

Performance Task #1 — Orbiting the Earth

Specific Expectations

- LC2.01 identify the standard forms for the equations of parabolas, circles, ellipses, and hyperbolas having centres at $(0, 0)$ and at (h, k)
- LC3.01 describe the importance, within applications, of the focus of a parabola, an ellipse, or a hyperbola (e.g., all incoming rays parallel to the axis of a parabolic antenna are reflected through the focus; the planets move in elliptical orbits with the sun at one of the foci)
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions

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**Time Required Functions and Relations
or Functions:** 45–60 min

Overview

This *Performance Task* allows students to demonstrate that they can

- determine the equation of a conic in standard form or in general form
- sketch graphs of conics
- pose and solve problems about conics
- report the solution to a problem clearly, using appropriate mathematical forms

Description of Task

This *Performance Task* requires students to create an equation and a diagram for the elliptical orbit of a satellite.

Management Suggestions

Students could work individually on this *Performance Task*.

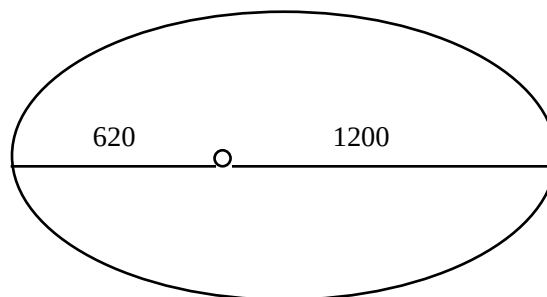
Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*.

Assess Learning Skills as students work on this *Performance Task*. Assess the ability of students to work independently and to organize their work. You might distribute the

Learning Skills Self-Assessment Checklists and ask students to fill out the Working Independently and the Organization sections. (See Assessment and Evaluation in the *Teacher Resource*.)

Guidelines for Solution



The Earth has a radius of 6.34×10^3 km, and the centre of the Earth is then the true focal point. So the satellite is 6960 km from the Earth's surface at the perigee and 7540 km from the apogee. The length of the major axis is the sum of these values, or 14 500 km.

Therefore, a is $14\,500 \div 2$, or 7250. c is $7250 - 620$, or 6630. Since $a^2 = b^2 + c^2$, the value of b is about 2933.5. Therefore, the equation of the ellipse is

$$\frac{x^2}{52\,562\,500} + \frac{y^2}{8\,605\,600} = 1$$

In students' solutions, the values of a and b may be reversed, because the orientation of the ellipse is not given.

Performance Task #2 — Conics and Foci

Specific Expectations

- LC2.01 identify the standard forms for the equations of parabolas, circles, ellipses, and hyperbolas having centres at $(0, 0)$ and at (h, k)
- LC2.03 determine the key features (e.g., the centre or the vertex, the focus or foci, the asymptotes, the lengths of the axes) of a conic whose equation is given in the form $ax^2 + by^2 + 2gx + 2fy + c = 0$, by hand in simple cases (e.g., $x^2 + 9y^2 - 6x + 36y - 36 = 0$)
- LC3.02 pose and solve problems drawn from various applications involving conics, and communicate the solutions with clarity and justification (Sample problem: A parabolic antenna is 320 m wide at a distance of 50 m above its vertex. Determine the distance above the vertex of the focus of the antenna)
- OC3.01 explain mathematical processes, methods of solution, and concepts clearly to others
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions

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Time Required Functions and Relations or Functions: 45–60 min

Materials and Equipment

- graphing calculator

Overview

This *Performance Task* allows students to demonstrate that they can

- investigate and determine the key features of conics
- determine the equation of a conic in standard form or in general form
- pose and solve problems about conics
- solve multi-step problems
- communicate and justify their reasoning

Description of Task

This *Performance Task* requires students to extend what they know about the key features of a conic to determine the eccentricity of the conic. Students are also required to make and confirm conjectures about the eccentricities of various types of conics.

Management Suggestions

Students could work in pairs or small groups on this *Performance Task*. Once they have completed the *Performance Task*, groups could compare results.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*.

Assess the Learning Skill Initiative as students work on this *Performance Task* using the related generic rubric or distribute the Initiative Self-Reflection Sheet to students. (See Assessment and Evaluation in the *Teacher Resource*.)

