
The Chapter Problem—Designing a Bridge

Answers

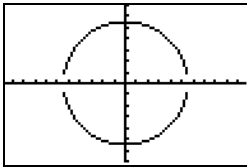
By the end of this chapter, students should be able to provide comprehensive, well-explained solutions to the problem.

Section 7.2U

CP1. *What is the radius? Write the equation of the curve.*

If the road is 11 m wide, the radius would have to be at least 5.5 m. Then the equation is $x^2 + y^2 = 30.25$.

CP2. *Graph the curve.*



CP3. *Determine if two trucks will be able to pass under the arch. Leave space between the trucks, as well as at each side of the road.*

Each truck is 3 m wide and there should be about 1 m between the edge of the road and each side of the truck. Then a point of the upper edge of the right truck is (4, 4). Substitute this point in the equation of the circle to get $4^2 + 4^2 = 32$. 32 is greater than 30.25. Therefore, either the circle should have a larger radius, or there should be less space between the centre of the road and the side of the truck. (Answers will vary, depending on students' assumptions about how much space there should be between each side of each truck and the edge of the road.)

CP4. *Is this a good design? Why or why not?*

The design would be better if there were more space around the sides of each truck. The radius of the semicircular arch could be much larger. For instance, the trucks will clear the arch easily if the radius is 8.5 m. In this case, the coordinates of the upper edge of the right truck are (5.5, 4). Substituting in the equation gives $(5.5)^2 + (4)^2 = 46.25$, and 46.25 is less than 72.25 or $(8.5)^2$.

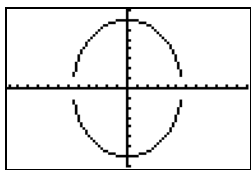
(Answers will vary but should be justified.)

Section 7.4U

CP5. What are the lengths of the major and the minor axes? Write the equation of the curve.

Try the semiellipse that has a vertical major axis of 14 and a horizontal minor axis of 11. Then the equation is $\frac{x^2}{30.25} + \frac{y^2}{49} = 1$, or, using the fractional form for b , $\frac{11}{2}$, $\frac{4x^2}{121} + \frac{y^2}{49} = 1$.

CP6. Graph the curve.



CP7. Determine whether two trucks will be able to pass, side by side, under the arch. There should be space between the trucks and at each side of the road. Adjust the major and minor axes so your arch is workable.

Each truck is 3 m wide and there should be about 1 m between the edge of the road and each side of the truck. Then a point of the upper edge of the truck is (4, 4). Substitute this in the equation of the ellipse, $\frac{4x^2}{121} + \frac{y^2}{49} = 1$,

$$\begin{aligned}\frac{4(4)^2}{121} + \frac{4^2}{49} &= \frac{64}{121} + \frac{16}{49} \\ &= \frac{3136}{5929} + \frac{1936}{5929} \\ &= \frac{5072}{5929}\end{aligned}$$

The value of this fraction is less than 1. Therefore, the point (4, 4) will lie within the semiellipse and the trucks will fit beneath the arch. (Answers will vary, depending on students' assumptions about how much space there should be between each side of the truck and the edge of the road.)

CP8. Is this a good design?

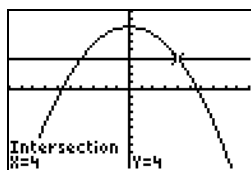
The responses to this question will depend on the individual design. Drawing the arch and the trucks passing beneath the arch will help students determine just how much space is available around the truck. Students may want to consider what happens if the truck veers onto the road's shoulder. Will there still be enough space?

Section 7.6U

CP9. *Decide where you will place the axes on the parabolic arch and then write the equation of the curve.*

Students will need to estimate the location of the vertex on the y -axis and the factor p in $x^2 = 4p(y - k)$. Initially, they should realize that p is negative. They might try graphing some curves using a graphing calculator and then decide which curve will provide adequate space for the truck, assuming point $(4, 4)$ is at the upper top edge of the right truck.

For instance, if they test the equation $x^2 = -4(y - 8)$, and find its intersection with the line $y = 4$, they can see that $(4, 4)$ lies directly on the curve.

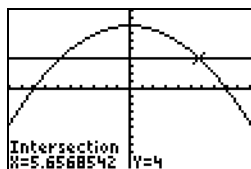


Therefore, it is necessary to experiment with different values of p and k . For instance, $x^2 = -8(y - 8)$ intersects $y = 4$ at the point $(5.7, 4)$, which would allow one truck to drive on the edge of the road.

Alternatively, estimate suitable x -intercepts and a vertex, and use these three points to find the equation of a parabola.

CP10. *Graph the curve.*

Here is the graph of $x^2 = -8(y - 8)$.



CP11. *Determine whether two trucks will be able to pass, side by side, under the arch. There should be space between the trucks and at each side of the road. You may have to adjust some of your dimensions. If so, justify your changes.*

See the graph in the answer to question CP11. It appears that two trucks would be able to pass under the arch.

CP12. *Is this is a good design?*

Answers will vary. Students might research which conic appears most frequently in the design of arches. Students might discuss other factors such as which arches offer the best support for a road or railway that travels across the arch.