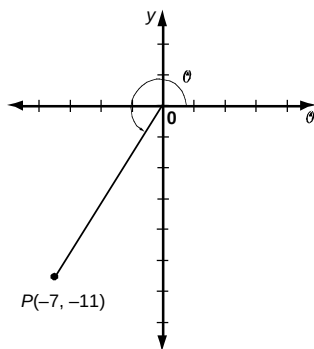


## Chapter 5 Test: Solutions

1. (a) The moon waxes to a full moon where all of the face is illuminated; and then it wanes to a new moon where none of the face is illuminated. This situation is an example of periodic phenomena, because the cycle of waxing and waning repeats itself over and over again.
- (b) As the ball bounces, its maximum height decreases with each bounce, until the ball stops bouncing. The ball may bounce repeatedly, but the cycle changes with each bounce, so this situation is not an example of periodic phenomena.
- (c) Each morning, the sun rises at a different place on the horizon, from the viewpoint of an observer who stands in the same place each day. It moves north until it reaches a maximum angle above due east and then moves south until it reaches a minimum angle below due east. This cycle repeats itself each year, so this situation is an example of periodic phenomena.

2. (a) iv. a positive rotation between  $\pi$  and  $\frac{3\pi}{2}$
- (b) vi. a positive rotation between  $\frac{\pi}{2}$  and  $\frac{3\pi}{2}$
- (c) v. a positive rotation between  $\frac{3\pi}{2}$  and  $2\pi$
- (d) ii. a negative rotation between  $-\pi$  and  $-\frac{3\pi}{2}$
- (e) iii. a negative rotation between 0 and  $-\pi$
- (f) i. a negative rotation between 0 and  $-\frac{\pi}{2}$

3.



$$\begin{aligned}\tan \theta &= \frac{-11}{-7} \\ \theta &= \tan^{-1}\left(\frac{-11}{-7}\right) \\ \theta &\doteq 57.5^\circ\end{aligned}$$

In quadrant III,  $\theta = 180^\circ + 57.5^\circ$ , or  $238^\circ$ , to the nearest degree.

$$\begin{aligned}r &= \sqrt{(-7)^2 + (-11)^2} \\ &= \sqrt{170} \\ &\doteq 13\end{aligned}$$

Then  $P(x, y) = P(13 \cos 238^\circ, 13 \sin 238^\circ)$ .

4. (a)  $\frac{-7\pi}{4} = \frac{-7(180^\circ)}{4}$   
 $= 315^\circ$

(b)

$$\frac{\text{degrees}}{180^\circ} = \frac{\text{radians}}{\pi}$$

$$\frac{150^\circ}{180^\circ} = \frac{x}{\pi}$$

$$x = \frac{150^\circ \pi}{180^\circ}$$

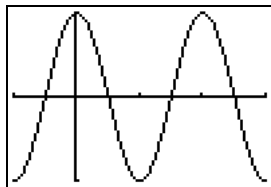
$$= \frac{5\pi}{6}$$

5. Students can either draw the graph by hand or use a graphing calculator. In either case, the graph should clearly show the domain,  $-\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$ , with intervals of  $\frac{\pi}{4}$ .

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WINDOW
Xmin=-1.570796
Xmax=4.71238898
Xscl=.78539816...
Ymin=-1
Ymax=1
Yscl=1
Xres=1

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6. (a)  $\theta = \tan^{-1} \frac{1}{\sqrt{3}}$   
 $\theta = 30^\circ$

The tangent function is positive in quadrants I and III.

Then, for  $0^\circ \leq \theta \leq 360^\circ$ , the angle in quadrant I is  $30^\circ$  and the angle in quadrant III is  $180^\circ + 30^\circ = 210^\circ$ .

$$\cos x = -0.875$$

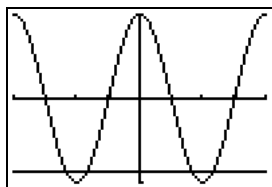
(b)  $x = \cos^{-1}(-0.875)$   
 $x \doteq 2.6$  radians

The cosine function is negative in quadrants II and III. Then, for  $-2\pi \leq x \leq 2\pi$ ,

$x = 2.6$  and  $x = -2.6$ . The related acute angles are

$$\pi + (\pi - 2.6) = 3.7 \text{ and } -\pi - (\pi - 2.6) = -3.7$$

Confirm by graphing  $y = \cos x$  and  $y = -0.875$ .



(c)

$$7 = 8 + 4 \sin(2\theta)$$

$$4 \sin(2\theta) = -1$$

$$\sin 2\theta = -\frac{1}{4}$$

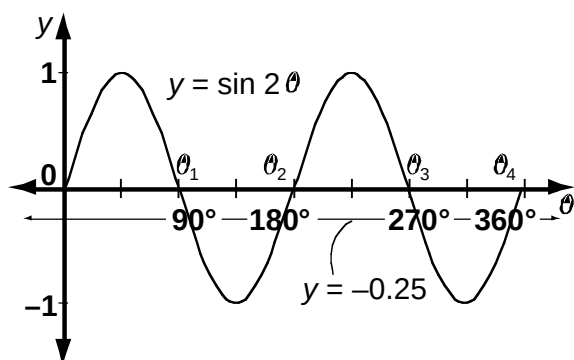
$$2\theta \doteq \sin^{-1}\left(-\frac{1}{4}\right)$$

$$2 \doteq -14.477\,512\,19^\circ$$

$$\theta = -7.2^\circ$$

This angle is not in the given domain. Use the sketch of  $y = \sin 2\theta$  and  $y = -\frac{1}{4}$ ,

$\theta = -7.2^\circ$ , the domain  $0^\circ \leq \theta \leq 360^\circ$ , the symmetries of the curve, and the period of  $180^\circ$  to determine the four values of  $\theta$ .



