
The Chapter Problem—What Time Is It? Answers

By the end of this chapter, students should be able to provide comprehensive, well-explained solutions to the problem.

The formula for the distance, x , that the pendulum swings from the centre point, or rest position,

is $x = M \cos\left(t\sqrt{\frac{980}{l}}\right)$, where

l is the length of the pendulum,

M is the maximum value of x , and

t is the time, in seconds, since the pendulum was set in motion.

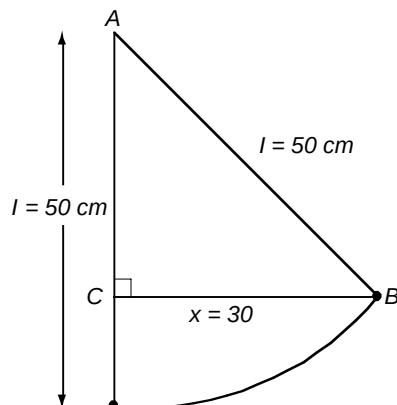
The period of a pendulum is the time the pendulum takes to return to its point of release. The period represents two swings, one to the right and one to the left.

In this problem, the pendulum (as in a clock) moves continuously, because of the yoke, as described in the original problem on page 494. The formula uses radian measure.

Section 6.1

CPI. Suppose a 50-cm long pendulum is pulled back 30 cm from the rest position. Determine the angle that the pendulum passes through on the first swing. That is, find the angle from its starting point to the point where it changes direction.

$l = 50$ cm and $x = 30$ cm



$$\sin A = \frac{30}{50}$$

$$\sin A = 0.6$$

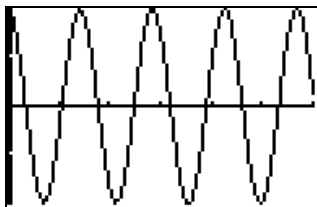
$$\angle A \doteq 37^\circ$$

$$2\angle A = 74^\circ$$

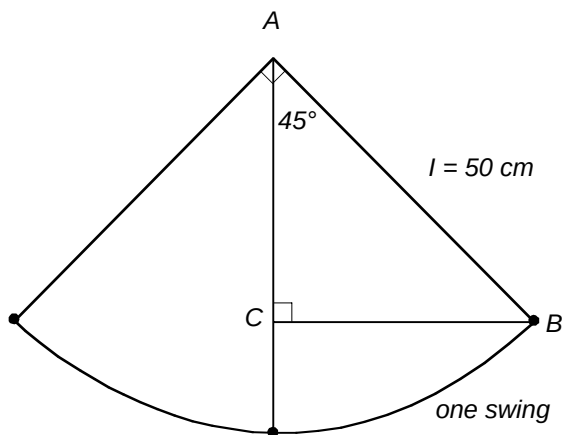
The angle from the starting point to the point where it changes direction is about 37° .

The angle for one complete swing of the pendulum is 74° .

(Note: Here is a graph of $x = 30 \cos\left(t\sqrt{\frac{980}{50}}\right)$ with domain 0 s to 6 s.)



- CP2.** How far must the pendulum be pulled back to pass through
(a) a right angle?



For a complete swing through 90° , use $\angle A = 45^\circ$, $l = 50$ cm and $x = ?$

$$\sin 45^\circ = \frac{x}{50}$$

$$x = 50 \sin 45^\circ$$

$$x \doteq 35$$

Pull the pendulum back about 35 cm for the pendulum to pass through 90° .

- (b)** an angle of 120° ?

For a complete swing through 120° , use $\angle A = 60^\circ$, $l = 50$ cm and $x = ?$

$$\sin 60^\circ = \frac{x}{50}$$

$$x = 50 \sin 60^\circ$$

$$x \doteq 43$$

Pull the pendulum back about 43 cm for the pendulum to pass through 120° .

CP3. How long will the pendulum take to complete the swing? one period?

(Refer to section 5.6.)

