
Chapter 7 Test: Solutions

1. (a) $(x-1)^2 - 2(y+3)^2 = 18$

This equation describes a hyperbola, since $ab < 0$, with centre $(1, -3)$.

(b) $16x^2 + 9y^2 - 18y - 135 = 0$

This equation describes an ellipse since $ab > 0$. To find the centre, change the equation to standard form, which is $\frac{x^2}{9} + \frac{(y-1)^2}{16} = 1$. Therefore, the centre is $(0, 1)$.

(c) $\frac{(x-2)^2}{4} + \frac{(y+1)^2}{4} = 1$

This equation describes a circle since $a = b$. The centre is $(2, -1)$.

2. (a) the locus of points that are 6 units from the point $(2, -5)$

Geometric solution: The locus is a circle with centre $(2, -5)$ and radius 6.

Algebraic solution: The equation for this locus is $(x-2)^2 + (y+5)^2 = 36$.

(b) the locus of points equidistant from points $A(0, 4)$ and $B(-2, 0)$

Geometric solution: The locus is the perpendicular bisector of line segment AB .

Algebraic solution: First determine the midpoint of the line segment, which is $(-1, 2)$. Then find the slope of AB .

$$\text{slope}_{AB} = \frac{(4-0)}{[0-(-2)]}$$

Therefore, the slope of AB is 2. The slope of the perpendicular bisector is then $-\frac{1}{2}$.

Next find the equation of the line with slope $-\frac{1}{2}$ that passes through point $(-1, 2)$.

$$\begin{aligned}y &= -\frac{1}{2}x + b \\2 &= \left(-\frac{1}{2}\right)(-1) + b \\b &= 2 - \frac{1}{2} \\b &= \frac{3}{2}\end{aligned}$$

The equation of the line in slope-intercept form is $y = -\frac{1}{2}x + \frac{3}{2}$. In general form, the equation is $x + 2y - 3 = 0$.

- (c) the locus of points equidistant from point (0, 4) and line $y = -4$.

Geometric solution: The locus is a parabola with focus (0, 4). The directrix is the line $y = -4$.

Algebraic solution: Since the distance from the origin to the focus is the same as the distance from the origin to the directrix, we know that the parabola's vertex is at the origin. Since the directrix is horizontal, we know that the equation is in the form $x^2 = 4py$.

In this case, $p = 4$, and the equation is $x^2 = 16y$.

3. Assume that the x -axis is the bridge and the minimum point of the centre cable is (0, 1). So the model for the parabola is $x^2 = 4p(y - 1)$.

Now, use the point, (35, 35), at the height of the tower. Substitute this point in the equation and solve for p .

$$\begin{aligned}x^2 &= 4p(y - 1) \\35^2 &= 4p(35 - 1) \\1225 &= 4p(34) \\1225 &= 136p \\p &= \frac{1225}{136} \\p &\doteq 9\end{aligned}$$

Therefore, the model for the parabola is $x^2 = 36(y - 1)$.

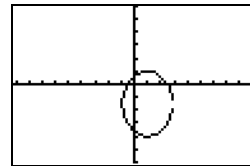
4. (a) First rearrange the equation in standard form by completing the square.

$$\begin{aligned}9x^2 + 4y^2 - 18x + 16y &= 11 \\9(x^2 - 2x) + 4(y^2 + 4y) &= 11 \\9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) &= 11 + 9 + 16 \\9(x - 1)^2 + 4(y + 2)^2 &= 36 \\\frac{(x - 1)^2}{4} + \frac{(y + 2)^2}{9} &= 1\end{aligned}$$

This equation represents an ellipse with centre (1, -2). The vertical major axis is 6 units long and the horizontal minor axis is 4 units long.

By $a^2 = b^2 + c^2$, $c = \sqrt{5}$.

The vertices are (1, 1) and (1, -5). The foci are $(1, -2 + \sqrt{5})$ and $(1, -2 - \sqrt{5})$.



- (b) Since this figure is an ellipse, not a hyperbola, there are no asymptotes.

The equation $81x^2 - 36y^2 = 2916$ represents a hyperbola. Divide by 2916 to get the equation in standard form.

$$\frac{x^2}{36} - \frac{y^2}{81} = 1$$

Thus, the vertices are $(-6, 0)$ and $(6, 0)$. The asymptotes are $y = 1.5x$ and $y = -1.5x$.

To find the foci, use $c^2 = a^2 + b^2$.

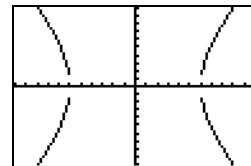
$$c^2 = a^2 + b^2$$

$$c^2 = 6^2 + 9^2$$

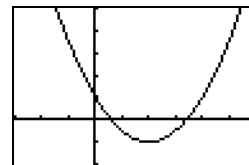
$$c^2 = 117$$

$$c = \pm\sqrt{117}$$

Therefore, the foci are $(\sqrt{117}, 0)$ and $(-\sqrt{117}, 0)$.



5. First determine that the conic is a parabola and put the equation in standard form, $(x - 2)^2 = 2(y + 1)$. Now draw the graph.



- (a) Many lines will intersect the parabola in exactly two points. One such line is $y = -x + 1$. To find the intersection points and verify that there are exactly two intersection points, substitute $y = -x + 1$ in the equation of the parabola.

$$\begin{aligned}(x - 2)^2 &= 2(y + 1) \\(x - 2)^2 &= 2(-x + 1 + 1) \\x^2 - 4x + 4 &= -2x + 4 \\x^2 - 2x &= 0 \\x(x - 2) &= 0\end{aligned}$$

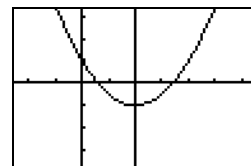


Therefore, $x = 0$ and $x = 2$. When $x = 0$, $y = 1$ and, when $x = 2$, $y = -1$. The intersection points are $(0, 1)$ and $(2, -1)$.

- (b) Again there are many lines that will intersect the parabola at just one point. The simplest line might be a horizontal or vertical line through the vertex, for example, the vertical line $x = 2$.

Substitute $x = 2$ in the equation of the parabola.

$$\begin{aligned}(x - 2)^2 &= 2(y + 1) \\(2 - 2)^2 &= 2(y + 1) \\0 &= 2(y + 1) \\0 &= y + 1 \\y &= -1\end{aligned}$$



Therefore, the intersection point is $(2, -1)$ for the line $x = 2$.