
Check Your Understanding: Answers

1. *How can you tell when the graph of a set of data is periodic?*

In the graph of periodic data, a congruent shape repeats itself.

2. *List five real-life situations that can be modelled by periodic functions. Explain why they are periodic.*

Five possible examples.

- i. The temperature during one year is periodic, because temperature rises to a maximum in the summer and falls to a minimum (negative numbers) in the winter.
- ii. The number of hours of daylight increases to a maximum in the spring and then decreases to a minimum in the winter. The pattern is the same every year.
- iii. The motion of the pendulum of a grandfather clock is periodic, because it repeatedly moves back and forth through the same distance.
- iv. The sale of ice cream could be periodic if it rises to a maximum in the hottest month of the year and falls to a minimum in the coldest month, and follows this pattern each year.
- v. A healthy person's blood pressure has a low of 80 mm of mercury to a high of 120 mm of mercury as the heart beats over and over again.

3. (a) *Sketch a sinusoidal curve.*

Students can show a sketch of a sine function or a cosine function with any values of a , k , b , and d .

- (b) *Name the primary trigonometric functions that are models of sinusoidal curves.*

The sine and cosine functions are models of sinusoidal curves; they are congruent shapes, but separated by a phase shift.

- (c) *How are the functions the same?*

If $y = a \sin k(\theta + b) + d$ and $y = a \cos k(\theta + b) + d$, then both have the same maximum and minimum values, the same axis of the curve, the same shape, and the same period.

- (d) *How are the functions different?*

The sine and cosine functions are separated by a phase shift. In other words, the graph of the cosine function becomes the graph of the sine function if you horizontally translate (shift) the cosine function right by 90° . For example, $\sin \theta = \cos (\theta - 90^\circ)$ and $\sin 2\theta = \cos 2(\theta - 45^\circ)$.

4. *Consider a set of data that can be modelled using a sinusoidal function. Describe the relationship between*

- (a) *the maximum and minimum values and the axis of the function*

The axis of the function is halfway between the maximum and minimum values of the function or $\frac{\text{maximum value} + \text{minimum value}}{2}$.

(b) *the maximum and minimum values and the amplitude of the curve*

The amplitude is half the difference between the maximum and minimum values or
$$\frac{\text{maximum value} - \text{minimum value}}{2}.$$

5. *Explain the following sentence: Sinusoidal functions have period lengths that are multiples of 360° or 2π .*

The period of each base sinusoidal curve, $y = \sin \theta$ or $y = \cos \theta$, is 360° or 2π . The period becomes $\frac{360^\circ}{|k|}$ or $\frac{2\pi}{|k|}$ for $y = \sin k\theta$ or $y = \cos k\theta$.

6. **(a)** *Define the meaning of radian measure and explain its use.*

Radian measure is an angle measure which compares arc length, a , and the radius, r , of circle, $\theta = \frac{a}{r}$. One radian is the angle measure when $a = r$. One radian is about 57° .

(b) *Describe how to convert between degree measure and radian measure.*

Substitute in and solve the proportion $\frac{\text{radians}}{\pi} = \frac{\text{degrees}}{180^\circ}$.

7. *Point P(x, y) is on the terminal arm of an angle θ in standard position. State the primary trigonometric functions.*

$$\sin \theta = \frac{y}{r}, \cos \theta = \frac{x}{r}, \text{ and } \tan \theta = \frac{y}{x}, \text{ where } x \neq 0 \text{ and } r = \sqrt{x^2 + y^2}$$

8. **(a)** *Explain why the tangent function is not defined for all values of the independent variable.*

The independent variable, x , can never be 0, because division by 0 is undefined.

(b) *What occurs at the point where the tangent function is not defined? How does this feature help when sketching the graph?*

Vertical asymptotes occur when $x = 0$ on the circle of rotation. Use the vertical asymptotes to sketch one complete cycle of the tangent curve.

9. *Describe the transformation on $y = \cos \theta$ for each function. Use a sketch in your description.*

(a) $y = \cos 2\theta$

The cosine curve is horizontally compressed to half its period (each x -coordinate is multiplied by $\frac{1}{2}$). There are two cycles in the period 2π .

(b) $y = -\frac{1}{2} \cos \theta$

The cosine curve is vertically compressed by a factor of $\frac{1}{2}$ and then reflected about the θ -axis (each y -coordinate is multiplied by $-\frac{1}{2}$).

(c) $y = \cos \frac{\theta}{2}$

The cosine curve is horizontally stretched by a factor of 2 (each x -coordinate is multiplied by 2). There is half a cycle for the period 2π .

(d) $y = \cos \theta + 2$

The cosine curve is translated up 2 units.

(e) $y = \cos (\theta + 2)$

The cosine curve is translated to the left 2 units.

10. *What transformations are needed to obtain the graph of $y = a \sin k(\theta + b) + d$ from the graph, $y = \sin \theta$ under each set of conditions?*

(a) $a > 1, k > 1, b < 0, d > 0$

a vertical stretch, a horizontal compression, a horizontal shift to the right, and a vertical shift up

(b) $-1 < a < 0, 0 < k < 1, b > 0, d < 0$

a vertical compression and a reflection about the θ -axis, a horizontal stretch, a horizontal shift left, and a vertical shift down

11. *Explain how to use the domain of a periodic function to determine all values of θ when $\sin \theta = 0.5$.*

Solve $\theta = \sin^{-1} 0.5$ to get $\theta = 30^\circ$. Use this angle as the related acute angle and the CAST rule to determine any other angles with a positive sine for the domain $0^\circ \leq \theta \leq 360^\circ$. For this domain, θ also equals 150° (sine is positive in quadrant II). Then all values of θ can be represented by $30^\circ + 360^\circ n, n \in \mathbf{I}$, and $150^\circ + 360^\circ n, n \in \mathbf{I}$.