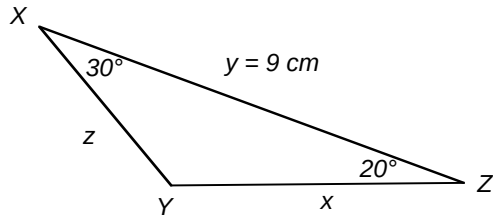


Chapter 6 Test: Solutions

1. (a) In $\triangle XYZ$, $\angle X = 30^\circ$, $\angle Z = 20^\circ$, and $y = 9$ cm.



$$\angle Y = 130^\circ (180^\circ - 30^\circ - 20^\circ)$$

$$\frac{x}{\sin x} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{x}{\sin 30^\circ} = \frac{9}{\sin 130^\circ} = \frac{z}{\sin 20^\circ}$$

$$x = \frac{9 \sin 30^\circ}{\sin 130^\circ}$$

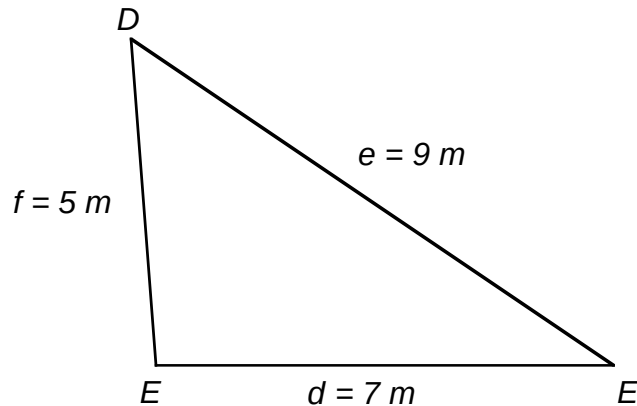
$$x = 5.874\,332\,802$$

$$z = \frac{9 \sin 20^\circ}{\sin 130^\circ}$$

$$z = 4.018\,280\,294$$

Therefore, $\angle Y = 130^\circ$, $x \doteq 6$ cm, and $z \doteq 4$ cm.

- (b) In $\triangle DEF$, $d = 7$ m, $e = 9$ m, and $f = 5$ m.



$$7^2 = 9^2 + 5^2 - 2(9)(5) \cos D$$

$$49 = 81 + 25 - 90 \cos D$$

$$-57 = -90 \cos D$$

$$0.633\,333\,333\,3 = \cos D$$

$$\angle D = \cos^{-1}(0.633\,333\,333\,3)$$

$$\angle D = 50.703\,519\,76^\circ$$

$$\frac{\sin E}{e} = \frac{\sin D}{d}$$

$$\frac{\sin E}{9} = \frac{\sin 50.70351976^\circ}{7}$$

$$\sin E = \frac{9 \sin 50.70351976^\circ}{7}$$

$$\sin E = 0.994\,987\,437\,1$$

$$\angle E = \sin^{-1}(0.994\,987\,437\,1)$$

$$\angle E = 84.260\,829\,52^\circ$$

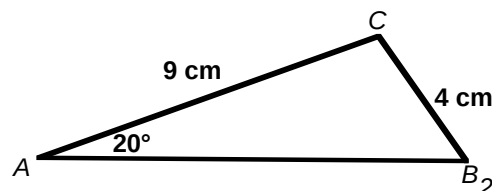
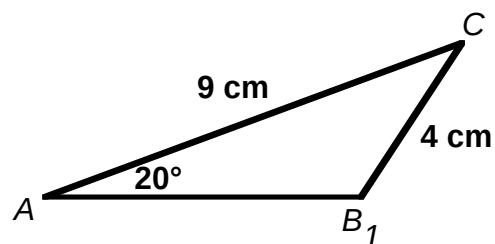
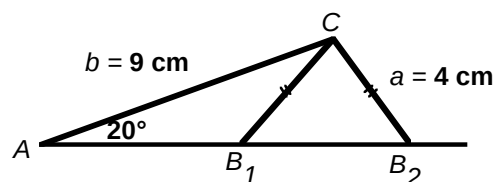
$$\begin{aligned}\angle F &= 180^\circ - 50.703\,519\,76^\circ - 84.260\,829\,52^\circ \\ &= 45.035\,650\,72^\circ\end{aligned}$$

Therefore, $\angle D \doteq 50.7^\circ$, $\angle E \doteq 84.3^\circ$, and $\angle F \doteq 45^\circ$.

