

The Chapter Problem—Fundraising: Answers

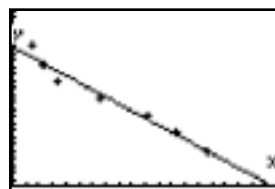
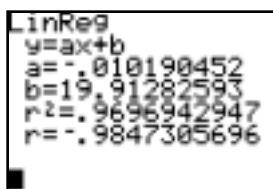
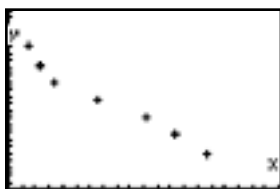
By the end of this chapter, students should be able to provide comprehensive, well-explained solutions to the problem.

Section 4.2

CP1. Use the survey data on page 300 and the total number of students in the school to estimate the number of T-shirts that should sell at each price.

Survey Number (out of 100)	93	78	64	42	21	15	9
Whole School Number (out of 1582)	$1582 \times 0.93 = 1471$	1234	1012	664	332	237	142
Price of T-shirt (\$)	5.00	7.50	10.00	12.50	15.00	17.50	20.00

CP2. Enter your data from CP1 into a graphing calculator. Do a linear regression to obtain a demand function relating price to the number of T-shirts sold. Round the coefficients to two decimal places.

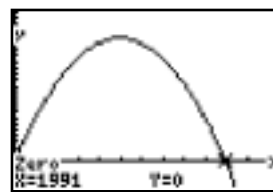
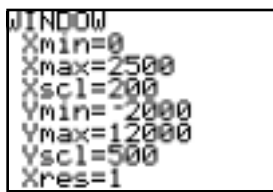
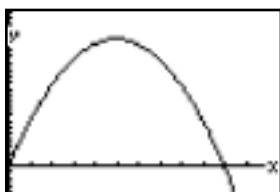


The demand function is $p(x) = -0.01x + 19.91$.

CP3. Write the revenue function, based on your demand function from CP2.

The revenue function is $R(x) = -0.01x^2 + 19.91x$.

CP4. Graph the revenue function and describe its shape. What is its domain?



The graph of the revenue function is a parabola which opens down.

The domain is the set of positive integers less than 1991.

Section 4.3

CP5. Use the data in the first table on page 300 to write a cost function for each supplier.

Custom Cottons: $C(x) = 3.90x + 250$

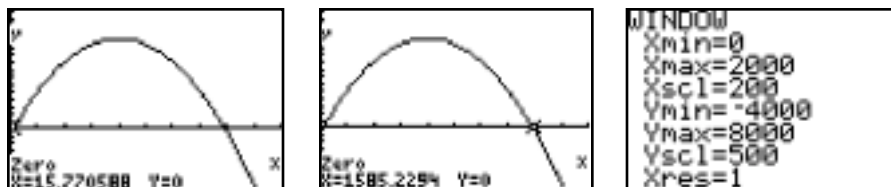
Tailored Tees: $C(x) = 2.80x + 400$

Tops to Go: $C(x) = 4.50x + 120$

- CP6.** Use your revenue function from CP3 and the cost function for the first supplier in question CP5 to find the corresponding profit function.

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= -0.01x^2 + 19.91x - (3.90x + 250) \\ &= -0.01x^2 + 16.01x - 250 \end{aligned}$$

- CP7.** Graph your profit function from question CP6. State the domain and determine the break-even points.



The domain is the set of positive integers less than 1585.

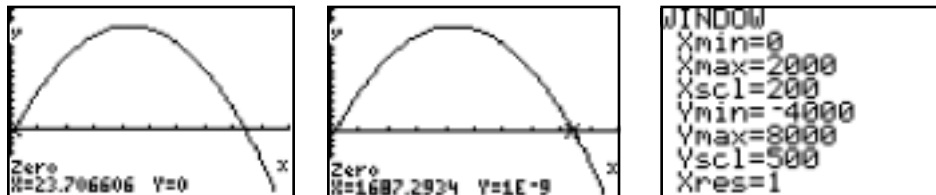
The student council would break even at sales of 16 T-shirts and at 1585 T-shirts.

For sales of more than 16 T-shirts and less than 1585 T-shirts, the student council would make a profit.

- CP8.** Repeat questions CP6 and CP7 for the other suppliers.

Tailored Tees

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= -0.01x^2 + 19.91x - (2.80x + 400) \\ &= -0.01x^2 + 17.11x - 400 \end{aligned}$$



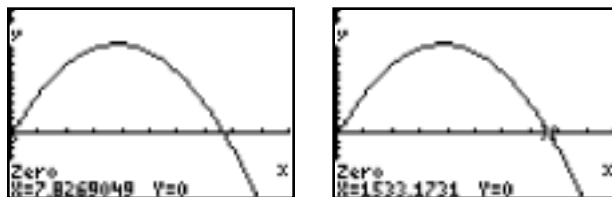
The domain is the set of positive integers less than 1687.

The student council would break even at sales of 24 T-shirts and at 1687 T-shirts.

For sales of more than 24 T-shirts and less than 1687 T-shirts, the student council would make a profit.

Tops to Go

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= -0.01x^2 + 19.91x - (4.50x + 120) \\ &= -0.01x^2 + 15.41x - 120 \end{aligned}$$



The domain is the set of positive integers less than 1533. The student council would break even at sales of 8 T-shirts and at 1533 T-shirts. For sales of more than 8 T-shirts and less than 1533 T-shirts, the student council would make a profit.

CP9. Which supplier would you recommend?