Guidelines for Solution

Students will choose various equations of ellipses and hyperbolas for determining their eccentricities, e . Once they have investigated various ellipses and hyperbolas, they should note that $e < 1$ for an ellipse and $e > 1$ for a hyperbola. Further investigation will show that $e = 1$ for a parabola.

Solutions should show the original conjecture, further investigation to verify the conjecture, investigation of other conics, and a summary of findings.

Performance Task #3 — What Are the Equations?

Specific Expectations

- LC2.01 identify the standard forms for the equations of parabolas, circles, ellipses, and hyperbolas having centres at $(0, 0)$ and at (h, k)
- LC2.04 sketch the graph of a conic whose equation is given in the form $ax^2 + by^2 + 2gx + 2fy + c = 0$
- LC3.03 solve problems involving the intersections of lines and conics
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions
- OC3.05 use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve)

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Time Required Functions and Relations or Functions: 50–70 min

Materials and Equipment

- graphing calculator or graphing software

Overview

This *Performance Task* allows students to demonstrate that they can

- investigate and determine the key features of conics
- determine the equation of a conic in standard form or in general form
- sketch graphs of conics
- use technology as an aid in problem solving
- solve multi-step problems
- communicate and justify their reasoning

Description of Task

This *Performance Task* requires students to create equations of conics and lines that will generate a given design. As well, students are required to determine point(s) of intersection between the conics and line(s).

Management Suggestions

Students could work individually or in pairs to create the equations, (a). They should use graphing software, if possible, to test their equations. You might remind students to record their thinking as they work so that they can report the process that they went through to create the equations, (c).

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*.

Assess the Learning Skills Independence and Initiative using the related generic rubrics. Or distribute the Learning Skills Self-Assessment Checklists and ask students to fill out the Independence and Initiative sections. (See Assessment and Evaluation in the *Teacher Resource*.)

Guidelines for Solution

Answers will vary. Solutions should include the following:

- a description of creating the equations, including how to find the vertices of the hyperbola and the centres of the circles
- the algebraic and graphical solutions to three equations and a discussion of the different methods
- the addition of another feature to the design and the appropriate equation for this feature
- one point of intersection between a line with a conic

Performance Task #4 — What Are the Solutions?

Specific Expectations

- LC1.05 determine equations for conics from their locus definitions, by hand for simple particular cases [e.g., determine the equation of the locus of points the sum of whose distances from $(-3, 0)$ and $(3, 0)$ is 10]
- LC2.01 identify the standard forms for the equations of parabolas, circles, ellipses, and hyperbolas having centres at $(0, 0)$ and at (h, k)
- LC2.03 determine the key features (e.g., the centre or the vertex, the focus or foci, the asymptotes, the lengths of the axes) of a conic whose equation is given in the form $ax^2 + by^2 + 2gx + 2fy + c = 0$, by hand in simple cases (e.g., $x^2 + 9y^2 - 6x + 36y - 36 = 0$)
- LC2.04 sketch the graph of a conic whose
- OC3.03 communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions

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Time Required Functions and Relations or Functions: 50–70 min

Materials and Equipment

- graph paper
- graphing technology
- *The Geometer's Sketchpad*

Overview

This *Performance Task* allows students to demonstrate that they can

- investigate and determine the key features of conics
- determine the equation of a conic in standard form or in general form
- sketch graphs of conics
- solve multi-step problems
- report the solution to a problem clearly, using appropriate mathematical forms

Description of Task

This *Performance Task* requires students to determine equations for various conics. These questions bring together many different concepts about the equations and key features of conics.

Management Suggestions

Students could work individually or in small groups on this *Performance Task*. If students work in small groups, ask each group to present the solution to one problem.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*.

Assess the Learning Skills Independence and Organization as students work on this *Performance Task* using the related generic rubrics. (See Assessment and Evaluation in the *Teacher Resource*.)

Guidelines for Solution

Each situation should be modelled both graphically and algebraically.

