

Chapter 1

Practise, Apply, Solve 1.1, page 21

1. (a) 1, 5, 25, 125, **625, 3125, 15 625, 78 125**
- (b) 16, 12, 8, 4, **0, -4, -8, -12**
- (c) 8, 14, 20, 26, **32, 38, 44, 50**
- (d) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \frac{1}{36}, \frac{1}{49}, \frac{1}{64}$
- (e) 9, 98, 987, 9 876, **98 765, 987 654, 9 876 543, 98 765 432**
- (f) -3, 6, -12, 24, **-48, 96, -192, 384**
- (g) 5, 4, 9, 13, 22, 35, 57, **92, 149**
- (h) 6, 16, 36, 66, **106, 156, 216, 286**
- (i) 1, 2, 2, 4, 8, **32, 256, 8192, 2 097 152**
- (j) 1, 8, 27, 64, **125, 216, 343, 512**
- (k) 0.3, 0.5, 0.7, 0.9, **1.1, 1.3, 1.5, 1.7**
- (l) 12, 6, 3, 1.5, **0.75, 0.375, 0.1875, 0.093 75**

2. (a) not predictable
- (b) 1024, 1026, 1028, 1030, **1032, 1034, 1036, 1038**
- (c) not predictable
- (d) 48, 96, 192, 384, **768, 1536, 3072, 6144**
- (e) \$42 000.00, \$43 680.00, \$45 427.20, \$47 244.29,
\$49 134.06, \$51 099.42, \$53 143.40, \$55 269.14
- (f) not predictable

3. (a) $t_1 = (1) + 4 = 5, t_2 = (2) + 4 = 6, t_3 = (3) + 4 = 7,$
 $t_4 = (4) + 4 = 8, t_5 = (5) + 4 = 9$
- (b) $t_1 = (1)^2 - 2 = -1, t_2 = (2)^2 - 2 = 2, t_3 = (3)^2 - 2 = 7,$
 $t_4 = (4)^2 - 2 = 14, t_5 = (5)^2 - 2 = 23$
- (c) $t_1 = 2^1 = 2, t_2 = 2^2 = 4, t_3 = 2^3 = 8, t_4 = 2^4 = 16,$
 $t_5 = 2^5 = 32$
- (d) $t_1 = 5(1) - 2 = 3, t_2 = 5(2) - 2 = 8, t_3 = 5(3) - 2 = 13,$
 $t_4 = 5(4) - 2 = 18, t_5 = 5(5) - 2 = 23$
- (e) $t_1 = (1 + 2)(1 - 2) = (3)(-1) = -3,$
 $t_2 = (2 + 2)(2 - 2) = (4)(0) = 0, t_3 = (3 + 2)(3 - 2) = (5)(1) = 5,$
 $t_4 = (4 + 2)(4 - 2) = (6)(2) = 12,$
 $t_5 = (5 + 2)(5 - 2) = (7)(3) = 21$
- (f) $t_1 = \frac{5}{1} = 5, t_2 = \frac{5}{2}, t_3 = \frac{5}{3}, t_4 = \frac{5}{4}, t_5 = \frac{5}{5} = 1$
- (g) $t_1 = 3(-2)^{1+1} = 3(-2)^2 = 3(4) = 12,$
 $t_2 = 3(-2)^{2+1} = 3(-2)^3 = 3(-8) = -24,$
 $t_3 = 3(-2)^{3+1} = 3(-2)^4 = 3(16) = 48,$
 $t_4 = 3(-2)^{4+1} = 3(-2)^5 = 3(-32) = -96,$
 $t_5 = 3(-2)^{5+1} = 3(-2)^6 = 3(64) = 192$
- (h) $t_1 = (-1)^1 = -1, t_2 = (-1)^2 = 1, t_3 = (-1)^3 = -1,$
 $t_4 = (-1)^4 = 1, t_5 = (-1)^5 = -1$
- (i) $t_1 = -3(1) + 7 = 4, t_2 = -3(2) + 7 = 1, t_3 = -3(3) + 7 = -2,$
 $t_4 = -3(4) + 7 = -5, t_5 = -3(5) + 7 = -8$

