of sinusoidal functions
- use radian measure in applications of trigonometric functions in applications of trigonometric functions
- solve problems involving models of sinusoidal functions in various applications
- communicate mathematical reasoning clearly
- use graphing technology effectively
- solve multi-step problems

Description of Task

This *Performance Task* relates trigonometric functions to music. Students extend their knowledge of simple transformations of trigonometric functions by adding functions together to simulate the waves produced on an oscilloscope by musical instruments.

Management Suggestions

Students could work in pairs on this *Performance Task*, but solutions should be individual.

This *Performance Task* could also be a good independent study project for students interested in music and technology. See also the *Research Extension*.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*. This rubric also has information for assessing the presentation (Extension).

Assess the Learning Skills Teamwork and Independence using the related generic rubrics. Or distribute the Learning Skills Self-Assessment Checklists and ask students to fill out the Teamwork and Independence sections. (See Assessment and Evaluation in the *Teacher Resource*.)

Research Extension

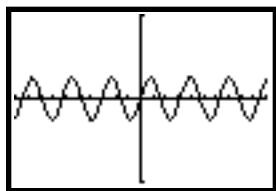
This *Performance Task* will be much more effective if a simple oscilloscope and keyboard are used to demonstrate the waves produced by musical instruments. The technology and/or music departments in your school may have the equipment and expertise to “hook up” a demonstration, which is relatively simple to do. In fact, there may be a student in the class who can get the equipment and do the hook-up.

Students can then design their own “instrumental sine curves” and test out which instruments produce their sine waves.

Guidelines for Solution

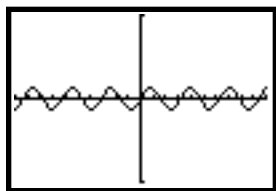
Part (a)

$$y = \sin(2x)$$

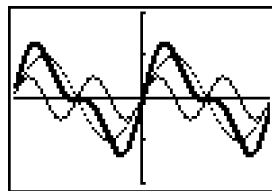
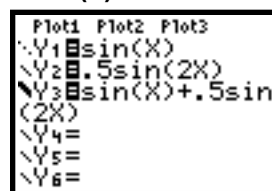


Part (b)

The amplitude is $\frac{1}{2}$.

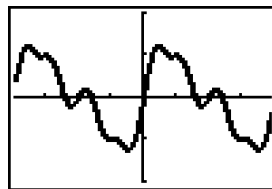


Part (c)



Part (d)

i. $y = \sin x + \frac{1}{2} \sin(2x) + \frac{1}{4} \sin(4x)$



ii.

$$y = \sin x + \frac{1}{3} \sin(3x) + \frac{1}{5} \sin(5x) + \frac{1}{7} \sin(7x)$$

