
The Chapter Problem—Controlling Non-Native Plant Populations Answers

By the end of this chapter, students should be able to provide comprehensive, well-explained solutions to the problem. Students should also be able to develop several terms of a sequence and look for a pattern that leads to a general term for the sequence. Some students might consider creating an appropriate spreadsheet to generate data.

Section 1.1

CP1. *How many of the original 100 knapweed seeds will germinate?*

4% or 4 of the original 100 seeds will germinate in the first year.

CP2. *How many knapweed seeds will be added to the seedbank?*

The remaining seeds that do not germinate will be added to the seedbank, so 96% or 96 of the 100 seeds will be added in the first year.

CP3. *Of the seeds that do germinate, how many will grow to maturity in the first year?*

Twenty-five percent of the germinated seeds will grow to maturity, so 25% of 4 is 1 and one new plant will grow to maturity in the first year.

CP4. *How many new seeds will the mature plants add to the seedbank?*

The one new plant will produce 1000 new seeds.

CP5. *Use your answers for questions 1 to 4 to complete the table.*

Year	Number of Seeds in the Seedbank	Number of Seeds that Germinate	Number of Remaining Seeds	Total Number of Plants	Number of New Seeds Produced
0	100	4	96	1	1 000
1	1096	44	1 052	12	11 960
2	13 010	520	12 490	142	142 100
3	154 600	6 183	148 400	1 688	1 688 000
4	1 825 000	73 050	1 753 000	19 940	19 940 000
5	21 370 000	854 800	20 520 000	233 600	233 600 000

(Answers are rounded.)

CP6. *Describe any pattern in the Total Number of Plants column.*

Students may be surprised by the increasingly rapid growth in the numbers. They should recognize that the growth is nonlinear, and they may be able to recognize that each term is an approximate multiple of the previous term. The value of this column is always about $\frac{1}{4}$ of the value in column 3 (Number of Seeds that Germinate) of the same row.

CP7. Describe any pattern in the Number of Seeds in the Seedbank column.

The growth is nonlinear. Some students may use the word *exponential* or identify a multiplication factor between successive terms. The value in each row of this column is the sum of the corresponding value of column 4 and column 6 of the previous row.

Students may choose to use a spreadsheet. Here is the spreadsheet.

0	100	4	96	1	1000
1	1096	43.84	1052.16	10.96	11 960
2	130 12.16	520.4864	12 491.67	142.081 6	142 081.6
3	154 573.3	6182.931	148 390.3	1687.814	1 687 814
4	1 836 205	73 448.19	1 762 756	20 049.86	20 049 861
5	21 812 618	872 504.7	20 940 113	238 176	2.38E+08

Section 1.3

CP8. Recall that the knapweed plant lives for five years and that the seeds of the plant are viable in soil for eight years. Use this information to extend the table on page 25 to include years 6 to 10.

Year	Number of Seeds in the Seedbank	Number of Seeds that Germinate	Number of Remaining Seeds	Total Number of Plants	Number of New Seeds Produced
6	2.59E+	10 364 646	2.49E+08	2 829 338	2.83E+09
7	3.08E+	1.23E+08	2.95E+09	33 610 228	3.36E+10
8	3.66E+	1.46E+09	3.51E+10	3.99E+08	3.99E+11
9	4.34E+	1.74E+10	4.17E+11	4.74E+09	4.74E+12
10	5.16E+	2.06E+11	4.95E+12	5.63E+10	5.63E+13

CP9. Write a recursive formula for the sequence of the total number of plants in each year.

For the first five years, in year n , the total number of plants is $t_n = t_{n-1} + 0.25(0.04)$ (number of seeds in seedbank in year n)

CP10. Write a recursive formula for the sequence of the number of seeds in the seedbank each year.

The number of seeds in seedbank in year n is

$$n = 0.96 [\text{number of seeds in year } (n - 1)] + 1000 [\text{number of plants in year } (n - 1)]$$

Section 1.6

CP11. Is the plant sequence you found in CP9 arithmetic? Explain.

No. There is no identifiable common difference.

CP12. Is the seed sequence you found in CP10 arithmetic? Explain.

No. There is no identifiable common difference.

Section 1.7

CP13. Are either of the plant or seed sequences you found in CP9 and CP10 geometric? Explain.

It appears that there is a common factor of about 11.85 linking successive terms in each sequence, so the sequences are geometric.

CP14. Find the general term for the plant sequence.

In the Total Number of Plants column, there appears to be a factor of about 11.8 or 11.9 linking successive terms, so the entries in the column are part of a geometric sequence with $t_n = 11.85^n$, where n is the year n .

CP15. Find the general term for the seed sequence.

In the Number of Seeds in the Seedbank column, a factor of 11.85 again links successive terms, so $t_n = 1096(11.85)^{n-1}$.

Section 1.9

CP16. Use your model to predict the number of plants present after 5 years.

$$\begin{aligned}t_n &= 11.85^n \\t_5 &= 11.85^5 \\&= 233\,663\end{aligned}$$

CP17. Use your model to predict the number of seeds present after 2 years 3 months.

Two years 3 months is 2.25 years, so

$$\begin{aligned}t_n &= 1096(11.85)^{n-1} \\t_{2.25} &= 1096(11.85)^{1.25} \\&= 24\,097\end{aligned}$$

CP18. What factors make question CP17 unreasonable for this situation?

Because the seeds germinate once a year, 2.25 years is not realistic. All of the relations in the model rely on annual data.

Section 1.11

CP19. Use your model to predict the number of plants and seeds that exist after ten years.

After ten years, the expected number of plants is $t_{10} = 11.85^{10}$, or about 5.5×10^{10} plants. After ten years, the expected number of seeds is $t_{10} = 1096(11.85)^9$, or about 5.1×10^{12} seeds.

CP20. How do the values in CP19 compare with those in the table? Explain the similarities or differences.

For the plants, the values are different. During calculations, it is reasonable to round because the original data are estimates, but rounding will cause discrepancies between the numbers.

CP21. Use your model, along with graphing technology, to determine when the number of plants exceeds 500 trillion.

Use graphing technology to find the zero of the corresponding function

$y = 11.85^x - 5 \times 10^{12}$. The number of plants will exceed five hundred trillion after about 11.8 years.

CP22. If the site will support 105 mature knapweed plants per square metre, how many hectares would be affected after ten years? (1 ha = 10 000 m²)

The anticipated number of plants after 10 years is 5.5×10^{10} . The number of knapweed plants supported by 1 ha is $105(10\,000) = 1.05 \times 10^6$. The area affected after ten years is

$$\frac{5.5 \times 10^{10}}{1.05 \times 10^6} = 5.23 \times 10^4 \text{ ha.}$$