
Chapter 1 Test: Solutions

1. (a) The sequence is arithmetic with first term 11 and common difference -4 . The general term is

$$\begin{aligned}t_n &= 11 + (n - 1)(-4) \\&= 15 - 4n\end{aligned}$$

- (b) The sequence is geometric with first term 5 and common ratio 2. The general term is $t_n = 5(2)^{n-1}$.

- (c) The sequence is neither arithmetic nor geometric. The general term is $t_n = (2n - 1)^2$.

2. (a) The general term is

$$\begin{aligned}t_n &= 11 + (n - 1)(-4) \\&= 15 - 4n\end{aligned}$$

The recursive formula is $t_n = t_{n-1} - 4$, $t_1 = 11$

- (b) The general term is $t_n = 5(2)^{n-1}$.

The recursive formula is $t_n = 2t_{n-1}$, $t_1 = 5$.

- (c) The general term is $t_n = (2n - 1)^2$.

The recursive formula is $t_n = \left(\sqrt{t_{n-1}} + 2\right)^2$, $t_1 = 1$.

3. (a) $A = 3000(1.04)^5$
 $= 3649.96$

Possible situation: \$3000 invested at 4%/a, compounded annually, is worth \$3649.96 at the end of five years.

- (b) $8 = 256(0.5)^t$

$$\frac{1}{32} = \left(\frac{1}{2}\right)^t$$

$$2^{-5} = 2^{-t}$$

$$5 = t$$

Possible situation: It takes five half-life periods for 256 g of radioactive material to decay to 8 g.

- (c) $5500 = P\left(1 + \frac{0.03}{12}\right)^{24}$
 $= 5180.09$

Possible situation: \$5180.09 must be deposited now to produce \$5500 in two years with interest at 3%/a, compounded monthly.

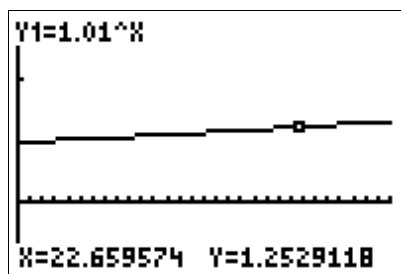
4. (a) $A = 8000 \left(1 + \frac{0.04}{4} \right)^{12}$
 $= 9014.60$

After three years, the value of the investment will be \$9014.60.

- (b) Let t represent the number of quarterly conversion periods.

$$10000 = 8000(1.01)^t$$

To solve for t , use graphing technology and interpolate with TRACE.



It will take about 23 quarterly conversion periods, that is, in the sixth year, for the value of the investment to surpass \$10 000.

It is also possible to rewrite the equation as $y = 8000(1.01)^x - 10\,000$ and use technology to find the zeros of the resulting graph.

5. (a) Choose several pairs of successive terms. If the first differences are identical, the sequence is arithmetic. Divide each pair. If the ratios are identical, the sequence is geometric.
- (b) The graph of an arithmetic sequence is a set of points that suggests a straight line and that shows a linear relation. The graph of a geometric sequence is a set of points suggesting a curve, or an exponential relation.

6. (a) The growth rate is 1.2 and the sequence is 225, 270, 324, ...

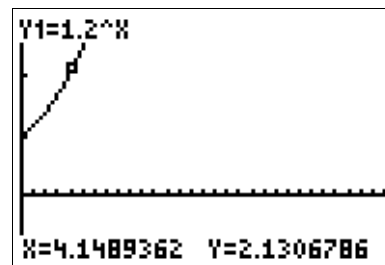
Let n represent the year. Then
 $t_n = 225(1.2)^{n-1995}$.

$$\begin{aligned} t_{2005} &= 225(1.2)^{2005-1995} \\ &= 225(1.2)^{10} \\ &\doteq 1393 \end{aligned}$$

If the growth continues at the same rate, there will be 1393 deer in 2005.

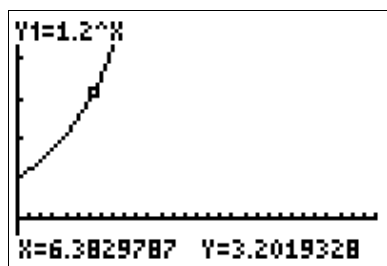
(b) $500 = 225(1.2)^{n-1995}$
 $2.222\,222\,222 \dots = 1.2^{n-1995}$

To solve for t , use graphing technology and interpolate with the TRACE feature.



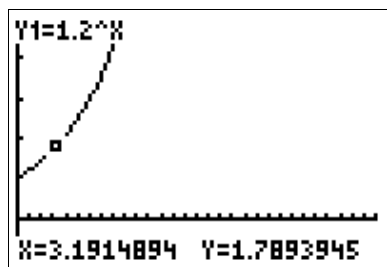
The herd will exceed 500 after just over four years, that is, when $n - 1995 = 4$, or in 1999.

$$\begin{aligned} \text{(c)} \quad 700 &= 225(1.2)^{n-1995} \\ n-1995 &= 6.4 \\ n &\doteq 2001 \end{aligned}$$



The first cull should occur in 2001.

Starting with 2001, let a be the number of years that passes before the next cull.



$$\begin{aligned} 700 &= 400(1.2)^a \\ 1.75 &= 1.2^a \\ a &= 3 \end{aligned}$$

The second cull will occur three years later in 2004.

$$\begin{aligned} 7. \text{ (a)} \quad \frac{(x^3y^4)^2}{x^5y^{-1}} &= \frac{x^6y^8}{x^5y^{-1}} \\ &= xy^9 \end{aligned}$$

$$(b) \quad (x^8y^{12})^{\frac{2}{3}} = x^{\frac{16}{3}}y^8$$

$$\begin{aligned} \text{(c)} \quad \frac{(x^{3a+1})(x^{a-3})}{\sqrt[4]{x^8}} &= \frac{x^{4a-2}}{x^2} \\ &= x^{4a-4} \end{aligned}$$

$$\begin{aligned} 8. \text{ (a)} \quad 4^{2-x} &= 8^{x+1} \\ (2^2)^{2-x} &= (2^3)^{x+1} \\ 2^{4-2x} &= 2^{3x+3} \\ 4-2x &= 3x+3 \\ 5x &= 1 \\ x &= \frac{1}{5} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 3^{2x} - 4(3^x) + 3 &= 0 \quad \text{Let } 3^x = a. \\ a^2 - 4a + 3 &= 0 \\ (a-3)(a-1) &= 0 \\ a = 3 \text{ and } a = 1 \\ \therefore 3^x = 3 \text{ and } 3^x = 1 \\ x = 1 \text{ and } x = 0 \end{aligned}$$