2. In $\triangle ABC$, $\angle A = 20^\circ$, $a = 4$ cm, and $b = 9$ cm.

$$\begin{aligned}b \sin A &= 9 \sin 20^\circ \\ &= 3.078\,181\,29\end{aligned}$$

Therefore, $b \sin A < a < b$, and there are two triangles to consider.



For obtuse $\triangle AB_1C$, use the sine law to calculate $\angle B_1$. Find $\angle C$ and determine c using the sine law.

For acute $\triangle AB_2C$, $\angle B_2$ is the supplement of $\angle B_1$. Recalculate $\angle C$ and again determine c using the sine law.

$$\begin{aligned}3. \quad (\text{a}) \quad \sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{5\pi}{6} \sin \frac{\pi}{4} &= \frac{\sqrt{3}}{2} \left(\frac{1}{\sqrt{2}} \right) - \left(-\cos \frac{\pi}{6} \right) \left(\frac{1}{\sqrt{2}} \right) \\ &= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{\sqrt{3}}{2} \left(\frac{1}{\sqrt{2}} \right) \\ &= \frac{2\sqrt{3}}{2\sqrt{2}} \\ &= \frac{\sqrt{3}}{\sqrt{2}}\end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \cos 60^\circ \cos 210^\circ - \sin 45^\circ \sin 135^\circ &= \frac{1}{2}(-\cos 30^\circ) - \frac{1}{\sqrt{2}}(\sin 45^\circ) \\
 &= \frac{1}{2}\left(\frac{-\sqrt{3}}{2}\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) \\
 &= \left(\frac{-\sqrt{3}}{4}\right) - \frac{1}{2} \\
 &= \frac{-\sqrt{3}-2}{4}
 \end{aligned}$$

4. (a) $2 \sin^2 \theta - 1 = 0, 0 \leq \theta \leq 2\pi$

$$\sin^2 \theta = \frac{1}{2}$$

$$\sin \theta = \pm \frac{1}{\sqrt{2}}$$

$$\sin \theta = +\frac{1}{\sqrt{2}} \text{ in quadrants I and II. } y = 1 \text{ and } r = \sqrt{2}. \sin \theta = -\frac{1}{\sqrt{2}} \text{ in quadrants III and IV.}$$

$$y = -1 \text{ and } r = \sqrt{2}. \text{ These are sides in the right isosceles special triangle. So } \theta = \frac{\pi}{4}, \theta = \frac{3\pi}{4},$$

$$\theta = \frac{5\pi}{4}, \text{ and } \theta = \frac{7\pi}{4} \text{ for } 0 \leq \theta \leq 2\pi.$$

(b) $2 \tan^2 \alpha - \tan \alpha - 3 = 0, 0 \leq \alpha \leq 360^\circ$

$$2 \tan^2 \alpha - \tan \alpha - 3 = 0$$

$$(2 \tan \alpha - 3)(\tan \alpha + 1) = 0$$

$$2 \tan \alpha - 3 = 0$$

$$\tan \alpha + 1 = 0$$

$$\tan \alpha = \frac{3}{2}$$

$$\tan \alpha = -1$$

The tangent function is positive in quadrants I and III.

$$\alpha = \tan^{-1}\left(\frac{3}{2}\right)$$

$$\text{and } \alpha = 180^\circ + 56.309\,932\,47^\circ$$

$$\alpha = 56.309\,932\,47^\circ$$

$$\alpha = 236.309\,932\,47^\circ$$

The tangent function is negative in quadrants II and IV.

$$\alpha = \tan^{-1}(-1) \quad \text{and} \quad \alpha = 315^\circ - 180^\circ$$

$$\alpha = -45^\circ \text{ or } 315^\circ$$

$$\alpha = 135^\circ$$

So $\alpha \doteq 56.3^\circ$, $\alpha \doteq 236.3^\circ$, $\alpha = 315^\circ$, and $\alpha = 135^\circ$.

