
The Chapter Problem—How Much Daylight?: Answers

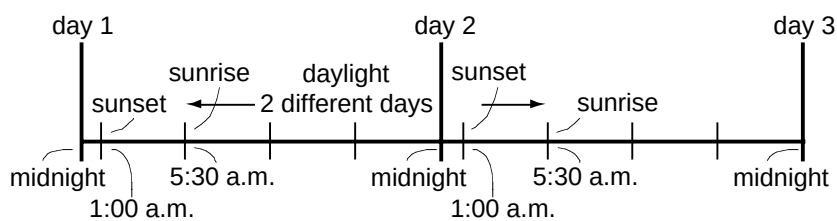
By the end of this chapter, students should be able to provide comprehensive, well-explained solutions to the problem.

Section 5.1

CP1. *How can Naomi use the data in the table to schedule her guided fishing trips and maximize the possible catch within the fishing regulations on catch limits per day?*

Fishing regulations usually specify the maximum number of each type of fish someone can catch each day during a fishing season. Students may like to do some research to find the fishing regulations for different parts of Ontario. There are a number of practical factors that will influence Naomi's schedule, but the best time to go fishing is during the day. From the table, June, July, and August have the highest average number of daylight hours.

For example,



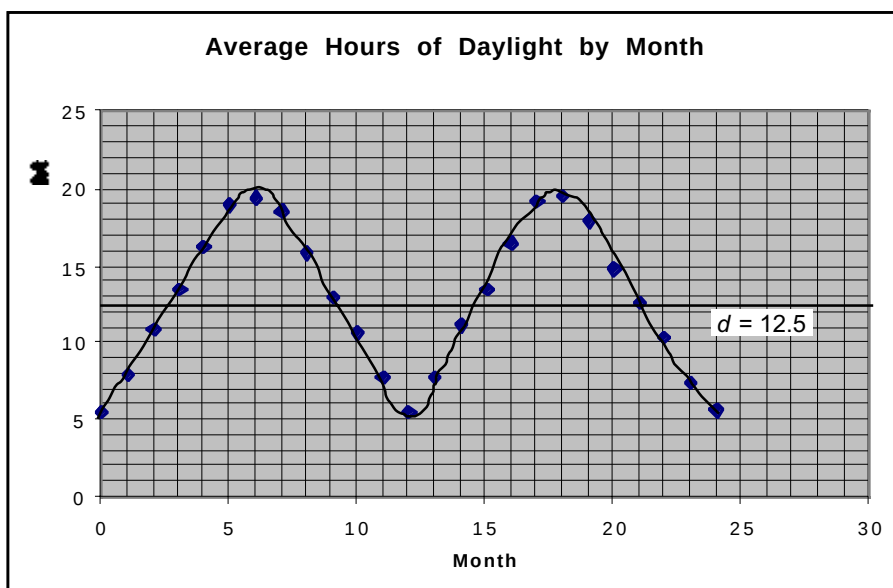
CP2. *Explain why the data is periodic. What is the period of the data?*

The data is periodic, because the pattern of increasing and then decreasing daylight of the first year repeats itself in the second year. The period is 12 months.

CP3. *State the domain and range of the data.*

The domain is {January, February, March, April, May, June, July, August, September, October, November, December}, where January = 0 and December = 11. The range is 5.5 to 19.6, measured in hours of daylight.

CP4. (a) Graph average hours of daylight versus month. Refer to the start of the data as $t = 0$.



(b) What is the equation of the axis of the curve?

The axis of the curve is $d = \frac{(19.6 + 5.5)}{2}$. Therefore, $d = 12.5$, which means that there is an average of 12 h of daylight each month.

(c) What are the maximum and minimum values?

The maximum value is 19.6. The minimum value is 5.5.

(d) What is the amplitude of the graph?

