
CHECK YOUR UNDERSTANDING

1. *What is a function? Give an example of a relation that is a function and another example of a relation that is not a function.*

A function is a relation where there is a unique value of the dependent variable for every value of the independent variable. For example, a function is defined by

$y = (x - 3)^2 + 1$, or $\{(1, 2), (2, 5), (3, 7)\}$. The relations $x^2 + y^2 = 25$, $y = \pm\sqrt{x+4}$, and $\{(1, 2), (1, 5), (3, 7)\}$ are not functions.

2. *A function can be described or defined in a number of ways. List the different ways and explain how you would use each to determine if a relation is a function.*

- A table: Each value in the first column, which represents the independent variable, should be different if the relation is a function.
- An arrow diagram: Only one arrow should lead from each member of the domain to a member of the range if the relation is a function. More than one arrow can lead to the same member of the range.
- A graph: Any vertical line does not cross the graph at more than one point if the relation is a function.
- An equation: Each value substituted for x will determine one value for y if the relation is a function.

3. *How can you use the graph of a relation to decide if the relation is a function?*

Imagine or draw a vertical line anywhere on the graph. For the relation to be a function, this vertical line will not cross the graph in more than one place.

4. *How do you find the domain and range of a relation?*

The domain of a function is the set of all the possible values for the independent variable. The domain may be thought of as the “width” of the graph. The range is the set of all possible values for the dependent variable, and it may be thought of as the “height” of the graph.

5. *How do you write the domain and the range in set notation?*

For the domain, use set notation to write the values of the variable, if applicable, give the restrictions, if any, and describe the kinds of numbers in the domain. Do the same for the range.

6. *How are the rules for solving a linear inequality algebraically different from the rules for solving a linear equation?*

For a linear inequality, reverse the sign when multiplying or dividing by a negative quantity. Multiplying by 0 turns the inequality into an equation.

7. *How do you write the solution to a linear inequality?*

Use set notation to describe all possible values for the variable. Include the restriction on the variable and the appropriate number set.

8. *Describe a non-algebraic method for solving a linear inequality.*

Graph the function. Use the restrictions on the value of the function to “read” the corresponding restrictions on the independent variable, that is, the solution.

9. *What is the inverse of a relation?*

The inverse of the relation is the “reverse” of the original relation. The inverse of a relation is also the set of all ordered pairs that results from interchanging the x - and y -coordinates of the original relation.

10. *How do you graph the inverse of a relation using the graph of the relation?*

Reflect the graph of the original relation in the line $x = y$.

11. *How do you determine the equation of the inverse from the equation of a relation?*

Interchange the variables.

12. *Why might $f^{-1}(x)$ be different from the equation of the inverse?*

When solving the equation of the inverse, you may not have isolated y correctly. To find $f^{-1}(x)$, y must be completely isolated.

13. *Point (a, b) is on the graph of f . Express each of the coordinates of the point using function notation.*

$$f(a) = b \text{ and } f^{-1}(b) = a$$

14. *Why might it be difficult to graph the inverse of a relation on a graphing calculator in function mode (**Func**)? What can you do instead?*

When the inverse is not a function, you can enter the equation of the inverse, for example, $y = \sqrt{x}$, but the graphing calculator will graph the part of the relation that is a function. To graph the inverse, it may be necessary to enter more than one equation in the equation editor. (In this case, $y = \sqrt{x}$ and $y = -\sqrt{x}$.) Or, use parametric equations.

15. *Compare the domain and range of a relation with the domain and range of the inverse relation.*

The domain and range of the original relation become the range and domain of the inverse, respectively. The variables are interchanged in the restrictions, as well as in the equation.

16. *Why might the inverse of a function not be a function? How can you check whether the inverse of a function is a function?*

The inverse will not be a function if at least two points of the original relation have the same y -value. Both the original relation and the inverse will be functions if every x -value of the original relation is paired with a unique y -value. This relation is called a one-to-one relation.

17. *Why is the inverse of $y = x^2$ not $y = \sqrt{x}$? Explain.*

The function $y = x^2$ is a complete parabola, but $y = \sqrt{x}$ is only one branch of the parabola. Placing the restriction $x \geq 0$ on $y = x^2$ yields the inverse $y = \sqrt{x}$, which is a function.

- 18.** *The equation of a function is expressed in terms of a base function and involves multiplication. What type of transformation could be applied to the graph of the base function to get the graph of the new function?*

Stretch the graph of the base function if the factor is less than 1; shrink the graph if the factor is greater than 1. Reflect the graph in the x -axis if the factor is negative. $f(kx)$ stretches the function $f(x)$ horizontally by a factor of $\frac{1}{k}$, and $af(x)$ stretches the function $f(x)$ vertically by a factor of $|a|$. For $a < 0$, reflect the graph in the x -axis.

- 19.** *The equation of a function is expressed in terms of a base function and involves addition. What type of transformation could be applied to the graph of the base function to get the graph of the new function?*

Keep the shape of the graph, but translate the graph. For the graph of $y = f(x) + q$, shift the graph vertically by q units. For $q < 0$, the direction of translation is down. For $q > 0$, the direction of translation is up.

- 20.** *How does expressing a function in terms of a base function help you to determine its domain and range?*

Apply the same transformations to the domain and range of the original function to get the domain and range of the new function.