

Chapter 4 Test: Solutions

$$1. \quad (a) \quad \frac{m^2 - 2m}{m^2 + 5m - 14} = \frac{m(m-2)}{(m+7)(m-2)}, m \neq -7, 2$$

$$= \frac{m}{m+7}, m \neq -7, 2$$

(b) The LCD is $(x-4)(x+3)(x-3)$.

$$\frac{4}{x^2 - x - 12} - \frac{3}{x^2 - 9} = \frac{4}{(x-4)(x+3)} - \frac{3}{(x-3)(x+3)}$$

$$= \frac{4}{(x-4)(x+3)} \times \frac{x-3}{x-3} - \frac{3}{(x-3)(x+3)} \times \frac{x-4}{x-4}$$

$$= \frac{4(x-3) - 3(x-4)}{(x-4)(x+3)(x-3)}$$

$$= \frac{4x - 12 - 3x + 12}{(x-4)(x+3)(x-3)}$$

$$= \frac{x}{(x-4)(x+3)(x-3)}$$

$$= \frac{x}{x^3 - 4x^2 - 9x + 36}, x \neq -3, 3, 4$$

$$(c) \quad \frac{x^2 - 3x - 4}{x^2 + 5x + 6} \times \frac{x^2 - 4}{x^2 - 2x - 3} \div \frac{x^2 - 4x}{(x+3)^2} = \frac{(x-4)(x+1)}{(x+2)(x+3)} \times \frac{(x-2)(x+2)}{(x-3)(x+1)} \div \frac{x(x-4)}{(x+3)^2}$$

$$= \frac{(x-4)(x+1)}{(x+2)(x+3)} \times \frac{(x-2)(x+2)}{(x-3)(x+1)} \times \frac{(x+3)^2}{x(x-4)}$$

$$= \frac{(x-2)(x+3)}{x(x-3)}$$

$$= \frac{x^2 + x - 6}{x^2 - 3x}, x \neq -3, -2, -1, 0, 3, 4$$

$$2. \quad (a) \quad i. \quad s(t) = -4.9t^2 + 34.3t - 3$$

$$= -4.9(t^2 - 7t) - 3$$

$$= -4.9 \left[t^2 - 7t + \left(\frac{49}{4} \right) - \left(\frac{49}{4} \right) \right] - 3$$

$$= -4.9 \left[t - \left(\frac{7}{2} \right) \right]^2 + (4.9) \left(\frac{49}{4} \right) - 3$$

$$= -4.9(t - 3.5)^2 + 57.025$$

Since $-4.9 < 0$, the function has a maximum value. The vertex coordinates are $(3.5, 57.025)$.

$$ii. \quad f(x) = (x+4)(x-3)$$

$a = 1$. Since $1 > 0$, the function has a minimum value. The x -coordinate of the vertex is

$\frac{(\pm 4 + 3)}{2} = -0.5$. The corresponding y -coordinate is $f(-0.5) = -12.25$. The vertex coordinates are $(-0.5, -12.25)$.

(b) i. The revenue function is

$$\begin{aligned} R(x) &= x(-5x + 18) \\ &= -5x^2 + 18x \end{aligned}$$

The profit function is

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= -5x^2 + 18x - (2x + 9) \\ &= -5x^2 + 16x - 9 \end{aligned}$$

Complete the square.

$$\begin{aligned} P(x) &= -5 \left[x^2 - \left(\frac{16}{5} \right) x \right] - 9 \\ &= -5(x^2 - 3.2x + 2.56 - 2.56) - 9 \\ &= -5(x - 1.6)^2 + (5)(2.56) - 9 \\ &= -5(x - 1.6)^2 + 3.8 \end{aligned}$$

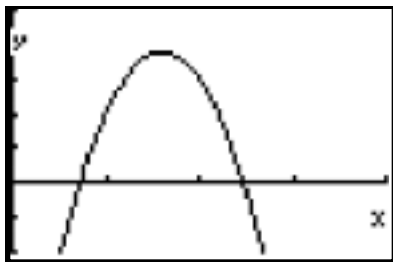
The profit will reach a maximum when $x = 1.6$, that is, when 1600 items are sold.

ii. The break-even quantities are the zeros of the profit function, $P(x) = -5x^2 + 16x - 9$. Using the quadratic formula,

$$\begin{aligned} x &= \frac{-16 \pm \sqrt{16^2 - 4(-5)(-9)}}{2(-5)} \\ &= \frac{-16 \pm \sqrt{76}}{-10} \end{aligned}$$

$$x = 0.728\ 220\ 211\ 3 \text{ or } x = 2.471\ 779\ 789$$

The break-even quantities are 728 items and 2472 items.



