
Check Your Understanding

1. *Must a sequence have a recognizable pattern? Explain.*

A sequence does not have to have a recognizable pattern, but the terms in a sequence must follow an order.

2. *Can any sequence be represented by a formula or by a general term?*

No. A formula or a general term can represent only a sequence with a recognizable pattern.

3. *Explain why the set of natural numbers is used in the general term of a sequence.*

Because the terms of the sequence are ordered, they can be counted using natural numbers. The general term expresses each term with respect to its position in the sequence, that is, with respect to the natural number with which the term is paired.

4. *How do you decide whether a list of numbers represents an arithmetic sequence or a geometric sequence? Use examples in your explanation.*

First find the first differences and the ratios of a few pairs of successive terms. If the differences are constant, the sequence is arithmetic, for example, for the sequence 3, 6, 9, 12, 15, ... , $6 - 3 = 3$ and $15 - 12 = 3$. If the ratios are equal, the sequence is geometric, for example, for the sequence 5, 10, 20, 40, 80, ... , $\frac{10}{5} = 2$ and $\frac{40}{20} = 2$.

5. *In any arithmetic sequence,*

- (a) *what is true about the rate of change between consecutive terms of the sequence?*

The rate of change is constant. The differences between successive terms is always the same.

- (b) *what type of relationship exists between n and t_n ?*

The value of the term, t_n , is a function of the term number, n . Find the value using $t_n = a + (n - 1)d$, where n is the term number.

6. *In any geometric sequence,*

- (a) *what is true about the rate of change between consecutive terms of the sequence?*

The rate of change is not constant, because the sequence exhibits exponential growth.

- (b) *what type of relationship exists between n and t_n ?*

The term number less 1 is an exponent in the expression for the value of a term in a geometric sequence. $t_n = ar^{n-1}$

7. *Write the recursive formula for*

- (a) any arithmetic sequence

$$t_n = t_{n-1} + d$$

- (b) any geometric sequence

$$t_n = (t_{n-1})(r)$$

8. Explain why an investment of \$1000 will be worth more in ten years if the interest is compounded annually, but is not simple.

When interest is compounded, the value of the investment grows in each compounding period and interest is calculated on the amount at the beginning of the conversion period that includes interest earned during the previous conversion period. In each period, the amount of interest as well as the amount invested increases.

9. What is the difference between the amount of an investment and the present value of an investment? Use an example in your explanation.

The amount, or future value, of an investment includes interest accumulated over time, whereas the present value does not include interest.

For example, if \$1000 is deposited today, the present value is \$1000, but the *amount* in three years, earning interest of 8%/a, compounded quarterly, would be $A = 1000(1.02)^{12}$ or \$1268.24.

Alternatively, to accumulate \$5000 in six years in an account paying 8%/a, compounded semiannually, the present value, that is, the deposit required today, is $5000 = P(1.04)^{12}$.

$$P = \frac{5000}{(1.04)^{12}} \text{ or } \$3122.99$$

10. In your own words, define the *n*th root of a number *a*. Express the *n*th root of *a* using exponents.

If *n* identical factors are multiplied together to produce *a*, then one of those factors is the *n*th root of *a*.

The *n*th root of *a* can be expressed as $a^{\frac{1}{n}}$.

11. Without using a calculator, evaluate $25^{-\frac{3}{2}}$ using two different methods. Verify both answers by using a calculator.

Method 1:

$$\begin{aligned} 25^{-\frac{3}{2}} &= \left(25^{\frac{1}{2}}\right)^{-3} \\ &= 5^{-3} \\ &= \left(\frac{1}{5}\right)^3 \\ &= \frac{1}{125} \end{aligned}$$

Method 2:

$$\begin{aligned} 25^{-\frac{3}{2}} &= \frac{1}{25^{\frac{3}{2}}} \\ &= \frac{1}{(\sqrt{25})^3} \\ &= \frac{1}{5^3} \\ &= \frac{1}{125} \end{aligned}$$

Checking with a calculator shows that both answers are correct and equal 0.008.

12. List and describe the steps for simplifying the expression $\left(\frac{1}{c}\right)^{n+m} (c^3)^m$.

Express each bracket using the common base c . Remove the brackets by multiplying the interior exponents by the exterior exponents. Multiply two terms with a common base by retaining the base and adding the exponents..

$$\begin{aligned}\left(\frac{1}{c}\right)^{n+m} (c^3)^m &= c^{-n-m} c^{3m} \\ &= c^{2m-n}\end{aligned}$$

13. How can you decide whether an exponential equation should be solved algebraically or by graphing?

An algebraic solution is possible if all parts of the equation can be expressed with a common base. Otherwise, solve by graphing.

14. Solve $6(3^{2x-1}) = 486$ algebraically. Verify your answer by solving this equation using graphing technology.

$$6(3^{2x-1}) = 486$$

$$3^{2x-1} = 81$$

$$3^{2x-1} = 3^4$$

$$\therefore 2x - 1 = 4$$

$$2x = 5$$

$$x = 2.5$$

