
Challenge 1: Solution Guide

MANAGEMENT SUGGESTION

Challenge 1 invites a numeric and concrete approach, while Challenge 2 invites an algebraic or general approach. If you use only one challenge, select Challenge 1 for the Functions course and Challenge 2 for the Functions and Relations course.

The sequence is 1, 2, 4, 8, ... The general term is $t_n = 2^{n-1}$.

1. Because students do not meet the formula for finding the sum of the terms in a sequence (a series) until the next chapter, they may generate the number of wheat grains on each square for the first ten squares and then add the terms. To calculate the total mass, multiply.

$$1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 = 1023$$

The total number of wheat grains is 1023. The mass of one grain is 0.0648 g.

$$1023(0.0648) \doteq 66.3$$

The mass of 1023 wheat grains is about 66.3 g.

Here is the solution using the formula for a geometric series, which is introduced in Chapter 2. To calculate the total number of wheat grains on the first ten squares, find S_{10} .

$$a = 1, r = 2, \text{ and } n = 10$$

$$\begin{aligned} S_n &= \frac{a(r^n - 1)}{r - 1} \\ S_{10} &= \frac{1(2^{10} - 1)}{2 - 1} \\ &= 1023 \end{aligned}$$

Calculate the mass as above.

2. The yearly world production in grams is $1\,000\,000(582\,500\,000) = 5.825 \times 10^{14}$ g. The total number of wheat grains on the chessboard is

$$\begin{aligned} S_{64} &= \frac{1(2^{64} - 1)}{2 - 1} \\ &\doteq 1.84 \times 10^{19} \end{aligned}$$

The mass of this number of wheat grains is $1.84 \times 10^{19}(0.0648) \doteq 1.192 \times 10^{18}$ g.

Calculate the number of years that it would take to fill the chessboard.

$$\begin{aligned} \frac{\text{mass of wheat grains on chessboard}}{\text{mass of yearly production}} &= \frac{1.192 \times 10^{18}}{5.825 \times 10^{14}} \\ &\doteq 2050 \end{aligned}$$

It would take 2050 years to fill the chessboard completely.

3. The selling price for wheat is often quoted per bushel. Research shows that the current price of wheat is \$150/t. The mass of number of wheat grains on the completely filled chessboard is 1.84×10^{19} (0.0648) $\div 1.192 \times 10^{18}$ g. One metric tonne equals 1×10^6 g.

$$\frac{1.192 \times 10^{18}}{1 \times 10^6} = 1.192 \times 10^{12}$$

There are 1.192×10^{12} t of wheat grains for the reward. Calculate the cash value of the inventor's reward.

$$1.192 \times 10^{12} (150) = 179\,250\,000\,000\,000$$

The cash value of the inventor's reward is \$179 250 000 000 000.

CHAPTER 1: RUBRIC FOR CHALLENGE 1

Assessment Category	Scores and Criteria			
	Passable level of achievement – below provincial standard LEVEL 1	Moderate level of achievement – below, but approaching provincial standard LEVEL 2	High level of achievement – at provincial standard LEVEL 3	Very high to outstanding level of achievement – above provincial standard LEVEL 4
Knowledge and Understanding				
Demonstration of understanding of a concept	Attempted solution demonstrates limited understanding of geometric sequences.	Solution demonstrates some evidence of understanding of geometric sequences.	Solution demonstrates sufficient evidence to show understanding of geometric sequences.	Detailed solution demonstrates thorough understanding of geometric sequences.
Thinking, Inquiry, Problem Solving				
Selection and sequencing of a variety of tools and strategies to solve a problem	The student selects and uses a few appropriate tools to make a limited attempt to solve the problem.	The student selects some appropriate tools and uses these to solve part of the problem.	The student chooses appropriate tools and uses these to solve most of the problem.	The student chooses appropriate tools and incorporates these in the execution of the plan to solve problem and verify solution.
Making conclusions and inferences	Conclusions are somewhat connected with the student's solution but are not supported.	Conclusions are consistent with the student's solution but are not fully supported.	Conclusions are supported by evidence and aspects of the problem are integrated with the conclusions.	Conclusions are supported and convincing. Conclusions integrate all aspects of the problem.
Communication				
Correct use of mathematical symbols, labels, and conventions	The student uses few mathematical conventions.	The student correctly uses some mathematical conventions.	The student correctly uses most mathematical conventions.	The student correctly uses mathematical conventions.
Degree of clarity in explanations and justifications	Explanations and justifications lack clarity.	Explanations and justifications are understandable but lack clarity.	Explanations and justifications are clear.	Explanations and justifications are clear and thorough.
Application				
Use of a selected tool or strategy	Selected tool or strategy is used with some inaccuracies.	Selected tool or strategy is used, but solution may contain minor errors.	Selected tool or strategy is used correctly.	Selected tool or strategy is used correctly and is verified or supported.

Challenge 2: Solution Guide

Consider revisiting this challenge after students have learned the formula for the sum of a geometric sequence (series) in chapter 2.

The original length of each side of the square is x units. The sequence representing the shaded area at each stage is then

$$\frac{1}{4}x^2, \frac{1}{4}\left(\frac{1}{4}x^2\right) + \frac{1}{4}x^2, \frac{1}{4}\left[\frac{1}{4}\left(\frac{1}{4}x^2\right)\right] + \frac{1}{4}\left(\frac{1}{4}x^2\right) + \frac{1}{4}x^2, \dots$$

To simplify the expressions, let $a = \frac{1}{4}x^2$.

Then the sequence is $a, \frac{1}{4}a + a, \left(\frac{1}{4}\right)^2 a + \frac{1}{4}a + a, \dots$

The general term is $t_n = a + \frac{1}{4}a + \left(\frac{1}{4}\right)^2 a + \dots + \left(\frac{1}{4}\right)^{n-1} a$.

Consider the shaded area after the fourth stage.

$$\begin{aligned} t_4 &= \left(\frac{1}{4}\right)^3 a + \left(\frac{1}{4}\right)^2 a + \frac{1}{4}a + a \\ &= \frac{a}{64} + \frac{a}{16} + \frac{a}{4} + \frac{a}{1} \\ &= \frac{a + 4a + 16a + 64a}{64} \\ &= \frac{85a}{64} \end{aligned} \quad \text{But } a = \frac{1}{4}x^2.$$
$$= \frac{85}{256}x^2$$

After four stages of shading, $\frac{85}{256}$ or about 33.2% of the figure is shaded.

Then consider the shaded area after six stages.

$$t_6 = a + \frac{1}{4}a + \left(\frac{1}{4}\right)^2 a + \left(\frac{1}{4}\right)^3 a + \left(\frac{1}{4}\right)^4 a + \left(\frac{1}{4}\right)^5 a$$

This term is the sum of a sequence with initial term a and $r = \frac{1}{4}$.

So

$$t_6 = \frac{a \left[\left(\frac{1}{4} \right)^5 - 1 \right]}{\frac{1}{4} - 1}$$

$$= \frac{a \left(\frac{1 - 4^5}{4^5} \right)}{-\frac{3}{4}}$$

$$= \frac{a(-1023)}{-3(256)}$$

$$= \frac{341}{256} a$$

But $a = \frac{1}{4}x^2$, so

$$t_6 = \frac{341 \left(\frac{1}{4}x^2 \right)}{256}$$

$$= \frac{341}{1024} x^2$$

Therefore, $\frac{341}{1024}$ or 33.3% of the original figure is shaded after six stages of shading. Students might predict the fraction shaded after many repetitions.