3. (a) $10(x^2 - 5x + 5) = x^2 - 8x + 1$

$$10x^2 - 50x + 50 = x^2 - 8x + 1$$

$$9x^2 - 42x + 49 = 0$$

$$(3x - 7)^2 = 0$$

The equation has one double root.

You could also use the discriminant.

$$\begin{aligned} b^2 - 4ac &= (-42)^2 - 4(9)(49) \\ &= 0 \end{aligned}$$

(b) If the function $f(x) = x^2 - (k + 2)x + 2k + 4$ does not have zeros, $b^2 - 4ac < 0$.

$$[-(k + 2)]^2 - 4(1)(2k + 4) < 0$$

$$k^2 + 4k + 4 - 8k - 16 < 0$$

$$k^2 - 4k - 12 < 0$$

$$(k - 6)(k + 2) < 0$$

$$-2 < k < 6$$

4. (a) One root is $3 - i$. The other root is $3 + i$. The equation is

$$[x - (3 - i)] \times [x - (3 + i)] = 0$$

$$(x - 3 + i)(x - 3 - i) = 0$$

$$(x - 3)^2 - i^2 = 0$$

$$x^2 - 6x + 9 + 1 = 0$$

$$x^2 - 6x + 10 = 0$$

(b) $4x^2 - 4x + 3 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(3)}}{2(4)}$$

$$= \frac{4 \pm \sqrt{-32}}{8}$$

$$= \frac{4 \pm \sqrt{32}i^2}{8}$$

$$= \frac{4 \pm 4\sqrt{2}i}{8}$$

Roots are $\frac{1}{2} + \frac{\sqrt{2}}{2}i$ and $\frac{1}{2} - \frac{\sqrt{2}}{2}i$, or $0.5 + 0.7i$ and $0.5 - 0.7i$, to one decimal place.

5. For $z = 2 - i$ and $w = 2 + 3i$,

$$\begin{aligned}
 zw - \frac{z}{w} &= (2 - i)(2 + 3i) - \frac{2 - i}{2 + 3i} \\
 &= 4 + 4i - 3i^2 - \frac{2 - i}{2 + 3i} \times \frac{2 - 3i}{2 - 3i} \\
 &= 7 + 4i - \frac{4 - 8i + 3i^2}{4 - 9i^2} \\
 &= 7 + 4i - \frac{1 - 8i}{13} \\
 &= 7 + 4i - \left(\frac{1}{13} - \frac{8}{13}i \right) \\
 &= \frac{90}{13} + \frac{60}{13}i \\
 &= 30 \left(\frac{3}{13} + \frac{2}{13}i \right) \\
 &\approx 6.9 + 4.6i
 \end{aligned}$$

6. (a) $P \times V = k$, where P represents pressure, V represents volume, and k is a constant.

Substitute $P = 20.5$ and $V = 250$ in this equation.

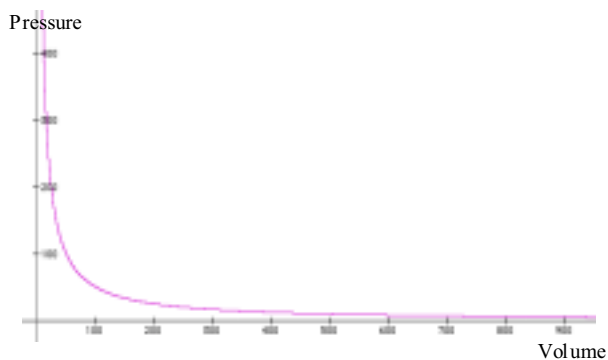
$$\begin{aligned}
 P \times V &= 20.5 \times 250 \\
 &= 5125
 \end{aligned}$$