Compare $x = M \cos\left(t\sqrt{\frac{980}{l}}\right)$ to $y = a \cos k\theta$, whose period is $\frac{2\pi}{k}$.

The period is $\frac{2\pi}{\sqrt{\frac{980}{l}}}$ or $2\pi \div \sqrt{\frac{980}{l}}$ for any length of this pendulum.

At the moment the pendulum is released, $M = 30$ cm and $l = 50$ cm.

$$x = 30 \cos\left(t\sqrt{\frac{980}{50}}\right)$$

$$x = 30 \cos(t\sqrt{19.6})$$

The period is

$$\begin{aligned} 2\pi \div k &= 2\pi \div \sqrt{19.6} \\ &= 1.419\,226\,895 \end{aligned}$$

One period equals two swings, so one swing is 0.5 periods.

$$1.4 \times 0.5 = 0.7$$

This pendulum takes about 0.7 s to complete one swing and about 1.4 s to complete one period.

Section 6.2

CP4. Assume the pendulum is 80 cm long. How far must it be pulled back from the centre or rest position (horizontal distance), to swing through an angle of $\frac{5\pi}{8}$?

This solution is similar to the solutions for questions CP2 (a) and CP2 (b).

For a complete swing through $\frac{5\pi}{8}$, use $\angle A = \frac{5\pi}{16}$. $l = 80$ cm and $x = ?$, and set the calculator's mode to radians.

$$\sin \frac{5\pi}{16} = \frac{x}{80}$$

$$x = 80 \sin \frac{5\pi}{16}$$

$$x = 66.517\,568\,98$$

Store this value in the calculator's memory for later use.

Pull the pendulum back about 66.5 cm from the centre so that the pendulum will swing through an angle of $\frac{5\pi}{8}$.

CP5. How long will this pendulum take to complete one swing? one period?

(The solution to this question is similar to the solution for question CP3.)

At the moment the pendulum is released, $M = 66.517\ 568\ 98$ cm and $l = 80$ cm.

$$x = M \cos\left(t \sqrt{\frac{980}{l}}\right)$$

$$x = 66.517\ 568\ 98 \cos\left(t \sqrt{\frac{980}{80}}\right)$$

The period is

$$\begin{aligned} 2\pi \div k &= 2\pi \div \sqrt{\frac{980}{80}} \\ &= 1.795\ 195\ 802 \end{aligned}$$

One period equals two swings, so one swing is 0.5 periods.

$$1.8 \times 0.5 = 0.9$$

This pendulum takes about 0.9 s to complete one swing and about 1.8 s to complete one period.

CP6. Does the pendulum need to be longer or shorter than 80 cm for one swing to occur in 1 s? Explain your reasoning.

Solution for the Functions and Relations course:

Consider the solution for question CP5.

The period is

$$\begin{aligned} 2\pi \div k &= 2\pi \div \sqrt{\frac{980}{80}} \\ &= 1.795\ 195\ 802 \end{aligned}$$

This pendulum takes about 1.8 s to complete one period and about 0.9 s to complete one swing.

For one swing to occur in 1 s, the time of the period, 1.8 s, must increase to about

2 s. Then the denominator in $\sqrt{\frac{980}{80}}$, which represents the length of the pendulum, must also increase to about 99.45 cm.

Solution for the Functions course:

Choose a pendulum that is shorter than 80 cm and choose another pendulum that is longer than 80 cm, so $l = 70$ cm and $l = 90$ cm, respectively.

For $l = 70$ cm, the period is

$$2\pi \div \sqrt{\frac{980}{70}} = 1.679\ 251\ 908$$

The period is about 1.7 s and one swing takes 0.5×1.7 , or 0.85 s.

For $l = 90$ cm, the period is

$$2\pi \div \sqrt{\frac{980}{90}} = 1.904\ 092\ 688$$

The period is about 1.9 s and one swing takes 0.5×1.9 , or 0.95 s.

Since the longer pendulum (90 cm) takes more time to complete one swing, the pendulum must be longer than 80 cm for one swing to take 1 s.

