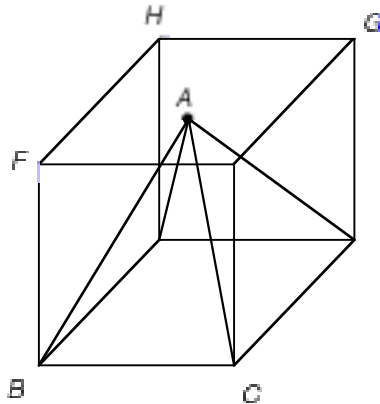


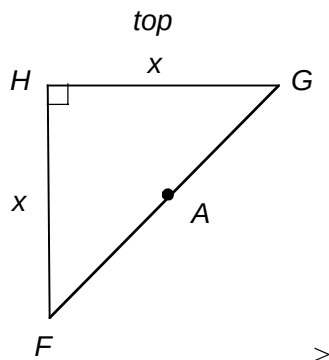
Challenge 1: Solution Guide

Management Suggestion

Students may be more comfortable expressing the side length of the cube as a number, rather than as a variable. A unit cube, where each edge is 1 unit long, is conventionally used, but using the unit cube here would lead to fractions. A better side length is 2 units.



- (a) Each edge of the cube is x units long. To calculate angles in $\triangle ABC$, find the lengths of the sides.



In $\triangle FHG$, $\angle H = 90^\circ$ and $FH = HG = x$.

By the Pythagorean theorem,

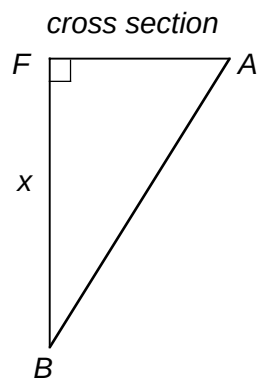
$$FG^2 = x^2 + x^2$$

$$FG^2 = 2x^2$$

$$FG = \sqrt{2}x$$

$$FA = \frac{1}{2}FG. \text{ So } FA = \frac{\sqrt{2}x}{2}.$$

In $\triangle ABF$, $\angle AFB = 90^\circ$ and $FB = x$.



By the Pythagorean theorem,

$$AB^2 = x^2 + \left(\frac{\sqrt{2}x}{2}\right)^2$$

$$AB^2 = x^2 + \frac{2x^2}{4}$$

$$AB^2 = \frac{3x^2}{2}$$

$$AB = \sqrt{\frac{3}{2}}x$$

In $\triangle ABC$, $AB = AC = \sqrt{\frac{3}{2}}x$ and $BC = x$. Now apply the cosine law.

$$BC^2 = AB^2 + AC^2 - 2(AB)(AC) \cos A$$

$$x^2 = \left(\sqrt{\frac{3}{2}}x\right)^2 + \left(\sqrt{\frac{3}{2}}x\right)^2 - 2\left(\sqrt{\frac{3}{2}}x\right)\left(\sqrt{\frac{3}{2}}x\right) \cos A$$

$$x^2 = \frac{3}{2}x^2 + \frac{3}{2}x^2 - 3x^2 \cos A$$

$$x^2 = 3x^2 - 3x^2 \cos A$$

$$-2x^2 = -3x^2 \cos A$$

$$\frac{2}{3} = \cos A$$

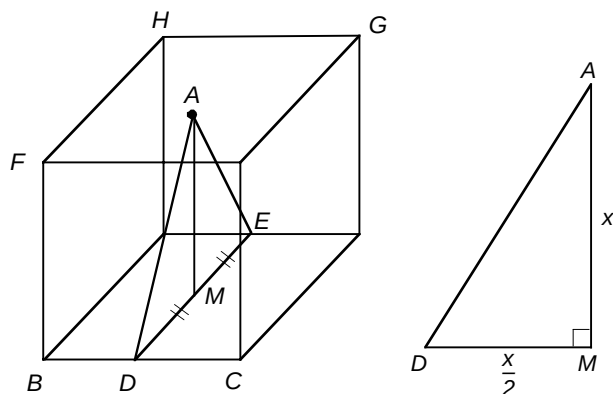
$$\angle A = \cos^{-1}\left(\frac{2}{3}\right)$$

$$\angle A = 48.189\,685\,1^\circ$$

Because $\triangle ABC$ is isosceles and $\angle BAC \doteq 48^\circ$, then $\angle ABC$ and $\angle ACB$ each equal $66^\circ \left(\frac{180^\circ - 48^\circ}{2}\right)$.

