
Check Your Understanding

1. *What is the difference between a sequence and a series?*

A series is the sum of the terms of a sequence.

2. *How can you decide whether a sum of terms represents an arithmetic series or a geometric series? Use examples in your explanation.*

You can tell whether a series is arithmetic or geometric series in exactly the same way that you can tell if a sequence is arithmetic or geometric. The series is arithmetic if the difference between pairs of consecutive terms is common. The series is geometric if the ratio between pairs of consecutive terms is common.

For the series $3 + 7 + 11 + 15 + \dots$, $t_4 - t_3 = 4$ and $t_3 - t_2 = 4$.

The sequence is arithmetic with $d = 4$.

For the series $3 + 12 + 48 + 192 + \dots$, $t_4 \div t_3 = 4$ and $t_3 \div t_2 = 4$.

The series is geometric with $r = 4$.

3. *Use Gauss's method to prove that the sum of all natural numbers from 1 to 60 is 1830.*

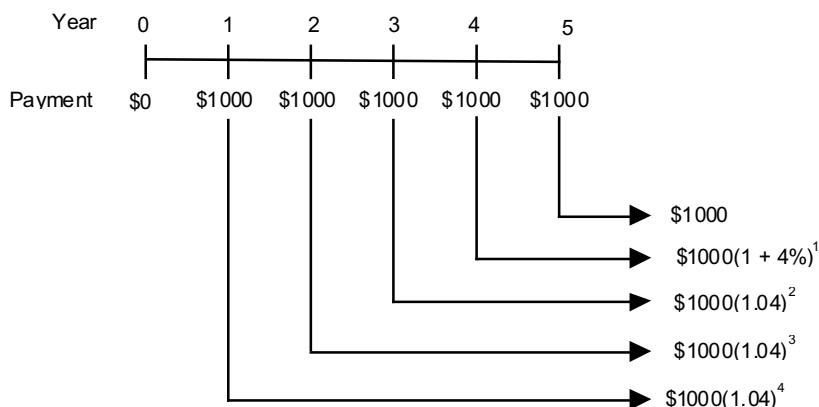
$$\begin{array}{rcllclclcl} S_{60} & = & 1 & + & 2 & + & 3 & + & \dots & + & 58 & + & 59 & + & 60 \\ S_{60} & = & 60 & + & 59 & + & 58 & + & \dots & + & 3 & + & 2 & + & 1 \\ \hline 2S_{60} & = & 61 & + & 61 & + & 61 & + & \dots & + & 61 & + & 61 & + & 61 \\ & & 2S_{60} & = & 60(61) & & & & & & & & & & \\ & & & & = & 3660 & & & & & & & & & \\ & & S_{60} & = & 3660 \div 2 & & & & & & & & & & \\ & & & & = & 1830 & & & & & & & & & \end{array}$$

4. *Describe the steps and calculations for amortizing a \$2000 loan. The monthly payment is \$320. The interest rate is 6%/a, compounded monthly.*

Calculate the interest on the outstanding balance. To calculate the principal paid, subtract this interest from the monthly payment. Subtract the principal paid from the outstanding balance. Repeat this process until the outstanding balance is 0 or close to 0. If the balance is close to 0, adjust the last payment. In this case, the initial outstanding balance is \$2000, the payment is \$320, and the interest is $6\% \div 12 = 0.005$. The amortization of the \$2000 loan will take six payments of \$320 each, plus a final payment of \$117.17. If using the TVM Solver, input $N = 0$, along with the other information. To get the number of payments, return to N and press **CLEAR** **ALPHA** **ENTER**.

5. At the end of each year, Charlotte deposits \$1000 in a savings account for five years. The interest rate is 4%/a, compounded annually. Explain and show how the amount of the sum of the deposits can be found by using the equation for the sum of a geometric series.

Draw a time line.



Consider the geometric series created by the sum of the accumulated amounts of the deposits. Substitute $a = 1000$, $r = 1.04$, and $n = 5$ to find S_n .

$$\begin{aligned}
 S_n &= \frac{a(r^n - 1)}{r - 1} \\
 S_5 &= \frac{1000(1.04^5 - 1)}{1.04 - 1} \\
 &\doteq \frac{1000(1.216652902 - 1)}{0.04} \\
 &= 5416.32
 \end{aligned}$$

Charlotte's five deposits of \$1000 accumulate to \$5416.32.