4. (a) $t_{10} = 5(10) - 12 = 38$
- (b) $t_{10} = 3(2^{10-5}) = 3(2^5) = 3(32) = 96$
- (c) $t_{10} = \frac{10}{10+1} = \frac{10}{11}$
- (d) $t_{10} = 10^2 - 10(10) = 100 - 100 = 0$
- (e) $t_{10} = \sqrt{4.9(10)} = \sqrt{49} = 7$

$$(f) t_{10} = \frac{2^{10-6}}{10-2} = \frac{2^4}{8} = \frac{16}{8} = 2$$

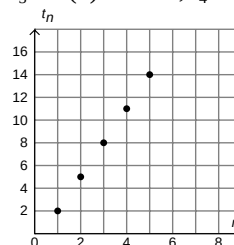
5. (a) $t_n = 3 + (n-1)4 = 4n - 1$
- (b) $t_n = 36 + (n-1)6 = 6n + 30$
- (c) $t_n = (4)3^{n-1}$
- (d) $t_n = (2)3^{-n} = \frac{2}{3^n}$
- (e) $t_n = 5(-1)^{n+1}$
- (f) $t_n = \sqrt{n}$
- (g) $t_n = 65 - (n-1)4 = -4n + 69$
- (h) $t_n = \frac{n+1}{n+2}$
- (i) $t_n = \frac{1}{n^2}$

$$6. (a) t_1 = (5-1)^1 = 4^1 = 4, t_2 = (5-2)^2 = 3^2 = 9,$$

 $t_3 = (5-3)^3 = 2^3 = 8, t_4 = (5-4)^4 = 1^4 = 1,$
 $t_5 = (5-5)^5 = 0^5 = 0, t_{10} = (5-10)^{10} = (-5)^{10} = 9\,765\,625$

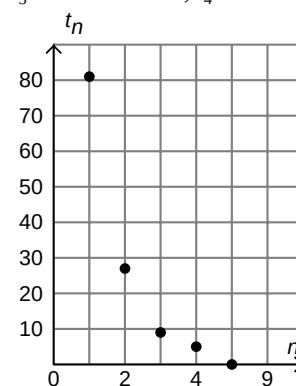
$$7. (a) t_1 = 3(1) - 1 = 2, t_2 = 3(2) - 1 = 5$$

 $t_3 = 3(3) - 1 = 8, t_4 = 3(4) - 1 = 11, t_5 = 5(3) - 1 = 14$



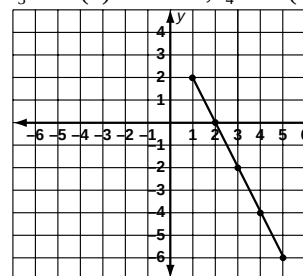
$$(b) t_1 = 3^{5-1} = 3^4 = 81, t_2 = 3^{5-2} = 3^3 = 27,$$

 $t_3 = 3^{5-3} = 3^2 = 9, t_4 = 3^{5-4} = 3^1 = 3, t_5 = 3^{5-5} = 3^0 = 1$

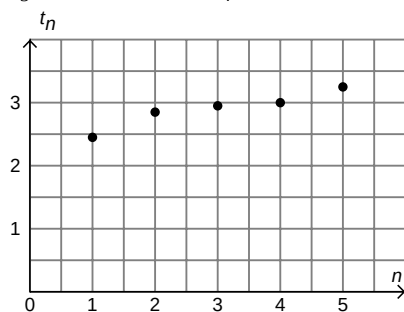


$$(c) t_1 = -2(1) + 4 = 2, t_2 = -2(2) + 4 = 0,$$

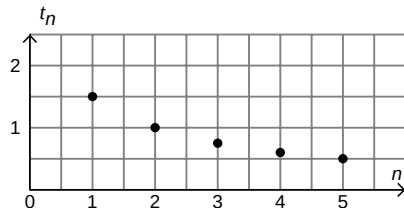
 $t_3 = -2(3) + 4 = -2, t_4 = -2(4) + 4 = -4, t_5 = -2(5) + 4 = -6$