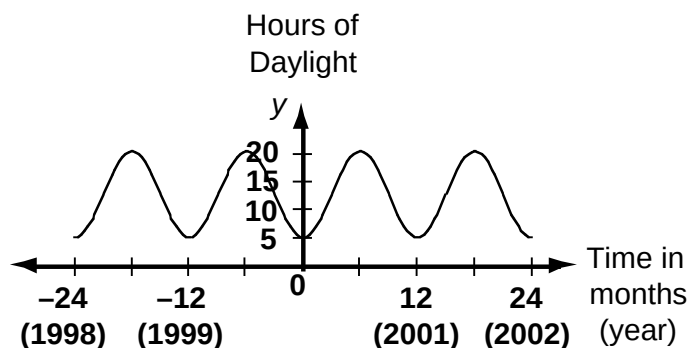
$$a = \frac{(19.6 - 5.5)}{2}$$

$$a \doteq 7$$

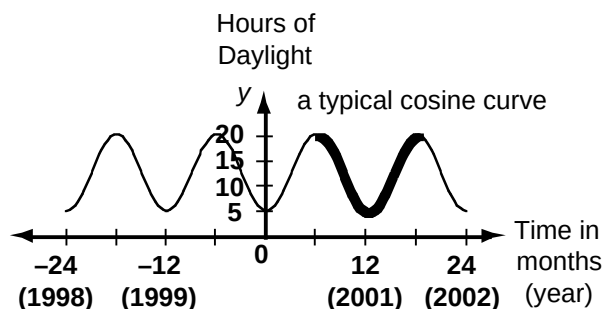
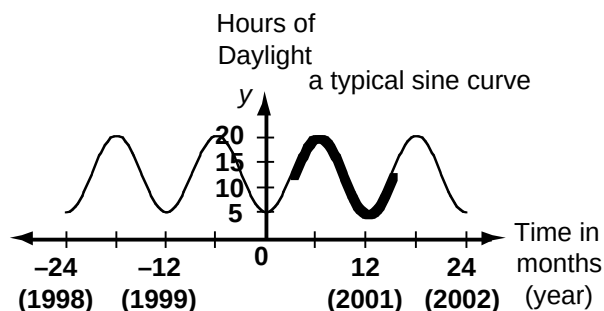
The amplitude, a , is 7, which means the maximum and minimum hours of daylight varies by about 7 h each month.

Section 5.3

CP5. Sketch the graph for all 25 months in the table.



- CP6.** Highlight one complete cycle for the graph that resembles a typical sine function. Highlight a cycle that resembles a typical cosine function.



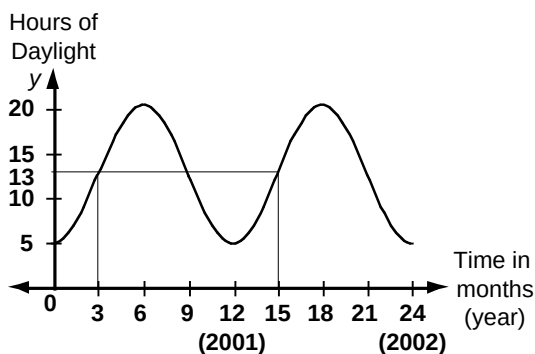
- CP7.** What type of periodic function could be used to model the relationship between time in months and average monthly hours of daylight?

Either a sine function or a cosine function could be used to model the relationship.

Section 5.4

- CP8.** Revisit your graph of the data for the two-year period in the table.

(a) Evaluate $f(3)$.



$f(3) \doteq 13$ (interpolated from the graph)

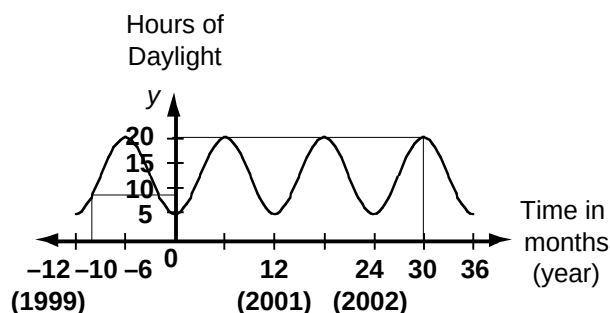
(b) Show that $f(3) = f(3 + 12)$ and explain why this should be true.

$f(3+12) \doteq f(15)$ and $f(15) \doteq 13$. At $f(3)$ and $f(3 + 12)$, there is about 13 h of daylight each month. $f(3) = f(3 + 12)$ should be true, because the average hours of daylight per month is a periodic function that repeats itself every 12 months.

(c) Explain why there are other values that are about the same as $f(3)$.

Each year, there are two times when there are about 13 hours of daylight: once as the function approaches a maximum and once again as the function moves toward a minimum. For this graph, there are four such times.

CP9. Extend the graph so that the values for $f(30)$ and $f(-10)$ can be extrapolated.



(a) Approximate the average number of daylight hours for $f(30)$.

$$f(30) \doteq 19.5$$

(b) Extend the graph to evaluate $f(-10)$.

$$f(-10) \doteq 11$$

(c) Which months correspond to $f(30)$ and $f(-10)$?

$f(30)$ corresponds to July and $f(-10)$ corresponds to March.

Section 5.6

CP10. Refer to the graph of the original data you made earlier.

(a) Explain how the axis of the graph and the concept of vertical shift are related. What does vertical shift mean in this problem?

