
Chapter 2 Test: Solutions

1. Anna's total earnings per quarter are $\frac{30000}{4} + \frac{30500}{4} + \frac{31000}{4} + \dots$.

In this arithmetic series, $a = \frac{30\,000}{4}$, $d = \frac{500}{4}$, and $n = 5(4)$ or 20.

$$\begin{aligned}S_n &= \frac{n}{2}[2a + (n-1)d] \\S_{20} &= \frac{20}{2}\left[2\left(\frac{30\,000}{4}\right) + 19\left(\frac{500}{4}\right)\right] \\&= 10(17\,375) \\&= 173\,750\end{aligned}$$

Anna will earn \$173 750 in five years.

2. Cheyenne's monthly earnings are 1000, $1000(1.05)$, $1000(1.05)^2$, ... Her total earnings for the last four months of her first year on the job, that is, t_9 to t_{12} , are $1000(1.05)^8 + 1000(1.05)^9 + 1000(1.05)^{10} + 1000(1.05)^{11}$. In this geometric series, $a = 1000(1.05)^8$, $r = 1.05$, and $n = 4$.

$$\begin{aligned}S_n &= \frac{a(r^n - 1)}{r - 1} \\S_4 &= \frac{1000(1.05)^8[(1.05)^4 - 1]}{1.05 - 1} \\&= \frac{1000(1.477\,455\,444)(1.215\,506\,25 - 1)}{0.05} \\&= 6368.02\end{aligned}$$

Cheyenne received \$6368.02 in the last four months of her first year on the job.

3. (a) See the diagram on text page 122 for a similar example. In this case, the maximum height of the bouncing ball after each bounce is $4(0.6)$, $4(0.6)^2$, $4(0.6)^3$, ...

After the fourth bounce, the vertical height is $4(0.6)^4$ m or 0.5184 m.

- (b) The sum of the downward distances after six bounces is $4 + 4(0.6) + 4(0.6)^2 + 4(0.6)^3 + \dots + 4(0.6)^6$. In this geometric series, $a = 4$, $r = 0.6$, and $n = 6$.

$$\begin{aligned}S_n &= \frac{a(r^n - 1)}{r - 1} \\S_6 &= \frac{4[(0.6)^6 - 1]}{0.6 - 1} \\&= \frac{4(0.046\,656 - 1)}{-0.4} \\&= 9.533\,44\end{aligned}$$

Store 9.533 44 in the calculator's memory.

The sum of the upward distances is the sum of the downward distances, less the original drop of 4 m, $9.533\ 44 - 4 = 5.533\ 44$ m. Therefore, the total distance travelled is

$9.533\ 44 + 5.533\ 44 = 15.06$ m, to two decimal places.

4. (a) There are eight payments, $N = 8$. The interest rate is $6.6\%/a$, $I\% = 6.6$. The present value is 0, $PV = 0$. The periodic payment is \$200, $PMT = -200$. P/Y and C/Y equal 12, because the payments are monthly and the interest conversion period is monthly. **END** is selected for PMT , because the payments are at the end of the month. The last step is to solve for **FV**, $FV = 1631.14$.

```

N=8
I%=6.6
PV=0
PMT=-200
FV=1631.1411
P/Y=12
C/Y=12
PMT:END BEGIN

```

- (b) There are eight payments, $N = 8$. The interest rate is $6.6\%/a$, $I\% = 6.6$. The present value is 0, $PV = 0$. The periodic payment is \$200, $PMT = -200$. P/Y and C/Y equal 12, because the payments are monthly and the interest conversion period is monthly. **BEGIN** is selected for PMT , because the payments are at the beginning of the month. The last step is to solve for **FV**, $FV = 1640.11$.

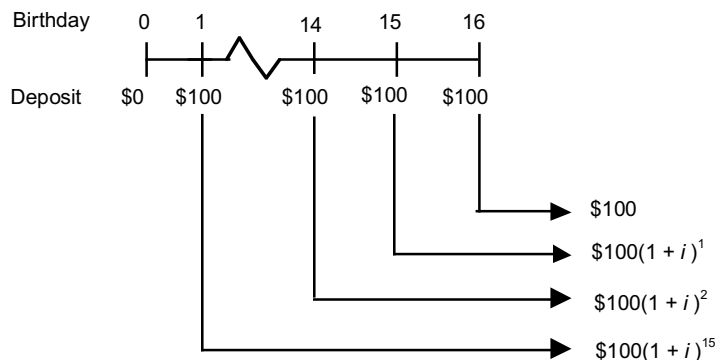
```

N=8
I%=6.6
PV=0
PMT=-200
FV=1640.11241
P/Y=12
C/Y=12
PMT:END BEGIN

```

When the deposit is made at the beginning of the month, that deposit earns interest for that month. In effect, an annuity due includes one more month's interest than an ordinary annuity.