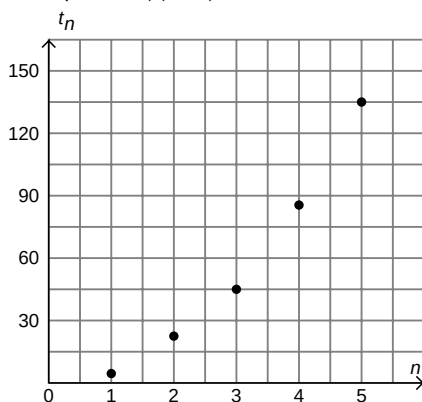
(d) $t_1 = \sqrt{1+5} = \sqrt{6}$, $t_2 = \sqrt{2+5} = \sqrt{7}$,
 $t_3 = \sqrt{3+5} = \sqrt{8}$, $t_4 = \sqrt{4+5} = 3$, $t_5 = \sqrt{5+5} = \sqrt{10}$



(e) $t_1 = \frac{3}{1+1} = \frac{3}{2}$, $t_2 = \frac{3}{2+1} = 1$,
 $t_3 = \frac{3}{3+1} = \frac{3}{4}$, $t_4 = \frac{3}{4+1} = \frac{3}{5}$, $t_5 = \frac{3}{5+1} = \frac{1}{2}$



(f)
 $t_1 = (2(1) - 1)(3(1)) = (1)(3) = 3$,
 $t_2 = (2(2) - 1)(3(2)) = (3)(6) = 18$,
 $t_3 = (2(3) - 1)(3(3)) = (5)(9) = 45$,
 $t_4 = (2(4) - 1)(3(4)) = (7)(12) = 84$,
 $t_5 = (2(5) - 1)(3(5)) = (9)(15) = 135$



8. Assumes that each line added must intersect with all other lines, and 3 or more cannot intersect at the same point.

(a) 2, 4, 7, 11, 16

(b) 2, 4, 7, 11, 16, 22, 29, 37, 46, 56

(c) The common difference increases by one for each term that is added. Writing out each term is an easy way of finding the tenth term. **Note:** $t_n = \frac{n^2 + n + 2}{2}$

9. (a) To 5 decimal places: 1.414 21, 1.189 21, 1.090 51, 1.044 27, 1.021 90, 1.010 89, 1.005 43, 1.002 71, 1.001 35, 1.000 68

(b) The terms appear to be approaching 1.

(c) $t_{1000} = 2^{\frac{1}{2^{1000}}} = 1 + \epsilon$, for some very, very small $\epsilon > 0$.
 Thus, t_{1000} will be 1.

10. The independent variable of a sequence uses only values from the set of natural numbers, instead of the real numbers, and so the points cannot always be joined with a line or a smooth curve.

11. (a) Examples: 1) {3, 6, 9, 12, 15, ...}
 2) {3, 6, 12, 24, 48, ...}
 3) {3, 6, 10, 15, 21, 28, ...}
 4) {3, 6, 0, -24, -75, -162, ...}

(b) 1) add three to each term, 2) multiply each term by two, 3) increase the common difference by one for each term, 4)

cube the term number and multiply by $\frac{-3}{2}$, then add the

square of the term number multiplied by $\frac{9}{2}$

(c) 1) $t_n = 3n$, 2) $t_n = (3)2^{n-1}$, 3) $t_{n+1} = t_n + n + 2$,

4) $t_n = \frac{-3}{2}n^3 + \frac{9}{2}n^2$

12. (a) $t_n = 5n - 15$, $t_2 = 5(2) - 15 = 10 - 15 = -5$,

$t_5 = 5(5) - 15 = 25 - 15 = 10$, $t_{10} = 5(10) - 15 = 50 - 15 = 35$

(b) $t_n = n^4$, $t_2 = 2^4 = 16$, $t_5 = 5^4 = 625$, $t_{10} = 10^4 = 10\,000$

(c) $t_n = \frac{5}{n}$, $t_2 = \frac{5}{2}$, $t_5 = \frac{5}{5} = 1$, $t_{10} = \frac{5}{10} = \frac{1}{2}$

(d) $t_n = (3n + 4)^2$, $t_2 = (6 + 4)^2 = 10^2 = 100$,

$t_5 = (15 + 4)^2 = 19^2 = 361$, $t_{10} = (30 + 4)^2 = 34^2 = 1156$

(e) $t_n = 2^{n+1}$, $t_2 = 2^{2+1} = 2^3 = 8$, $t_5 = 2^{5+1} = 2^6 = 64$,

$t_{10} = 2^{10+1} = 2^{11} = 2048$

(f) $t_n = (-1)^{n+3}$, $t_2 = (-1)^{2+3} = (-1)^5 = -1$,

$t_5 = (-1)^{5+3} = (-1)^8 = 1$, $t_{10} = (-1)^{10+3} = (-1)^{13} = -1$

(g) $t_n = \frac{n-1}{2n+2}$, $t_2 = \frac{2-1}{2(2)+2} = \frac{1}{6}$, $t_5 = \frac{5-1}{2(5)+2} = \frac{4}{12} = \frac{1}{3}$,

