

The equations for the two graphs are almost exactly the same. Their domains and ranges are nearly identical. The process for determining the line of best fit causes slight differences.

(g) If price = \$4.75/kg, then

$$f(4.75) = -3040(4.75) + 14\,700 = 260$$

Thus, if the price rises to \$4.75/kg, he can expect to sell only 260 kg of ground beef.

(h) If price = \$5/kg, then

$$f(5) = -3040(5) + 14\,700 = -500$$

Thus, if the price rises to \$5/kg, he will not sell any ground beef.

(i) If mass = 4000 kg, then

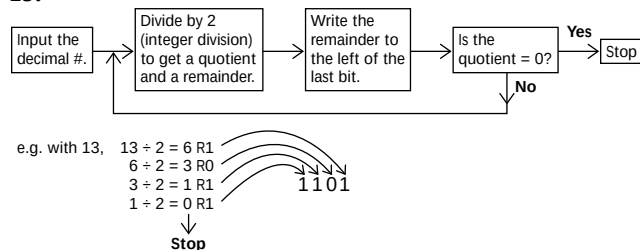
$$f^{-1}(4000) = -0.000\,314(4000) + 4.81 = 3.554$$

Using the second model, we get $f^{-1}(4000) = 3.554$. Thus, the price would have to be approximately \$3.55 in order to sell 4000 kg of ground beef.

18. (a) A postal code must be a function because each address in Canada must have a reliable, consistent postal code. One address cannot have more than one postal code.

(b) The inverse converts a postal code into its corresponding address. Several neighboring houses may have the same postal code, so one postal code produces several possible addresses. It is not a function.

19.



To convert binary to decimal, the number (at each stage) is multiplied by 2 and then a digit is added (1 or 0). To reverse this process, divide by 2, to get a quotient and a remainder. Write the remainder down as the next bit (moving left). Repeat using the quotient. Continue until the quotient is 0.

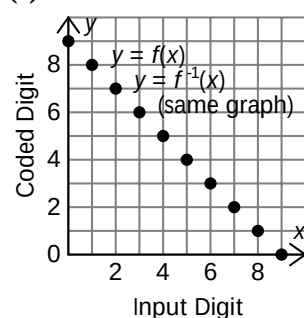
20. (a) 5 667 821 743

(b) The inverse is the same as the original code, so if 207 456 127 is received, then 792 543 872 was sent.

(c) $f(x) = 9 - x$, $\{x \mid x \leq 9, x \in \mathbf{W}\}$

(d) $f^{-1}(x) = 9 - x$, $\{x \mid x \leq 9, x \in \mathbf{W}\}$

(e)



21. (a) B --> 2, 0 --> 15, Y --> 25

BOY becomes 21525

(b) This is a function because each word can only be coded in one way.

(c) 21520201513 might be:

2	15	20	20	15	13
B	O	T	T	O	M

or

2	15	2	0	20	15	13
B	O	B	—	T	O	M

or many other combinations that are not necessarily English words.

(d) This is not a good encryption scheme because its inverse is not a function. Thus, if you know the encoded digits this will not necessarily determine the correct original message.

22. In the original soft drink machine example, a combination of coins and a button selection were the input, and the soft drink was the output.

(a) For the inverse relation, the independent variable is the soft drink received.

(b) For the inverse relation, the dependent variable is the combination of coins and a button selection.

(c) The domain of the inverse is the set of all the different drinks that the machine holds.

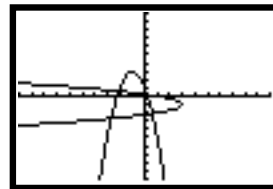
(d) The range of the inverse is the set of all possible combinations of coins and button selections that would yield a soft drink from the machine.

(e) The inverse is not a function because the size and type of soft drink received does not correspond to one unique combination of coins and button selections.

Practice 3.5, page 264

1. (a) $f(x) = -2(x + 1)^2 + 3$, $f^{-1}(x) = \pm \sqrt{\frac{x-3}{-2}} - 1$

i.