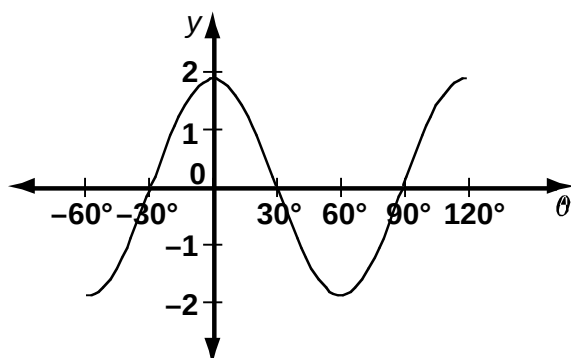
Then  $\theta = 90^\circ + 7.2^\circ$  or  $97.2^\circ$ ,

$\theta = 180^\circ - 7.2^\circ$  or  $172.8^\circ$ ,

$\theta = 97.2^\circ + 180^\circ$  or  $277.2^\circ$

$\theta = 172.8^\circ + 180^\circ$  or  $352.8^\circ$

7. Graph and join the points. The data is sinusoidal.



Use the general form of the cosine function,  $y = a \cos k(\theta + b) + d$ , for  $-60^\circ \leq \theta \leq 120^\circ$ .

The axis of the curve is  $y = 0$ . Then  $d = 0$ .

The amplitude,  $a$ , is 2 and the cosine curve is not reflected about  $y = 0$ . The phase shift,  $b$ , is 0. The period is  $120^\circ$ . Then  $120^\circ k = 360^\circ$  or  $k = 3$ . So the equation is  $y = 2 \cos 3\theta$ ,  $-60^\circ \leq \theta \leq 120^\circ$ .

8. (a) The maximum temperature occurs when  $\sin\left(\frac{\pi t}{6} + 4.2\right) = 1$ .

Then  $19.1(1) + 7.5 = 26.6$ . The maximum temperature is  $26.6^\circ\text{C}$ .

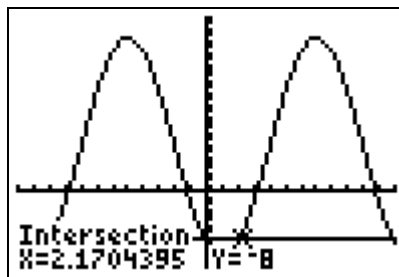
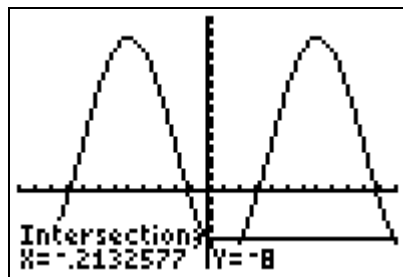
- (b)  $\frac{\pi t}{6} = 2\pi$  or  $t = 12$ . The period is 12 months or one year.

- (c) Factor  $\frac{\pi}{6}$  from  $\frac{\pi t}{6} + 4.2$  to get  $\frac{\pi}{6}\left[t + 4.2\left(\frac{6}{\pi}\right)\right]$ . The phase shift is  $\frac{25.2}{\pi}$ , or the function has been translated about eight months to the left of the base curve.

- (d) The graph of the base curve has been horizontally stretched by a factor of  $\frac{6}{\pi}$ , horizontally shifted to the left by  $\frac{25.2}{\pi}$ , or about 8, units (months), vertically expanded by a factor of 19.1, and vertically translated up by 7.5 units (months).

- (e) Solve the equation  $-8 = 19.1 \sin\left(\frac{\pi t}{6} + 4.2\right) + 7.5$  for  $t$  by graphing

$y = 19.1 \sin\left(\frac{\pi t}{6} + 4.2\right) + 7.5$  and  $y = -8$ . Use the intersect operation from the CALCULATE menu of the graphing calculator to locate the intersection points.



For  $t = 0$ , the month is January. Then the temperature is  $-8^\circ\text{C}$  in late December ( $t = -1$ ) and again in early March ( $t = 2$ ).

#### Alternative Solution:

$$\begin{aligned} -8 &= 19.1 \sin\left(\frac{\pi t}{6} + 4.2\right) + 7.5 \\ \frac{-15.5}{19.1} &= \sin\left(\frac{\pi t}{6} + 4.2\right) \\ \sin^{-1}\left(\frac{-15.5}{19.1}\right) &= \left(\frac{\pi t}{6} + 4.2\right) \\ \sin^{-1}\left(\frac{-15.5}{19.1}\right) &\doteq -0.9467 \end{aligned}$$

The sine function is negative in quadrants III and IV. Then the angle in quadrant III is  $\pi + 0.9467 = 4.088$  and the (positive) angle in quadrant IV is  $2\pi - 0.9467 = 5.336$ , to three decimal places.

$$\text{Then } \left(\frac{\pi t}{6} + 4.2\right) = 4.088 \text{ and}$$

$$\left(\frac{\pi t}{6} + 4.2\right) = 5.336$$

Solve these equations to get  $t = -0.21$  and  $t = 2.17$ , to two decimal places, which means late December and early March, respectively.