(b) We could calculate AD , but it is better to draw a perpendicular from A to M , the midpoint of DE .

Then we know that $FB = AM = x$ and $DM = \frac{1}{2}DE = \frac{x}{2}$.



In $\triangle ADM$,

$$\tan D = x \div \frac{x}{2}$$

$$\tan D = 2$$

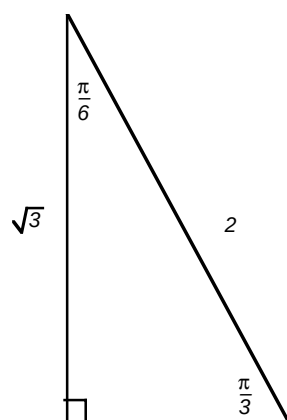
$$\angle D = \tan^{-1}(2)$$

$$\angle D = 63.434\ 948\ 82$$

Each triangular face of the pyramid meets the base at angle of about 63° .

(c) In $\triangle ADE$, $\angle D \doteq 63^\circ$. Then $\angle DAE = 180^\circ - 2(63^\circ)$ or 53° . The angle between opposite triangular faces is 53° .

Challenge 2: Solution Guide



- (a) Substitute $x = \frac{\pi}{3}$ and $y = \frac{\pi}{6}$ in each equation to prove the identity.

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

L.S.	R.S.
$\sin(x + y)$	$\sin x \cos y + \cos x \sin y$
$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$	$= \sin \frac{\pi}{3} \cos \frac{\pi}{6} + \cos \frac{\pi}{3} \sin \frac{\pi}{6}$
$= \sin \frac{\pi}{2}$	$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$
$= 1$	$= \frac{3}{4} + \frac{1}{4}$
	$= 1$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

L.S.	R.S.
$\cos(x + y)$	$\cos x \cos y - \sin x \sin y$
$= \cos\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$	$= \cos \frac{\pi}{3} \cos \frac{\pi}{6} - \sin \frac{\pi}{3} \sin \frac{\pi}{6}$
$= \cos \frac{\pi}{2}$	$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$
$= 0$	$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$
	$= 0$

Therefore, these relationships are true when $x = \frac{\pi}{3}$ and $y = \frac{\pi}{6}$.

(b) To develop an equivalent expression for $\sin 2x$, let $y = x$.

L.S.	R.S.
$\sin (x + y)$	$\sin x \cos y + \cos x \sin y$
$= \sin (x + x)$	$= \sin x \cos x + \cos x \sin x$
$= \sin 2x$	$= 2 \sin x \cos x$

Therefore, $\sin 2x = 2 \sin x \cos x$. To develop an equivalent expression for $\cos 2x$, let $y = x$.

L.S.	R.S.
$\cos (x + y)$	$\cos x \cos y - \sin x \sin y$
$= \cos (x + x)$	$= \cos x \cos x - \sin x \sin x$
$= \cos 2x$	$= \cos^2 x - \sin^2 x$
	$= (1 - \sin^2 x) - \sin^2 x$
	$= 1 - 2 \sin^2 x$

Therefore, $\cos 2x = \cos^2 x - \sin^2 x$ or $\cos 2x = 1 - 2 \sin^2 x$. Also, $\cos 2x = 2 \cos^2 x - 1$, if, instead, you had substituted the other form of the Pythagorean identity $1 - \cos^2 x = \sin^2 x$.

(c) Eliminate the ratios involving $2x$ by using the results from (b) and look for different forms of the Pythagorean identity.

L.S.	R.S.
$\frac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x}$	$\tan x$
$= \frac{1 - (\cos^2 x - \sin^2 x) + 2 \sin x \cos x}{1 + (\cos^2 x - \sin^2 x) + 2 \sin x \cos x}$	$= \frac{\sin x}{\cos x}$
$= \frac{1 - \cos^2 x + \sin^2 x + 2 \sin x \cos x}{1 + \cos^2 x - \sin^2 x + 2 \sin x \cos x}$	
$= \frac{\sin^2 x + \sin^2 x + 2 \sin x \cos x}{\cos^2 x + \cos^2 x + 2 \sin x \cos x}$	
$= \frac{2 \sin^2 x + 2 \sin x \cos x}{2 \cos^2 x + 2 \sin x \cos x}$	
$= \frac{2 \sin x (\sin x + \cos x)}{2 \cos x (\cos x + \sin x)}$	
$= \frac{\sin x}{\cos x}$	

Therefore, $\frac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x} = \tan x$.