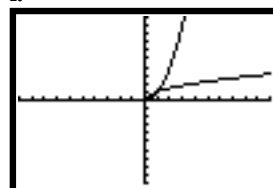
ii. $D = \{x \mid x \in \mathbf{R}\}$ and $R = \{y \mid y \leq 3, y \in \mathbf{R}\}$

iii. $D = \{x \mid x \leq 3, x \in \mathbf{R}\}$ and $R = \{y \mid y \in \mathbf{R}\}$

iv. The inverse is not a function. It fails the vertical line test.

(b) $g(x) = \sqrt{x}$, $g^{-1}(x) = x^2, x \geq 0$

i.



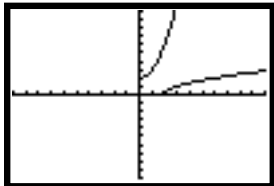
ii. $D = \{x \mid x \geq 0, x \in \mathbf{R}\}$ and $R = \{y \mid y \geq 0, y \in \mathbf{R}\}$

iii. $D = \{x \mid x \geq 0, x \in \mathbf{R}\}$ and $R = \{y \mid y \geq 0, y \in \mathbf{R}\}$

iv. The inverse is a function. It passes the vertical line test.

(c) $f(x) = \sqrt{x-2}$, $f^{-1}(x) = x^2 + 2, x \geq 0$

i.



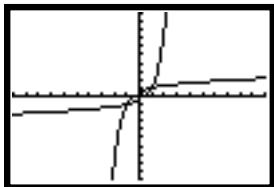
ii. $D = \{x \mid x \geq 2, x \in \mathbf{R}\}$ and $R = \{y \mid y \geq 0, y \in \mathbf{R}\}$

iii. $D = \{x \mid x \geq 0, x \in \mathbf{R}\}$ and $R = \{y \mid y \geq 2, y \in \mathbf{R}\}$

iv. The inverse is a function. It passes the vertical line test.

(d) $y = x^3$, $y^{-1}(x) = x^{\frac{1}{3}}$

i.



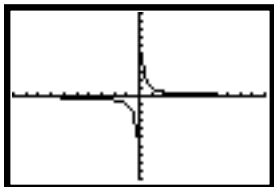
ii. $D = \{x \mid x \in \mathbf{R}\}$ and $R = \{y \mid y \in \mathbf{R}\}$

iii. $D = \{x \mid x \in \mathbf{R}\}$ and $R = \{y \mid y \in \mathbf{R}\}$

iv. The inverse is a function. It passes the vertical line test.

(e) $g(x) = \frac{1}{x}$, $g^{-1}(x) = \frac{1}{x}$

i.



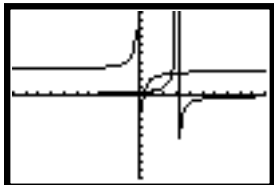
ii. $D = \{x \mid x \neq 0, x \in \mathbf{R}\}$ and $R = \{y \mid y \neq 0, y \in \mathbf{R}\}$

iii. $D = \{x \mid x \neq 0, x \in \mathbf{R}\}$ and $R = \{y \mid y \neq 0, y \in \mathbf{R}\}$

iv. The inverse is a function. Every value of x corresponds to a unique value of y .

(f) $f(x) = \frac{1}{3-x}$, $f^{-1}(x) = 3 - \frac{1}{x}$

i.



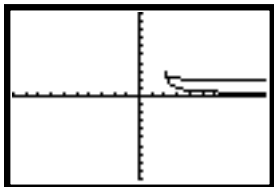
ii. $D = \{x \mid x \neq 3, x \in \mathbf{R}\}$ and $R = \{y \mid y \neq 0, y \in \mathbf{R}\}$

iii. $D = \{x \mid x \neq 0, x \in \mathbf{R}\}$ and $R = \{y \mid y \neq 3, y \in \mathbf{R}\}$

iv. The inverse is a function. It passes the vertical line test.