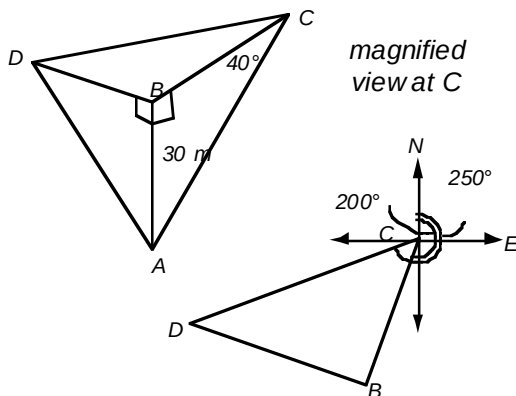
5. (a)

L.S.	R.S.
$\frac{\tan \beta}{\sin \beta}$	$\frac{\cos \beta}{1 - \sin^2 \beta}$
$= \frac{\sin \beta}{\cos \beta} \div \sin \beta$	$= \frac{\cos \beta}{\cos^2 \beta}$
$= \frac{1}{\cos \beta}$	$= \frac{1}{\cos \beta}$

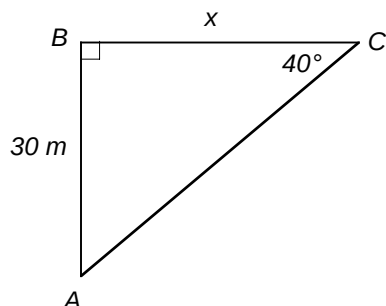
(b)

L.S.	R.S.
$\cos^4 \alpha - \sin^4 \alpha$	$1 - 2 \sin^2 \alpha$
$= (\cos^2 \alpha - \sin^2 \alpha) (\cos^2 \alpha + \sin^2 \alpha)$	
$= (\cos^2 \alpha - \sin^2 \alpha) (1)$	
$= 1 - \sin^2 \alpha - \sin^2 \alpha$	
$= 1 - 2 \sin^2 \alpha$	

6.

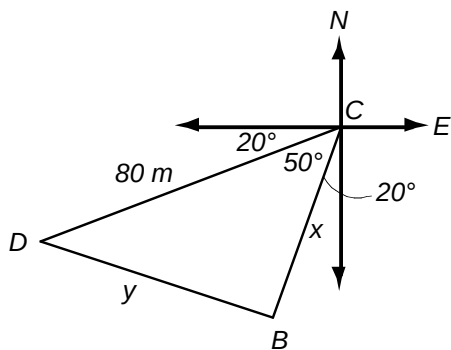


The distance from Ann to Dan is AD and the distance from the charter boat to Dan, CD , is 80 m. In vertical $\triangle ABC$, $BA = 30$ m, $\angle C = 40^\circ$, and $\angle B = 90^\circ$.



$$\begin{aligned} \tan 40^\circ &= \frac{30}{BC} \\ BC &= \frac{30}{\tan 40^\circ} \\ &= 35.752\ 607\ 78 \end{aligned}$$

In horizontal $\triangle BCD$, $DC = 80$ m, and $\angle DCB = 250^\circ - 200^\circ$ or 50° .



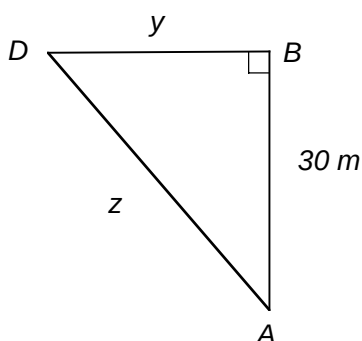
$$BD^2 = 80^2 + 35.752\ 607\ 78^2 - 2(80)(35.752\ 607\ 78) \cos 50^\circ$$

$$BD^2 = 6400 + 1278.248\ 963 - 5720.417\ 245 \cos 50^\circ$$

$$BD^2 = 4001.235\ 636$$

$$BD = 63.255\ 321\ 01$$

In vertical $\triangle DBA$, $AB = 30$ m and $\angle B = 90^\circ$.



$$AD^2 = DB^2 + 30^2$$

$$AD^2 = 63.255\ 321\ 01^2 + 900$$

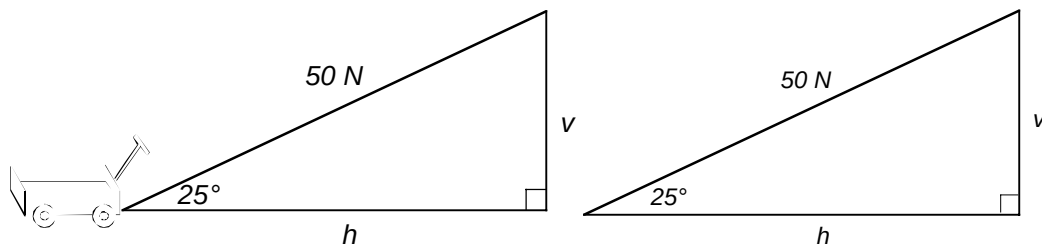
$$AD^2 = 4001.235\ 636 + 900$$

$$AD^2 = 4901.235\ 636$$

$$AD = 70.008\ 825\ 42$$

Compare the distance from Ann to Dan, AD , to the distance from the boat to Dan, CD . Therefore, Ann is farther away from Dan by about 10 m.

