

$$\begin{aligned}
 \text{(b)} \quad P(x) &= (x - 3.21)(14\,700 - 3040x) \\
 &= -3040x^2 + 14\,700x + 9758.4x - 47187 \\
 &= -3040x^2 + 24\,458.4x - 47\,187 \\
 &= -3040(x^2 - 8.0455x) - 47\,187 \\
 &= -3040(x^2 - 8.0455x + 16.182\,517\,56 - 16.182\,517\,56) \\
 &\quad - 47\,187 \\
 &= -3040(x - 4.022\,75)^2 + 49\,194.853\,39 - 47\,187 \\
 &= -3040(x - 4.02)^2 + 2008 \\
 \text{Thus, } P(x) &= y = -3040(x - 4.02)^2 + 2008
 \end{aligned}$$

so:

$$x = -3040(y - 2008)^2 + 2008$$

$$3040(y - 4.02)^2 = -x + 2008$$

$$(y - 4.02)^2 = \frac{-x + 2008}{3040}$$

$$y - 4.02 = \pm \sqrt{\frac{-x + 2008}{3040}}$$

$$y = 4.02 \pm \sqrt{\frac{-x + 2008}{3040}}$$

$$P^{-1}(x) = 4.02 \pm \sqrt{\frac{-x + 2008}{3040}}$$

(c) $P(x)$ requires price/kg and produces profit.
 $P^{-1}(x)$ requires total profit and produces price/kg.
 For a profit of \$1900, evaluate:

$$\begin{aligned}
 P^{-1}(1900) &= 4.02 \pm \sqrt{\frac{-1900 + 2008}{3040}} \\
 &\doteq 4.21 \text{ or } 3.83
 \end{aligned}$$

Thus, if he charges approximately \$4.21/kg or \$3.83/kg he can expect a total profit of \$1900.

If he charges \$3.83/kg, he will only make \$0.62 profit/kg, but will sell enough to make \$1900 in total profit.

If he charges \$4.21/kg, he will make more profit/kg, but will sell much less, resulting in the same total profit.

$$\text{(d)} \quad P(x) = -3040(x - 4.02)^2 + 2008$$

This is a parabola that opens downward with its vertex at (4.02, 2008). Thus, he should set the price at \$4.02/kg to get a maximum total profit of \$2008.

(e) If the supply cost drops to \$3.10/kg, then the new profit model would be: $P(x) = (x - 3.10)(14\,700 - 3040x)$

This has zeroes at $x = 3.1$ and $x = \frac{14\,700}{3040} \doteq 4.8355$.

The x-coordinate of the vertex is: $x = \frac{3.1 + 4.8355}{2} \doteq 3.97$.

$$P(3.97) = (3.97 - 3.1)(14\,700 - 3040(3.97)) \doteq 2290$$

Thus, if the supply cost drops to \$3.10/kg, the manager should set the price at \$3.97/kg. He would make about \$2290 in total profit.

(f)

Supply cost(\$/kg)	Zeroes	x-coordinate
3.21	$4.8355 + 3.21$	4.02
3.10	$4.8355 + 3.10$	3.97
2.50	$4.8355 + 2.50$	3.67
2.00	$4.8355 + 2.00$	3.42
4.00	$4.8355 + 4.00$	4.42
4.80	$4.8355 + 4.80$	4.82

x	$P(x)$	Profit: $P(x)$
4.02	$(x - 3.21)(14\,700 - 3040x)$	\$2008
3.97	$(x - 3.1)(14\,700 - 3040x)$	\$2290
3.67	$(x - 2.5)(14\,700 - 3040x)$	\$4146
3.42	$(x - 2)(14\,700 - 3040x)$	\$6110
4.42	$(x - 4)(14\,700 - 3040x)$	\$531
4.82	$(x - 4.8)(14\,700 - 3040x)$	\$0.94

(g) The main reason that demand rises for ground beef when the manager lowers his price is that customers from other stores come to his store to buy his beef. If supply costs drop, they would drop for everyone, so the formula $m(x)$ would no longer be valid. For example, if the supply cost of ground beef rose to \$4.84/kg, our current model suggests that no one would buy the beef, even if the manager only charged \$4.84/kg. While demand for ground beef would drop if the price rose that high, there would still be some people willing to buy it. If the manager sells it at his cost, he will get many customers from other stores because they are likely to try to earn a profit on it.