(g) $y = \frac{1}{\sqrt{x-2}}$, $y^{-1}(x) = \frac{1}{x^2} + 2, x \geq 2$

i.



ii. $D = \{x \mid x > 2, x \in \mathbf{R}\}$ and $R = \{y \mid y > 0, y \in \mathbf{R}\}$

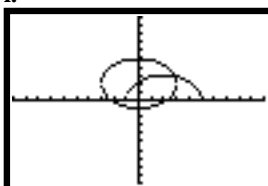
Part 2: Functions

iii. $D = \{x \mid x > 0, x \in \mathbf{R}\}$ and $R = \{y \mid y > 2, y \in \mathbf{R}\}$

iv. The inverse is a function. It passes the vertical line test.

(h) $f(x) = \sqrt{9-(x-2)^2}$, $f^{-1}(x) = \pm\sqrt{9-x^2} + 2$

i.



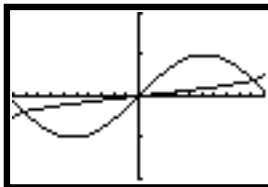
ii. $D = \{x \mid -1 \leq x \leq 5, x \in \mathbf{R}\}$ and $R = \{y \mid 0 \leq y \leq 3, y \in \mathbf{R}\}$

iii. $D = \{x \mid 0 \leq x \leq 3, x \in \mathbf{R}\}$ and $R = \{y \mid -1 \leq y \leq 5, y \in \mathbf{R}\}$

iv. The inverse is not a function. It fails the vertical line test.

(i) $g(x) = \sin(3x)$, $g^{-1}(x) = \frac{\sin^{-1}(x)}{3}$

i.



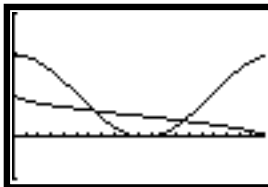
ii. $D = \{x \mid x \in \mathbf{R}\}$ and $R = \{y \mid -1 \leq y \leq 1, y \in \mathbf{R}\}$

iii. $D = \{x \mid -1 \leq x \leq 1, x \in \mathbf{R}\}$ and $R = \{y \mid y \in \mathbf{R}\}$

iv. The inverse is not a function. It fails the vertical line test.

(j) $f(x) = 1 + \cos(3x)$, $f^{-1}(x) = \frac{\cos^{-1}(x-1)}{3}$

i.

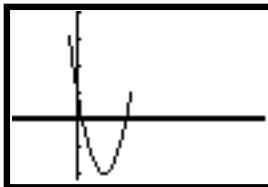


ii. $D = \{x \mid x \in \mathbf{R}\}$ and $R = \{y \mid 0 \leq y \leq 2, y \in \mathbf{R}\}$

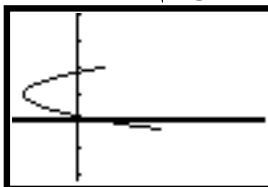
iii. $D = \{x \mid 0 \leq x \leq 2, x \in \mathbf{R}\}$ and $R = \{y \mid y \in \mathbf{R}\}$

iv. The inverse is not a function. It fails the vertical line test.

2. i.



ii. $f^{-1}(x) = \pm\sqrt{\frac{x+2}{3}} + 1$



iii. f:

$D = \{x \mid -0.3 \leq x \leq 2, x \in \mathbf{R}\}$

The vertex is at $(1, -2)$.

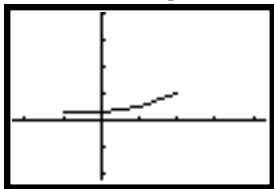
$f(-0.3) = 3.07$ and $f(2) = 1$

Thus, $R = \{y \mid -2 \leq y \leq 3.07, y \in \mathbf{R}\}$

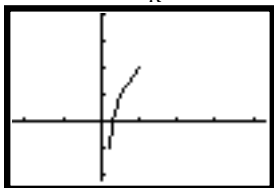
f^{-1} :

$D = \{x \mid -2 \leq x \leq 3.07, x \in \mathbf{R}\}$ and $R = \{y \mid -0.3 \leq y \leq 2, y \in \mathbf{R}\}$

iv. (a) $f(x) = \frac{1}{3-x}, -1 \leq x \leq 2$



$f^{-1}(x) = 3 - \frac{1}{x}$



f :