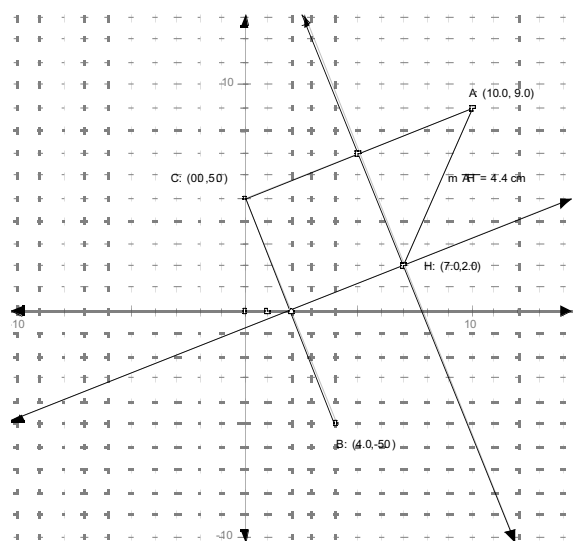
Part (a)

This problem could be solved algebraically or geometrically using *The Geometer's Sketchpad*. Use the diagram on the next page to visually locate the centre of the circle, $(7, 2)$.

In the diagram, $m \overline{AH} = 4.4$ cm means that the radius of the circle is 4.4 cm. Since this value does not relate to the units of the diagram, it is not very helpful. To find the exact value of the radius, solve algebraically by substituting one of the points in the distance formula.

The radius is then $\sqrt{58}$ and the equation of the circle is

$$(x - 7)^2 + (y - 2)^2 = 58.$$



Part (c)

Solutions will vary, but students should verify their solutions both algebraically and graphically.

Part (d)

Students will need to find the focal points, $(5, 0)$ and $(-5, 0)$, of both the hyperbola and the ellipse. They should include graphs of both conics and a full algebraic solution in their solutions.

Part (b)

The foci of this ellipse are $(3, 0)$ and $(-3, 0)$. The points on the curve where $x = 2$ are $\left(2, \pm \frac{\sqrt{336}}{5}\right)$.

Now use the distance formula to find the distance from each focus to the point $\left(2, \frac{\sqrt{336}}{5}\right)$. The

distance from $(3, 0)$ to $\left(2, \frac{\sqrt{336}}{5}\right)$ is $\frac{19}{5}$. The

distance from $(-3, 0)$ to $\left(2, \frac{\sqrt{336}}{5}\right)$ is $\frac{31}{5}$. Notice

that the sum of these values is 10, which is the value of $2a$.

Students should create graphs to support this solution.

Performance Task #5 — Blaise Pascal

Specific Expectations

- LC1.01 construct a geometric model (e.g., a diagram created by hand, a diagram created by using dynamic geometry software) to represent a described locus of points; determine the properties of the geometric model; and use the properties to interpret the locus (e.g., the locus of points equidistant from two fixed points is the right bisector of the line segment joining the two fixed points)
- LC1.02 explain the process used in constructing a geometric model of a described locus
- LC1.03 determine an equation to represent a described locus [e.g., determine the equation of the locus of points equidistant from $(-2, 7)$ and $(5, 4)$]
- LC1.04 construct geometric models to represent the locus definitions of the conics
- LC2.04 sketch the graph of a conic whose equation is given in the form $ax^2 + by^2 + 2gx + 2fy + c = 0$
- LC3.02 pose and solve problems drawn from various applications involving conics, and communicate the solutions with clarity and justification (Sample problem: A parabolic antenna is 320 m wide at a distance of 50 m above its vertex. Determine the distance above the vertex of the focus of the antenna)
- OC3.03 communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions

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Time Required Functions and Relations or Functions: 60–120 min

Materials and Equipment

- *The Geometer's Sketchpad*

Overview

This *Performance Task* allows students to demonstrate that they can

- construct and describe models to represent a described locus of points
- investigate and determine the key features of conics
- sketch graphs of conics
- pose and solve problems about conics
- use technology as an aid in problem solving
- solve multi-step problems
- communicate and justify their reasoning

Description of Task

This *Performance Task* requires students to explore several conics using *The Geometer's Sketchpad*. They must first pick points on a conic, draw lines connecting consecutive points, and then mark three points where these lines intersect. Next they must make a conjecture about the relationship between the three points and test their conjectures on other conics.