6. Each of the following represents an annuity. Write a problem that would describe each annuity.

(a) $400 + 400(1.06) + 400(1.06)^2 + \dots + 400(1.06)^{22} + 400(1.06)^{23}$

There are 24 payments of \$400 each. The positive exponents imply that each term may be a future value of a deposit. The interest rate is 6% per interest conversion period. A reasonable conversion period would be annual or semiannual. A quarterly or more frequent conversion period would give an unreasonably high annual interest rate. Answers will vary. Here is an example: Amad deposits \$400 every six months in an account that pays 12%/a, compounded semiannually. What is the total amount of his deposits after 12 years?

(b) $100(1.02)^{-48} + 100(1.02)^{-47} + 100(1.02)^{-46} + \dots + 100(1.02)^{-2} + 100(1.02)^{-1}$

There are 48 payments of \$100 each. The negative exponents imply each term may be a present value of a payment. The interest rate is 2% per conversion period. A reasonable interest conversion period would be quarterly. Answers will vary. Here is an example: How much must Marie deposit now so that she will receive \$100 every three months for 12 years? The interest rate is 8%/a, compounded quarterly.

7. Find the sum of each series in question 6 using the formula for the sum of a geometric series.

(a) $S_n = \frac{a(r^n - 1)}{r - 1}$, and $a = 400$, $r = 1.06$, and $n = 24$

$$\begin{aligned} S_{24} &= \frac{400[(1.06)^{24} - 1]}{1.06 - 1} \\ &\doteq \frac{400(4.048\,934\,641 - 1)}{0.06} \\ &= 20\,326.23 \end{aligned}$$

The value of the annuity is \$20 326.23.

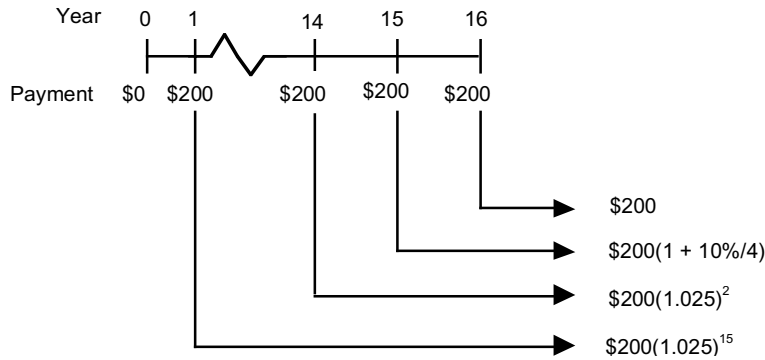
(b) $S_n = \frac{a(r^n - 1)}{r - 1}$, and $a = 100(1.02)^{-48}$, $r = 1.02$, and $n = 48$

$$\begin{aligned} S_{48} &= \frac{100(1.02)^{-48}[(1.02)^{48} - 1]}{1.02 - 1} \\ &\doteq \frac{100(0.386\,537\,608\,6)(2.587\,070\,385 - 1)}{0.02} \\ &= 3067.31 \end{aligned}$$

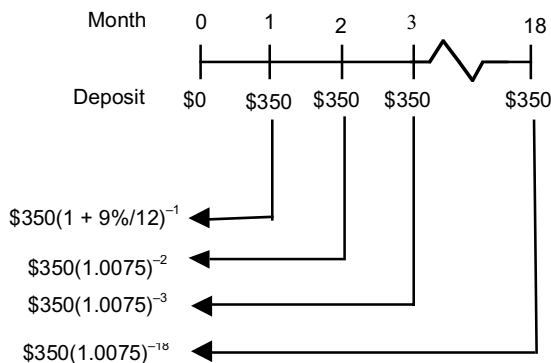
The value of the annuity is \$3067.31.

8. Draw a time line for each annuity.

(a) the accumulated value of \$200 deposited at the end of every 3 months for 4 years at 10%/a, compounded quarterly

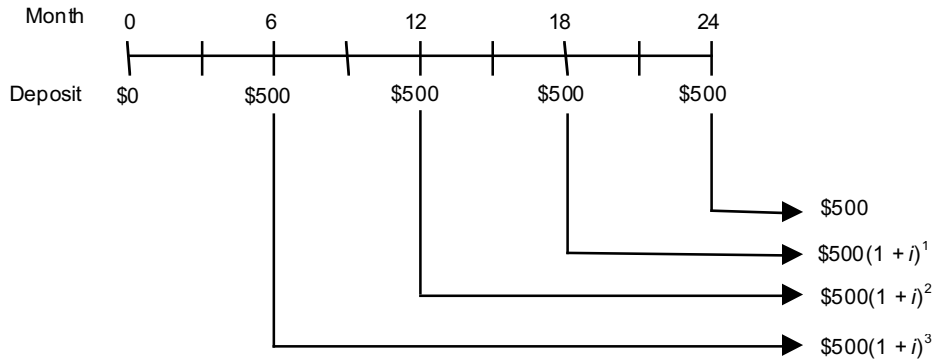


(b) the present value of \$350 deposited at the end of every month for 1.5 years at 9%/a, compounded monthly