$D = \{x \mid -1 \leq x \leq 2, x \in \mathbf{R}\}$

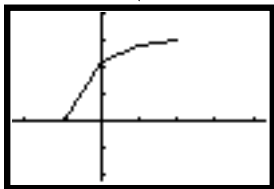
$f(-1) = 0.25$ and $f(2) = 1$

Thus, the $R = \{y \mid 0.25 \leq y \leq 1, y \in \mathbf{R}\}$

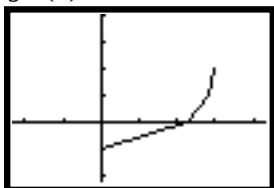
f^{-1} :

$D = \{x \mid 0.25 \leq x \leq 1, x \in \mathbf{R}\}$ and $R = \{y \mid -1 \leq y \leq 2, y \in \mathbf{R}\}$

(b) $g(x) = \sqrt{9 - (x-2)^2}, -1 \leq x \leq 2$



$g^{-1}(x) = \sqrt{9 - x^2} + 2$



g :

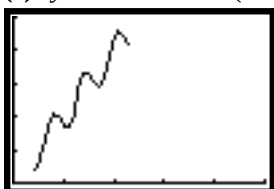
$D = \{x \mid -1 \leq x \leq 2, x \in \mathbf{R}\}$

$g(-1) = 0$ and $g(2) = 3$. Thus, the $R = \{y \mid 0 \leq y \leq 3, y \in \mathbf{R}\}$

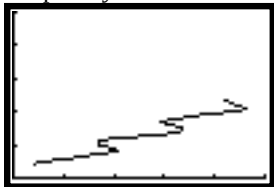
g^{-1} :

$D = \{x \mid 0 \leq x \leq 3, x \in \mathbf{R}\}$ and $R = \{y \mid -1 \leq y \leq 2, y \in \mathbf{R}\}$

(c) $y = 2x + 0.5 \sin(10x)$



Graph of y^{-1} :



y :

$D = \{x \mid 0.4 \leq x \leq 2.3, x \in \mathbf{R}\}$

Using the **max** function on the calculator we find that if $x \doteq 2.083$, then $y \doteq 4.625$

Using the **min** function on the calculator we find that if $x \doteq 0.430$, then $y \doteq 0.4019$

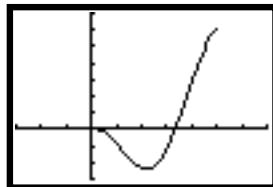
Thus, the $R = \{y \mid 0.4019 < y < 4.625, y \in \mathbf{R}\}$

y^{-1} :

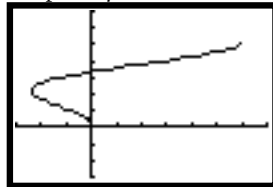
$D = \{x \mid 0.4019 < x < 4.625, x \in \mathbf{R}\}$ and

$R = \{y \mid 0.4 \leq y \leq 2.3, y \in \mathbf{R}\}$

(d) $f(x) = -1.2x \sin(x - 0.2), 0 \leq x \leq 5$



Graph of f^{-1} :



f :

$D = \{x \mid 0 \leq x \leq 5, x \in \mathbf{R}\}$

Using the **min** function on the calculator, we find that if $x \doteq 2.1978$, then $f(x) \doteq 2.4006$

$f(5) \doteq 5.97699$, and this is the highest point.

Thus, the $R = \{y \mid -2.4006 < y < 5.97699, y \in \mathbf{R}\}$

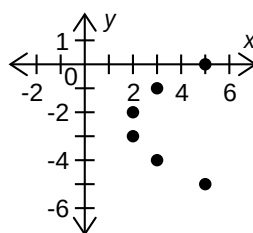
f^{-1} :

$D = \{x \mid -2.4006 < x < 5.97699, x \in \mathbf{R}\}$ and

$R = \{y \mid 0 \leq y \leq 5, y \in \mathbf{R}\}$

Practise, Apply, Solve 3.6, page 273

1. (a)



(b) A subset of f^{-1} which is a function is $\{(2, -3), (3, -4), (5, -5)\}$. There are several possibilities.

2. (a)