Management Suggestions

Students could work individually or in small groups on this *Task*.

Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*.

Assess the Learning Skills Teamwork and Initiative using the related generic rubrics. Or distribute the Learning Skills Self-Assessment Checklists and ask students to fill out the Teamwork and Initiative sections. (See Assessment and Evaluation in the *Teacher Resource*.)

Guidelines for Solution

Solutions will vary, because students will choose different conics to explore. Students should notice that the points of intersection of the line segments are collinear. They should choose properties of lines from previous work to verify that the points are truly collinear, for instance, points on the same line will have the same slopes.

Solutions should include *The Geometer's Sketchpad* sketches that show exploration and verification and an explanation of these processes.

Performance Task #6 — Constructing an Ellipse

Specific Expectations

- LC1.01 construct a geometric model (e.g., a diagram created by hand, a diagram created by using dynamic geometry software) to represent a described locus of points; determine the properties of the geometric model; and use the properties to interpret the locus (e.g., the locus of points equidistant from two fixed points is the right bisector of the line segment joining the two fixed points)
- LC1.02 explain the process used in constructing a geometric model of a described locus
- LC1.04 construct geometric models to represent the locus definitions of the conics
- LC1.05 determine equations for conics from their locus definitions, by hand for simple particular cases [e.g., determine the equation of the locus of points the sum of whose distances from $(-3, 0)$ and $(3, 0)$ is 10]
- LC2.01 identify the standard forms for the equations of parabolas, circles, ellipses, and hyperbolas having centres at $(0, 0)$ and at (h, k)
- LC3.02 pose and solve problems drawn from various applications involving conics, and communicate the solutions with clarity and justification (Sample problem: A parabolic antenna is 320 m wide at a distance of 50 m above its vertex. Determine the distance above the vertex of the focus of the antenna)
- OC3.03 communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms
- OC3.04 demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions

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**Time Required Functions and Relations
or Functions:** 60–120 min

Materials and Equipment

- *The Geometer's Sketchpad*

Overview

This *Performance Task* allows students to demonstrate that they can

- construct and describe models to represent a described locus of points
- investigate and determine the key features of conics
- use technology as an aid in problem solving
- solve multi-step problems
- report the solution to a problem clearly, using appropriate mathematical forms

Description of Task

This *Performance Task* requires students to explore a different method of constructing an ellipse. They should use the properties and locus definition of an ellipse to prove that the new figure is an ellipse.

Management Suggestions

Students could work individually or in pairs on this *Performance Task*. Invite students to present their solutions.

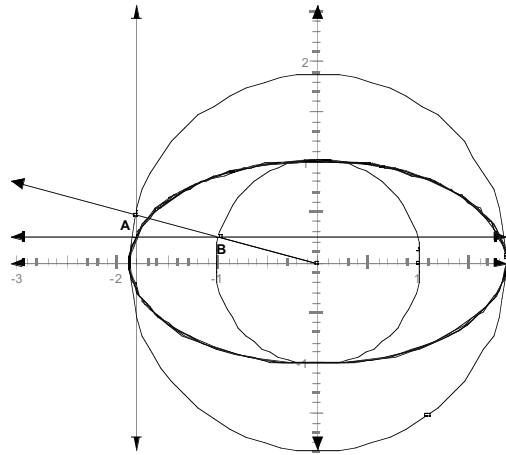
Assessment Suggestions

Assess Achievement using the rubric for this *Performance Task*. Enhance assessment of students' problem-solving skills by observing students as they work. Assess Communication skills by evaluating written solutions or by asking questions as students work.

Assess the Learning Skill Organization using the related generic rubric. (See Assessment and Evaluation in the *Teacher Resource*.)

Guidelines for Solution

Encourage students to explore the problem using *The Geometer's Sketchpad*. They might experiment with circles of different sizes. Once they construct the ellipse, they will need to plan how to prove that the constructed figure is an ellipse.



If r_1 is the radius of the smaller circle and r_2 is the radius of the larger circle, the equation of the ellipse is $\frac{x^2}{r_2^2} + \frac{y^2}{r_1^2} = 1$.

Students may choose points on the ellipse to verify this model or work with the locus definition of an ellipse.