For $P = 14$,

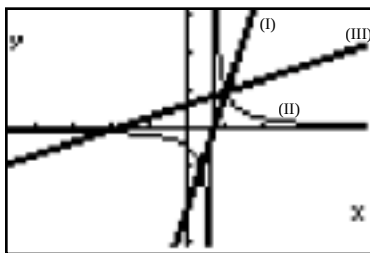
$$\begin{aligned}
 14 \times V &= 5125 \\
 V &= \frac{5125}{14} \\
 &= 366.0714286
 \end{aligned}$$

The volume is about 366.07 cm^3 when the pressure is 14 N/cm^3 .

The equation is $PV = 5125$, $P = \frac{5125}{V}$, or $V = \frac{5125}{P}$.



- (b) The graphs of $y = 3x - 2$ (I), $y = \frac{1}{3x-2}$ (II), and $y = \frac{(x+2)}{3}$ (III) are shown.



The domain and range for both $f(x)$ and $f^{-1}(x)$ are the set of real numbers. For $\frac{1}{f(x)}$, the domain is $\left\{x \mid x \in \mathbf{R}, x \neq \frac{2}{3}\right\}$, and the range is $\{y \mid y \in \mathbf{R}, y \neq 0\}$.

7. Let x represent the number of shirts in the case. Then the cost, to Sean, of each shirt is $\frac{504}{x}$. He sells $(x - 3)$ shirts for \$630, so the selling price of each shirt is $\frac{630}{(x-3)}$. Sean profits by \$9 on each shirt, so $\frac{630}{x-3} - \frac{504}{x} = 9$.

Solve for x .

$$\begin{aligned}\frac{630}{x-3} \times x(x-3) - \frac{504}{x} \times x(x-3) &= 9x(x-3) \\ 630x - 504(x-3) &= 9x^2 - 27x \\ 630x - 504x + 1512 &= 9x^2 - 27x \\ 9x^2 - 153x - 1512 &= 0 \\ 9(x^2 - 17x - 168) &= 0 \\ 9(x-24)(x+7) &= 0\end{aligned}$$

Reject the negative root since $x \geq 0$. So $x = 24$.

There were 24 shirts in the case.

8. (a) $p = x^2 - x + 1$ and $q = x^2 + x + 1$

Since $p^2 - 2pq + q^2 = (p - q)^2$,

L.S.	R.S.
$[(x^2 - x + 1) - (x^2 + x + 1)]^2$	$4x^2$
$= (x^2 - x + 1 - x^2 - x - 1)^2$	
$= (-2x)^2$	
$= 4x^2$	

(b) Answers will vary. Here is an example.

i. $(x^3 - 2x + 1)(x^3 - 1) = x^6 - 2x^4 + 2x - 1$, which has degree 6

ii. $(x^2 - 3x + 5)(x^5 - 1) = x^7 - 3x^6 + 5x^5 - x^2 + 3x - 5$, which has degree 7

The degree of a product of polynomials is the sum of the degrees of the polynomials.

9. If P has coordinates (x, y) , then the rectangle has area xy . Use the equation of the line on which P lies to obtain a relationship between x and y . This line has equation $y = \left(-\frac{b}{a}\right)x + b$.

Find the equation for the area of the rectangle and express this equation in factored form.

$$A = xy$$

$$= x \times \left[\left(-\frac{b}{a}\right)x + b \right]$$

$$= \left(-\frac{b}{a}\right)x^2 + bx$$

$$= \frac{-bx^2}{a} + \frac{abx}{a}$$

$$= \frac{-bx^2 + abx}{a}$$

$$= \frac{-b(x^2 - ax)}{a}$$

$$= \frac{-b[x(x - a)]}{a}$$

$$= \frac{-b}{a} x(x - a)$$

The maximum value for A occurs at the vertex. The x -coordinate of the vertex is $\frac{a}{2}$. The y -

coordinate of the vertex is $\left(-\frac{b}{a}\right)\left(\frac{a}{2}\right) + b = \frac{b}{2}$.

To maximize the area of the rectangle, the coordinates of P are $\left(\frac{a}{2}, \frac{b}{2}\right)$.