Section 6.6

CP7. A 70-cm long pendulum is released at a distance of 8 cm from rest.

(a) Determine the period and the time of one complete swing.

$$M = 8 \text{ cm and } l = 70 \text{ cm}$$

$$x = M \cos\left(t \sqrt{\frac{980}{l}}\right)$$

$$x = 8 \cos\left(t \sqrt{\frac{980}{70}}\right)$$

For $l = 70$ cm, the period is

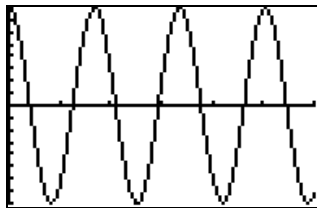
$$2\pi \div \sqrt{\frac{980}{70}} = 1.679\ 251\ 908$$

The period is about 1.7 s and one swing takes 0.5×1.7 , or 0.85 s.

(b) Graph the function that models this situation. Explain the meaning of positive and negative values of the dependent variable, x .

Use graphing technology to draw the graph of $x = 8 \cos\left(t \sqrt{\frac{980}{70}}\right)$.

The values of x are positive when the pendulum swings right of the rest position. The values of x are negative when the pendulum swings left of the rest position.



(c) Determine the pendulum's distance from the rest position after 2 s.

$t = 2$ s, $M = 8$ cm, and $l = 70$ cm. Set the calculator's mode to radians.

$$x = 8 \cos \left(t \sqrt{\frac{980}{70}} \right)$$

$$x = 8 \cos \left(2 \sqrt{\frac{980}{70}} \right)$$

$$x = 2.897\,896\,67$$

After 2 s, the pendulum is about 2.9 cm from the rest position. (During 2 s, the pendulum swings left, right, and then swings left again.)

(d) Determine the four possible values for t when the pendulum is 6 cm from the centre point.

Use graphing technology. An alternative algebraic solution follows. $M = 8$ cm, $l = 70$ cm, and $x = 6$ cm or $x = -6$ cm.

For $x = 6$,

$$x = 8 \cos \left(t \sqrt{\frac{980}{70}} \right)$$

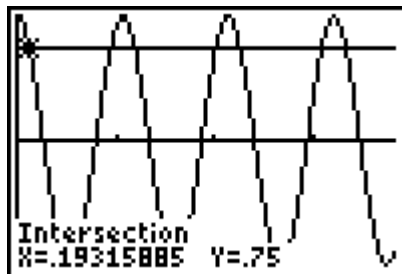
$$6 = 8 \cos \left(t \sqrt{\frac{980}{70}} \right)$$

$$\frac{6}{8} = \cos \left(t \sqrt{\frac{980}{70}} \right)$$

$$0.75 = \cos \left(t \sqrt{\frac{980}{70}} \right)$$

Graph $y = 0.75$ and $y = \cos \left(t \sqrt{\frac{980}{70}} \right)$ and determine the points of intersection.

At $t \doteq 0.193$ s, the pendulum is right of the centre point, moving left, and at $t \doteq 1.486$ s, the pendulum is left of the centre point, moving right.



Alternatively, for $x = 6$, graph $y = 6$ and $y = 8 \cos \left(t \sqrt{\frac{980}{70}} \right)$ and determine the points of intersection.

For $x = -6$,

$$x = 8 \cos \left(t \sqrt{\frac{980}{70}} \right)$$

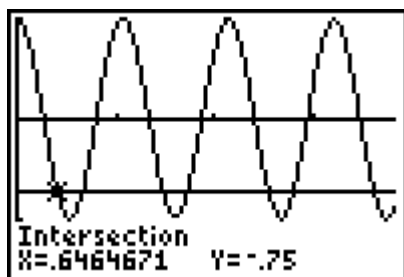
$$-6 = 8 \cos \left(t \sqrt{\frac{980}{70}} \right)$$

$$-\frac{6}{8} = \cos \left(t \sqrt{\frac{980}{70}} \right)$$

$$-0.75 = \cos \left(t \sqrt{\frac{980}{70}} \right)$$

Graph $y = -0.75$ and $y = \cos \left(t \sqrt{\frac{980}{70}} \right)$ and determine the points of intersection.