Based on the information about break-even points, there is very little difference between the three suppliers. If the supplier were Tailored Tees, the event would be profitable over a greater range of sales values. If the supplier were Tops to Go, the event becomes profitable after selling the least number of T-shirts, 8 T-shirts.

Section 4.4

CP10. Examine each profit function you obtained in CP6. Will the student council be able to raise \$7000 by selling T-shirts?

$$-0.01x^2 + 16.01x - 250 = 7000$$

$$-0.01x^2 + 16.01x - 7250 = 0$$

Calculate the discriminant.

$$\begin{aligned} b^2 - 4ac &= 16.01^2 - 4(-0.01)(-7250) \\ &= -33.6799 \end{aligned}$$

The equation has no real roots. It is not possible to raise \$7000 using Custom Cottons.

Similarly,

$$-0.01x^2 + 17.11x - 400 = 7000$$

$$-0.01x^2 + 17.11x - 7400 = 0$$

$$\begin{aligned} b^2 - 4ac &= 17.11^2 - 4(-0.01)(-7400) \\ &= -3.2479 \end{aligned}$$

The equation has no real roots. It is not possible to raise \$7000 using Tailored Tees.

$$-0.01x^2 + 15.41x - 120 = 7000$$

$$-0.01x^2 + 15.41x - 7120 = 0$$

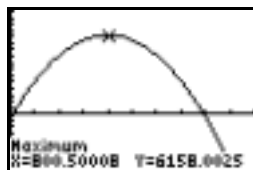
$$\begin{aligned} b^2 - 4ac &= 15.41^2 - 4(-0.01)(-7120) \\ &= -47.3319 \end{aligned}$$

The equation has no real roots. It is not possible to raise \$7000 using Tops to Go.

CP11. Determine the maximum amount the student council could raise on the basis of each supplier.

(Answer with answer for CP12.)

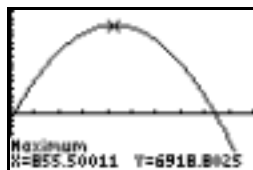
CP12. Find the number of T-shirts sold in each case in CP11.



Custom Cottons

The maximum profit is \$6158.00.

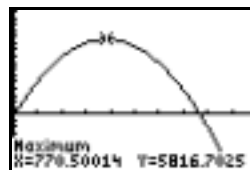
800 T-shirts sold



Tailored Tees

The maximum profit is \$6918.80.

855 T-shirts sold



Tops to Go

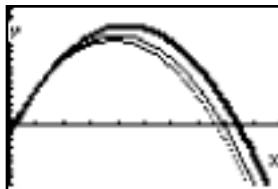
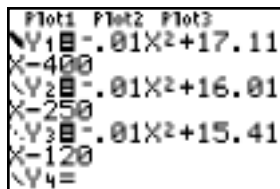
The maximum profit is \$5816.70.

770 T-shirts sold

CP13. Which T-shirt supplier would you choose? Explain your choice.

Tailored Tees provides the only opportunity to get close to the target of \$7000.

The graph of the profit function for Tailored Tees is higher than the graphs for the other two suppliers for most x -values.

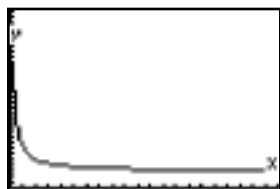


Section 4.8

CP14. If $C(x)$ is the cost of producing x items, then the average cost function is

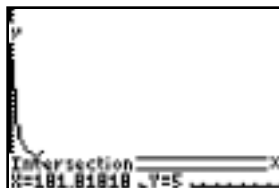
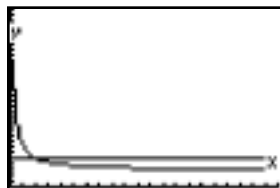
$AC(x) = \frac{C(x)}{x}$. Find the average cost function for your chosen supplier. Graph this function and state its domain.

For Tailored Tees, $C(x) = 2.80x + 400$, so $AC(x) = 2.80 + \frac{400}{x}$.



The domain is the set of positive integers.

CP15. How many T-shirts must be sold so that the average cost per T-shirt is less than \$5?



Using algebra,

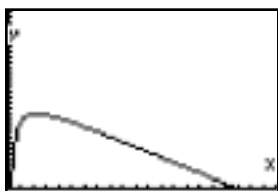
$$\begin{aligned} 2.80 + \frac{400}{x} &< 5 \\ 2.80x + 400 &< 5x \\ 400 &< 2.20x \\ x &> 181.8181818 \end{aligned}$$

182 T-shirts must be sold for the average cost per T-shirt to be less than \$5.

CP16. If $P(x)$ is the profit earned from producing and selling x items, then the average profit is modelled by the function $AP(x) = \frac{P(x)}{x}$. Find the average profit function for your chosen supplier. Graph this function and state the domain.

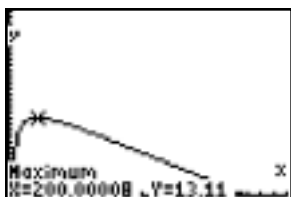
For Tailored Tees, $P(x) = -0.01x^2 + 17.11x - 400$, so

$$AP(x) = -0.01x + 17.11 - \frac{400}{x}.$$



The domain is the set of positive integers.

CP17. At what point is the average profit a maximum?



The average profit reaches a maximum when 200 T-shirts are sold. At this point, the average profit per T-shirt is \$13.11.

Section 4.11

CP18. Write a proposal, for this fundraiser, that the representative could present to the school council. Include all of your graphs and analyses.

The proposal will include student analyses for questions CP1 to CP17. Students might also want to include some discussion of the best price to ask for the T-shirts. They can use the demand function from CP2 for this calculation.