$t_{10} = \frac{10-1}{2(10)+2} = \frac{9}{22}$

(h) $t_n = 3n^2 + 2n - 1$, $t_2 = 3(2)^2 + 2(2) - 1 = 12 + 4 - 1 = 15$,

$t_5 = 3(5)^2 + 2(5) - 1 = 75 + 10 - 1 = 84$,

$t_{10} = 3(10)^2 + 2(10) - 1 = 300 + 20 - 1 = 319$

(i) $t_n = (n-1)^3$, $t_2 = (2-1)^3 = 1^3 = 1$, $t_5 = (5-1)^3 = 4^3 = 64$,

$t_{10} = (10-1)^3 = 9^3 = 729$

13. (a) Finite – the length of the rim is finite, so the pattern must also be finite

(b) Infinite – π is irrational

(c) Infinite – {1, 2, 3, ...} is infinite and so is {2(1), 2(2), 2(3), ...}

(d) Infinite – {1, 2, 3, ...} is infinite and so is $\{\sqrt{0}, \sqrt{1}, \sqrt{2}, \dots\}$

(e) Finite – {5, 25, 125, ..., $5^8 = 39\,625$ } only has 8 terms ($5^9 = 1\,953\,125$)

14. (a) After 1 year, $t_n = \$1000 + \$1000 \times 0.045 = \$1045$,

After 2 years, $t_n = \$1045 + \$1000 \times 0.045 = \$1090$

After 3 years, $t_n = \$1090 + \$1000 \times 0.045 = \$1135$

- (b) After n years,
 $t_n = \$1000 + \$1000 \times 0.045 + \dots + \$1000 \times 0.045$
 $= \$1000 + n \times \$1000 \times 0.045$
 $= \$1000 + \$45n$
 (c) $t_{10} = \$1000 + \$45(10) = \$1000 + \$450 = \$1450$

15. (a) $\{1, 8, 27, \dots\}$
 (b) 64, 125, 216
 (c) $t_n = n^3$
 (d) $t_{15} = 15^3 = 3375$

16. (a) $\{10, 5, 2.5, 1.25, 0.625, \dots\}$
 (b) $t_n = 20(0.5)^n$
 (c) $t_{10} = 20(0.5)^{10} = 0.0195 \text{ m}$
 (d) The ball will stop bouncing eventually, due to friction and so on. So, the sequence is finite. t_n is defined for all $n \in \mathbb{N}$ and the terms approach zero but will never reach it.

17. (a) $\{\$0.25, \$0.50, \$0.75, \$1.00, \$1.25, \dots\}$
 (b) $t_n = \$0.25n$
 (c) $t_{52} = \$0.25(52) = \$13.00$

18. (a) $t_7 = (2)^7 = 98$
 (b) $t_n = 2n^2$

19. (a)

Time (hours)	Bacteria
1	20
2	40
3	80
4	160
5	320
6	640
7	1280
8	2560

- (b) There are 2560 bacteria after 8 hours.
 (c) $b_n = 10(2^n)$ OR $b_n = 20(2^{n-1})$ OR $b_n = 5(2^{n+1})$
 (d) $b_{10} = 10(2^{10}) = 10(1024) = 10\,240$

20. The sequence given is $\{6, 12, 18, 24, \dots, 6n\}$. The numbers among 216, 575, 492, 750, 824, that are divisible by 6 are 216, 492, and 750.

21. (a) $\{6, 16, 30, \dots\}$
 (b) 48, 70, 96
 (c) There are n^2 surface squares with both top and bottom, and there are n surface squares $\times 4$ sides.
 $t_n = 4n + 2n^2 = 2n(n + 2)$
 (d) $t_{20} = 2(20)(20 + 2) = 40(22) = 880$

22. (a) $\{2, 8, 18, \dots\}$
 (b) 32, 50, 72
 (c) The length of each shape contains $2n$ cubes, the width contains n cubes, and the height of each shape is 1 unit. Thus, $t_n = 2n^2$.
 (d) $t_{20} = 2(20)^2 = 2(400) = 800$

23. (a) 24, 40
 (b) The number of toothpicks is $2h(h + 1)$, where h is the length of one side.
 (c) In stage 10, the number of toothpicks is $2(10)(11) = 220$.