At $t \doteq 0.646$ s, the pendulum is left of the centre point, moving right, and at $t \doteq 1.033$ s, the pendulum is left of the centre point, moving right.



Alternatively, for $x = -6$, graph $y = -6$ and $y = 8 \cos \left(t \sqrt{\frac{980}{70}} \right)$ and determine the points of intersection.

At about 0.193 s and 1.486 s, the pendulum is 6 cm right from the centre point. At about 0.646 s and 1.033 s, the pendulum is 6 cm left from the centre point. (Find more t -values by adding the period, which is about 1.679 s, to these four values.)

Alternative Solution:

$M = 8$ cm, $l = 70$ cm, and $x = 6$ cm or $x = -6$ cm. For $x = 6$,

$$6 = 8 \cos \left(t \sqrt{\frac{980}{70}} \right)$$

$$\frac{6}{8} = \cos \left(t \sqrt{\frac{980}{70}} \right)$$

$$0.75 = \cos \left(t \sqrt{\frac{980}{70}} \right)$$

$$0.75 = \cos(t\sqrt{14})$$

The cosine function is positive in quadrants I and IV. $\cos^{-1}(0.75) = 0.722\ 734\ 247\ 8$

$t\sqrt{14} \doteq 0.723$ radians in quadrant I, and $t\sqrt{14} \doteq 0.723 + \frac{3\pi}{2}$ or about 5.435 radians in

quadrant IV. Then $t \doteq 0.2$ s and $t \doteq 1.5$ s.

For $x = -6$, $-0.75 = \cos(t\sqrt{14})$. The cosine function is negative in quadrants II and III.

$$\cos^{-1}(-0.75) = 2.418\,858\,406$$

$t\sqrt{14} \doteq 2.419$ radians in quadrant II, and $t\sqrt{14} \doteq 2.419 + \frac{\pi}{2}$ or about 4 radians in quadrant

III. Then $t \doteq 0.646$ s and $t \doteq 1.07$ s.

At about 0.193 s and 1.486 s, the pendulum is 6 cm right from the centre point. At about 0.646 s and 1.07 s, the pendulum is 6 cm left from the centre point.

CP8. Use the model $x = M \cos\left(t\sqrt{\frac{980}{l}}\right)$ to determine an expression that represents the period of the function when $l = 70$ cm.

The period is independent of the value of M and is $2\pi \div \sqrt{\frac{980}{l}}$. For $l = 70$ cm,

$$\begin{aligned} 2\pi \div \sqrt{\frac{980}{70}} &= 2\pi \div \sqrt{14} \\ &= 1.679\,251\,908 \end{aligned}$$

(The period of the function is about 1.7 s.)

CP9. How long does the pendulum in a grandfather clock take to complete one full period? (A period is two complete swings.)

In any grandfather clock, 1 swing takes 1 s. Two complete swings (to the left and back to the right) takes 2 s. Therefore, a complete period takes 2 s.

CP10. Use your answers from CP8 and CP9 to create an equation to determine the length of the pendulum in a grandfather clock. Solve the equation.

A complete period is 2 s (from CP9). The period is also $2\pi \div \sqrt{\frac{980}{l}}$ (from CP8 or CP3).

So

$$2 = 2\pi \div \sqrt{\frac{980}{l}}$$

$$2\sqrt{\frac{980}{l}} = 2\pi$$

$$\sqrt{\frac{980}{l}} = \pi$$

$$\left(\sqrt{\frac{980}{l}}\right)^2 = \pi^2$$

$$\frac{980}{l} = \pi^2$$

$$980 = \pi^2 l$$

$$\frac{980}{\pi^2} = l$$

$$l = 99.294\ 759\ 97$$

Therefore, in any grandfather clock, the pendulum must be about 99.3 cm long.