- (c) the accumulated value of \$500 deposited at the end of every 6 months for 2 years at 12%/a, compounded quarterly

The semiannual interest rate, i , is equivalent to 12%/a, compounded quarterly.



9. Represent each annuity in question 8 as a geometric sum. Find the accumulated value of each sum.

- (a) The annuity is $200 + 200(1.025)^1 + 200(1.025)^2 + \dots + 200(1.025)^{15}$. For $R = 200$, $i = 0.025$, and $n = 4(4)$ or 16,

$$\begin{aligned}
 FV &= R \times \frac{(1+i)^n - 1}{i} \\
 &= 200 \times \frac{(1.025)^{16} - 1}{0.025} \\
 &\doteq 200 \times \frac{1.484\,505\,621 - 1}{0.025} \\
 &= 3876.04
 \end{aligned}$$

The value of the annuity is \$3876.04.

- (b) The annuity is $350(1.0075)^{-1} + 350(1.0075)^{-2} + 350(1.0075)^{-3} + \dots + 350(1.0075)^{-18}$.

Recall that R in the formula for present value does not equal a in the formula for the sum of a geometric series, S_n . For $R = 350$, $i = 0.0075$, and $n = 18$,

$$\begin{aligned}
 PV &= R \times \frac{1 - (1+i)^{-n}}{i} \\
 &= 350 \times \frac{1 - (1.0075)^{-18}}{0.0075} \\
 &\doteq 350 \times \frac{1 - 0.874\,156\,142}{0.0075} \\
 &= 5872.71
 \end{aligned}$$

The accumulated value of the annuity is \$5872.71.

(c) i/a , compounded semiannually = 12%/a, compounded quarterly

$$(1+i)^2 = \left(1 + \frac{12\%}{4}\right)^4$$

$$(1+i)^2 = (1.03)^4 \quad \text{Take the square root of each side.}$$

$$1+i = (1.03)^2$$

$$1+i = 1.0609$$

$$i = 0.0609$$

Then $2 \times 0.0609 = 0.1218$, and the nominal equivalent rate is 12.18%/a, compounded semiannually. The annuity is $500 + 500(1.0609)^1 + 500(1.0609)^2 + 500(1.0609)^3$. For $R = 500$, $i = 0.0609$, and $n = 4$,

$$\begin{aligned} FV &= R \times \frac{(1+i)^n - 1}{i} \\ &= 500 \times \frac{(1.0609)^4 - 1}{0.0609} \\ &\doteq 500 \times \frac{1.266\,770\,081 - 1}{0.0609} \\ &= 2190.23 \end{aligned}$$

The accumulated value of the annuity is \$2190.23.

- 10.** *The amount of an annuity with monthly payments will be worth more at 12%/a, compounded weekly, rather than at 12.5%/a, compounded annually. Explain and show why this statement is true.*

Compounding interest increases the value of an annuity. Interest at 12%/a, compounded weekly, would be equivalent to more than 12%/a, compounded annually. To determine how much more, calculate the equivalent nominal rate.

i/a , compounded annually = 12%/a, compounded weekly

$$(1+i)^1 = \left(1 + \frac{12\%}{52}\right)^{52}$$

$$1+i \doteq (1.002\,307\,692\,3)^{52}$$

$$1+i \doteq 1.127\,340\,987$$

$$i \doteq 0.127\,34$$

Therefore, 12.734%/a, compounded annually, is equivalent to 12%/a, compounded weekly. 12.734%/a, compounded annually, is more than 12.5%/a, compounded annually.

- 11.** *A family asks for your advice for minimizing the total cost of purchasing, and then repaying the mortgage of, a new home. Name at least four things that you would advise them to do.*

To minimize the total cost of purchasing a house: make a down payment of at least 25% of the purchase price to avoid mortgage insurance; make payments weekly; pay down as much as possible and as often as possible; negotiate real estate fees, legal fees, and interest rates.