Practice 1.2, page 26

- (a) 6, 8, 10, 12, 14, 16, 18, 20, 22, 24
 (b) 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000
 (c) $3, 2, \frac{5}{3}, \frac{3}{2}, \frac{7}{5}, \frac{4}{3}, \frac{9}{7}, \frac{5}{4}, \frac{11}{9}, \frac{6}{5}$
 (d) 1, 1.414 213 562, 1.732 050 808, 2, 2.236 067 977, 2.449 489 743, 2.645 751 311, 2.828 427 125, 3, 3.162 277 660
 (e) $-7, -12, -17, -22, -27, -32, -37, -42, -47, -52$
 (f) 0.5, 0.25, 0.125, 0.0625, 0.031 25, 0.015 625, 0.007 812 5, 0.003 906 25, 0.001 953 125, 0.000 976 562 5
 (g) 6, 18, 54, 162, 486, 1458, 4374, 13 122, 39 366, 118 098
 (h) 1, 4, 27, 256, 3125, 46 656, 823 543, 16 777 216, 387 420 489, 10 000 000 000

Practise, Apply, Solve 1.3U, page 32

1. (a) $\{3, 12, 21, 30, 39, \dots\}$
 (b) $\{6, 24, 96, 384, 1536, \dots\}$
 (c) $\{36, 18, 9, 4.5, 2.25, \dots\}$
 (d) $\{22, 20, 18, 16, 14, \dots\}$
 (e) $\{3, 4, 12, 48, 576, \dots\}$
 (f) $\{-5, 7, 2, 9, 11, \dots\}$
2. (a) $t_1 = 4, t_2 = (4) + 5 = 9, t_3 = (9) + 5 = 14,$
 $t_4 = (14) + 5 = 19, t_5 = (19) + 5 = 24$
 (b) $t_1 = -3, t_2 = 2(-3) - 2 = -8, t_3 = 2(-8) - 2 = -18,$
 $t_4 = 2(-18) - 2 = -38, t_5 = 2(-38) - 2 = -78$
 (c) $t_1 = 8, t_2 = 8 \div 2 = 4, t_3 = 4 \div 2 = 2, t_4 = 2 \div 2 = 1,$
 $t_5 = 1 \div 2 = 0.5$
 (d) $t_1 = 4, t_2 = (4)^2 = 16, t_3 = (16)^2 = 256,$
 $t_4 = (256)^2 = 65\,536, t_5 = (65\,536)^2 = 4\,294\,967\,296$
 (e) $t_1 = 5, t_2 = \frac{5}{2}, t_3 = \frac{5}{6}, t_4 = \frac{5}{24}, t_5 = \frac{5}{120} = \frac{1}{24}$
 (f) $t_1 = 1, t_2 = 2, t_3 = (2)(1) = 2, t_4 = (2)(2) = 4, t_5 = (4)(2) = 8$
 (g) $t_1 = 2, t_2 = -1, t_3 = -1 + 2 = 1, t_4 = 1 + (-1) = 0,$
 $t_5 = 0 + 1 = 1$
 (h) $t_1 = 1, t_2 = 1, t_3 = 2(1) - 3(1) = -1, t_4 = 2(-1) - 3(1) = -5,$
 $t_5 = 2(-5) - 3(-1) = -7$
3. (a) $t_1 = 2, t_{n+1} = t_n + 5$
 (b) $t_1 = -2, t_{n+1} = -2t_n$
 (c) $t_1 = 10, t_{n+1} = \frac{1}{2}t_n$
 (d) $t_1 = 2, t_{n+1} = (t_n)^2$
 (e) $t_1 = 243, t_{n+1} = \frac{1}{3}t_n$
 (f) $t_1 = 34, t_{n+1} = t_n - 9$
4. (a) $t_1 = 3, t_n = t_{n-1} + 9$
 (b) $t_1 = 6, t_n = 4(t_{n-1})$