5. (a)



The annual interest rate, i , is equivalent to $8\%/a$, compounded quarterly.

$$(1+i)^1 = \left(1 + \frac{8\%}{4}\right)^4$$

$$1+i = (1.02)^4$$

$$1+i = 1.082\,432\,16$$

$$i = 0.082\,432\,16$$

$$FV = R \times \frac{(1+i)^n - 1}{i}, \text{ and } R = 100, i = 0.082\,43, \text{ and } n = 16$$

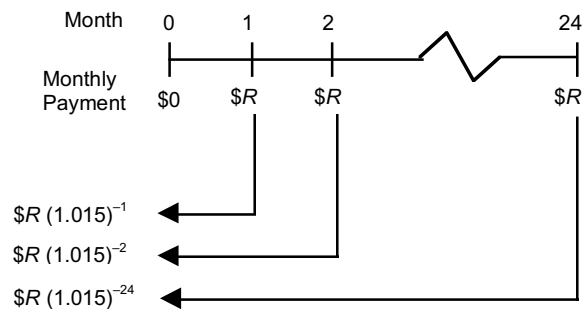
$$\begin{aligned} FV &= 100 \times \frac{(1.082\,432\,16)^{16} - 1}{0.082\,432\,16} \\ &= 100 \times \frac{3.551\,493\,243 - 1}{0.082\,432\,16} \\ &= 3095.26 \end{aligned}$$

The value of the annuity is \$3095.26.

5. (b)

	A	B	C	D	E	F	G
1	Period	Balance	Interest	Payment Made	New Balance		
2	1	0	0	100	=B2+C2+D2		
3	=A2+1	=E2	=B3*0.082 432 16	=\$D\$2	copy ↓		
4	copy ↓	copy ↓	copy ↓	copy ↓			
17	16				answer to (a)		

6. (a)



$$PV = 10\,000, i = \frac{18\%}{12} \text{ or } 0.015, n = 24, \text{ and } R = ?$$

The monthly payment would be \$499.24.

- (b) The loan repayment cost is $\$499.24(24) = \$11\,981.76$. Therefore, the finance charge is $\$11\,981.76 - \$10\,000 = \$1981.76$.

7. Option A, dealer financing: There are 60 payments, $N = 60$. The interest rate is 2.4%/a, $I\% = 2.4$. The selling price of the car is \$32 000, $PV = 32\ 000$. The loan will be paid off in the future, $FV = 0$. The payments and the interest are monthly, $P/Y = 12$ and $C/Y = 12$. Select **END**, because it is assumed the payments are made at the end of the month. The last step is to solve for the payment, $PMT = -566.51$.

```

N=60
I%=2.4
PV=32000
PMT=-566.50569...
FV=0
P/Y=12
C/Y=12
PMT: [ ] [ ] BEGIN

```

Under the dealer's financing plan, the monthly payment is \$566.51.

Option B, bank or other financing: Calculate the interest rate that would generate the same monthly payment of \$566.51.

Enter the new value, 29 000, for PV and solve for $I\%$.

```

N=60
I%=6.432782815
PV=29000
PMT=-566.50569...
FV=0
P/Y=12
C/Y=12
PMT: [ ] [ ] BEGIN

```

An interest rate of 6.43%/a, compounded monthly, generates the same monthly payment.

- (b) It is better to find alternative financing if the interest rate at a bank is less than 6.7%/a, compounded monthly. The advantage of dealer financing is convenience.
8. (a) For an amortization of 20 years, her monthly payment is \$1020.50.

```

N=240
I%=7.8
PV=125000
PMT=-1020.500...
FV=0
P/Y=12
C/Y=2
PMT: [ ] [ ] BEGIN

```

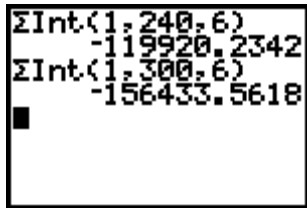
For an amortization of 25 years, there are $12 \times 25 = 300$ payments, $N = 300$. The interest rate is 7.8%/a, $I\% = 7.8$. The present value is \$125 000, $PV = 125\ 000$. The mortgage will be paid off in the future, $FV = 0$. There are 12 payments during each year, $P/Y = 12$. Interest is compounded semiannually, $C/Y = 2$. Select **END** for PMT. The last step is to solve for the payment, PMT . Her monthly payment is \$938.11.

```

N=300
I%=7.8
PV=125000
PMT=-938.11187...
FV=0
P/Y=12
C/Y=2
PMT: [ ] [ ] BEGIN

```

For 240 payments, the total interest paid is \$119 920.23. For 300 payments, the total interest paid is \$156 433.56.



The image shows a TI-83 Plus calculator screen with the following text displayed:
ΣInt(1,240,6)
-119920.2342
ΣInt(1,300,6)
-156433.5618
A small black cursor is visible on the line below the second result.

- (b) Although there is a difference of \$36 513.56 in total interest paid, there are other factors for Ms Burns to consider. These interest rates are assumed to be constant throughout the life of the mortgage. She must consider that interest rates change and that her financial status may change through marriage, inheritance, or loss of employment. Importantly, which monthly payment can she currently afford? In addition, although the monthly payment for the 25-year amortization is \$82.39 less each month, this amortization is 60 months longer.
9. The algebraic method requires the most steps. Correcting errors can be time consuming.

A spreadsheet is very useful if you need a printed copy of a repayment schedule. Most or all of the spreadsheet is visible on the computer screen. It is very easy and fast to change the parameters.

The TI-83 Plus is portable and easy to use, requiring the fewest entries. The repayment schedules are hard to read and the facility for printing may not be available.