7.



h is the horizontal force pulling the wagon forward; v is the vertical force tending to lift the wagon.

$$(a) \cos 25^\circ = \frac{h}{50}$$

$$h = 50 \cos 25^\circ$$

$$h = 45.315\ 389\ 53$$

$$(b) \sin 25^\circ = \frac{v}{50}$$

$$v = 50 \sin 25^\circ$$

$$v = 21.130\ 913\ 09$$

Therefore, a 45 N force pulls the wagon forward and a 21 N force tends to lift the wagon.

(c) Solution for the Functions and Relations course

The child holds the wagon at angle θ to the horizontal. h is the horizontal force, which is forward if positive and backward if negative.

$$\cos \theta = \frac{h}{50}$$

$$h = 50 \cos \theta$$

If $\theta = 0^\circ$, then $\cos \theta = 1$ and $h = 50$ N, which is the maximum forward force.

If $0 < \theta < 90^\circ$, then $\cos \theta$ decreases and h decreases. If $\theta = 90^\circ$, then $\cos \theta = 0$, $h = 0$ N, and there is no forward force. If $90^\circ < \theta < 180^\circ$, then $\cos \theta$ increases, but is negative. Then h also increases, but is negative, a backward force.

Therefore, the child can pull the wagon with a maximum forward force if the handle is horizontal. There is no force if the handle is vertical, and the force is backward if the handle leans backward.

Solution for the Functions course

The child holds the wagon at angle θ to the horizontal; h is the horizontal force, which is forward if positive and backward if negative. Vary the angle to determine the effect on the force in the forward direction.

$$\cos \theta = \frac{h}{50}$$

$$h = 50 \cos \theta$$

If $\theta = 0^\circ$, $h = 50 \cos 0^\circ$ or 50 N.

If $\theta = 30^\circ$, $h = 50 \cos 30^\circ$ or about 43 N.

If $\theta = 85^\circ$, $h = 50 \cos 85^\circ$ or 4 N.

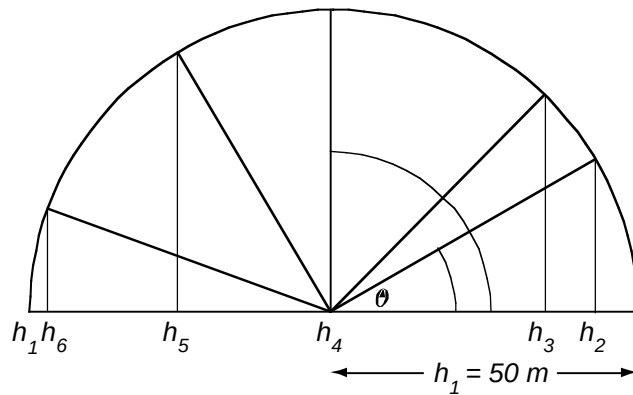
If $\theta = 90^\circ$, $h = 50 \cos 90^\circ$ or 0 N.

If $\theta = 120^\circ$, $h = 50 \cos 120^\circ$ or -25 N (a backward force).

Therefore, the child can pull the wagon with a maximum forward force if the handle is horizontal. There is no force if the handle is vertical, and the force is backward if the handle leans backward.

Alternative solution

The child holds the wagon at angle θ to the horizontal, and h is the horizontal force which is forward if positive and backward if negative.



When θ is 0° , the horizontal force (forward) is the full 50 N. As θ increases, the horizontal force (forward) decreases eventually to 0 when $\theta = 90^\circ$. As θ continues to increase, the horizontal force starts to increase, but in a backward direction.

Therefore, the child can pull the wagon with a maximum forward force if the handle is horizontal. There is no force if the handle is vertical, and the force is backward if the handle leans backward.