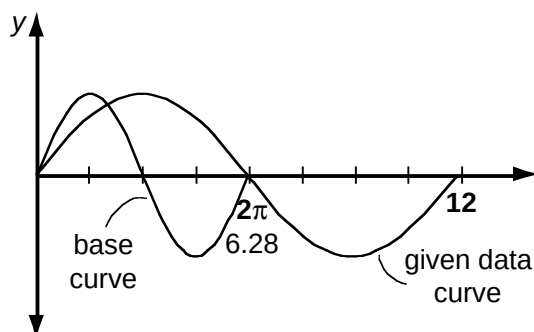
The horizontal axis of the base sinusoidal curve is $y = 0$. When a curve is vertically shifted, the axis of the curve is also shifted. Here, the vertical shift represents when half a day is daylight.

(b) What choices of starting position are possible when highlighting a cycle? What is the phase shift if a typical sine function were highlighted? if a typical cosine function were highlighted? What does phase shift mean in this problem?

A cycle can start anywhere. However, a typical sine curve would start in March, so the curve shifts right 2 units. The sine curve starts in March, which has about 12 h of daylight. For a typical cosine curve, the phase shift is right 6 units, and the curve starts at the end of June or early July. This point represents the day with the most hours of daylight.

(c) Compare the period of this data to the period of $y = \sin \theta$ or $y = \cos \theta$. How can you use this information to find the equation representing the data?

This data repeats itself every 12 months. The base sine function or the base cosine function repeats itself every 2π .



Then

$$k = \frac{2\pi}{12}$$

$$= \frac{\pi}{6}$$

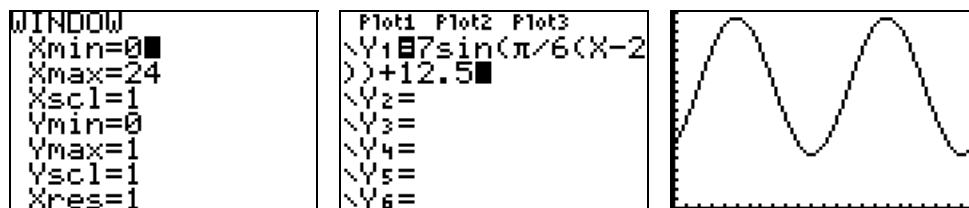
Section 5.7

CP11. Write an equation in the form $y = a \sin k(\theta + b) + d$ to model the average number of hours of daylight for the Far North community.

To write an equation in the form $y = a \sin k(\theta + b) + d$, determine a , k , b , and d .

$a = 7$, $d = 12.5$, and $k = \frac{\pi}{6}$. The curve is horizontally shifted right 2 units from the base sine curve. Then $b = -2$. Therefore, $d(t) = 7 \sin \frac{\pi}{6}(t - 2) + 12.5$.

CP12. Use a graphing calculator set to an appropriate scale to graph the equation. Confirm that the graph matches the graphs you drew in previous sections.



CP13. How many hours of daylight should be expected at month 252?

$$d(252) = 7 \sin \frac{\pi}{6}(252 - 2) + 12.5$$

$$\doteq 6.4$$

At month 252, there should be about 6.4 h of daylight.

Section 5.8

CP14. The equation $h(t) = 7 \sin \frac{\pi}{6}(t - 2) + 12.5$ is a model for the average number of hours of daylight per month, beginning at the turn of the millennium. Solve for t if $h(t) = 13$ and $0 \leq t \leq 240$.

$$13 = 7 \sin \frac{\pi}{6}(t - 2) + 12.5$$

$$7 \sin \frac{\pi}{6}(t - 2) = 13 - 12.5$$

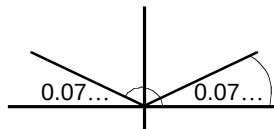
$$\sin \frac{\pi}{6}(t - 2) = \frac{0.5}{7}$$

$$\frac{\pi}{6}(t - 2) = \sin^{-1}\left(\frac{0.5}{7}\right)$$

and

$$\sin^{-1}\left(\frac{0.5}{7}\right) \doteq 0.7414894499$$

The more decimal places, the more accurate the answer for $0 \leq t \leq 240$.



For quadrant 1, $\theta \doteq 0.071\,489\,449\,9$. For quadrant 2,

$$\begin{aligned}\theta &\doteq \pi - 0.071\,489\,449\,9 \\ &= 3.070\,103\,204\end{aligned}$$

Then

$$\begin{aligned}\frac{\pi}{6}(t-2) &\doteq 0.071\,489\,449\,9 \\ t-2 &\doteq 0.136\,534\,79 \\ t &= 2.136\,534\,79\end{aligned}$$

and

$$\begin{aligned}\frac{\pi}{6}(t-2) &\doteq 3.070\,103\,204 \\ (t-2) &\doteq 5.863\,465\,21 \\ t &= 7.863\,465\,21\end{aligned}$$

The period of the curve is 12 months. Then $t = 2.136\,547\,9 + 12n$, $n \in \mathbf{I}$, $0 \leq n \leq 20$ and $t = 7.863\,465\,21 + 12n$, $n \in \mathbf{I}$, $0 \leq n \leq 20$.