$$20. \text{ Given } -2 < x < 3 \text{ and } f(x) = \frac{1}{x}$$

Substitute -2 and 3 into $f(x)$ to find the upper and lower bounds of the range.

$$f(-2) = -\frac{1}{2} \text{ and } f(3) = \frac{1}{3}$$

The value of y can be anything greater than $\frac{1}{3}$ or anything less than $-\frac{1}{2}$.

$$\text{Range} = \{y \mid y > \frac{1}{3} \text{ or } y < -\frac{1}{2}, y \in \mathbb{R}\}$$

Practise, Apply, Solve 3.7, page 285

1. The Base Function determines the shape that is preserved after the transformations. For example if $f(x) = x^2$, then the graph of $y = 3f(x - 2) + 7$ will be the same shape as $f(x)$, but will be stretched and translated into a new position. The reflections, translations and stretches do not change the basic shape of the graph.

$$\begin{array}{l}
 2. \quad \boxed{x} \rightarrow \boxed{+2} \rightarrow \boxed{f} \rightarrow \boxed{y} \\
 \boxed{-1} \leftarrow \boxed{-2} \leftarrow \boxed{(1, 5)} \rightarrow \boxed{5} \\
 \boxed{0} \leftarrow \boxed{-2} \leftarrow \boxed{(2, 3)} \rightarrow \boxed{3} \\
 \boxed{1} \leftarrow \boxed{-2} \leftarrow \boxed{(3, 7)} \rightarrow \boxed{7}
 \end{array}$$

Thus, $(-1, 5)$, $(0, 3)$, and $(1, 7)$ are three points on g .

$$3. \text{ (a) } (1, 4), (2, 10), (3, 14), (4, 16) \text{ satisfy } y = 2g(x)$$

$$\begin{array}{ccc}
 x & \xrightarrow{\boxed{g}} & \xrightarrow{\boxed{\times 2}} y \\
 1 & & 4 \\
 2 & & 10 \\
 3 & & 14 \\
 4 & & 16
 \end{array}$$

(b) (1, 4), (2, 7), (3, 9) and (4, 10) satisfy $y = g(x) + 2$

x	----->	$\boxed{g}$	----->	$\boxed{+2}$	----->	y
1						4
2						7
3						9
4						10

(c) (-1, 2), (0, 5), (1, 7) and (2, 8) satisfy $y = g(x + 2)$

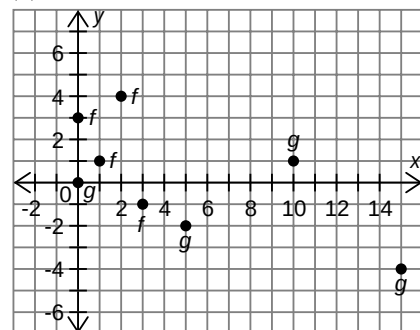
x	----->	$\boxed{+2}$	----->	$\boxed{g}$	----->	y
-1						2
0						5
1						7
2						8

(d) (0.5, 2), (1, 5), (1.5, 7) and (2, 8) satisfy $y = g(2x)$

x	----->	$\boxed{\times 2}$	----->	$\boxed{g}$	----->	y
0.5						2
1						5
1.5						7
2						8

4. $f = \{(0, 3), (1, 1), (2, 4), (3, -1)\}$
 $g = \{(0, 0), (5, -2), (10, 1), (15, -4)\}$

(a)

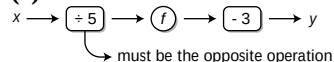


(b)

x	----->	$\boxed{\div 5}$	----->	$\boxed{f}$	----->	$\boxed{-3}$	----->	y
0								0
5								-2
10								1
15								-4

Multiply the x-coordinates of f by 5 and subtract 3 from the y-coordinates to get g .

(c)



(d) $g(x) = f\left(\frac{x}{5}\right) - 3$

(e) Divide x-coordinates of g by 5; add 3 to the y-coordinates

(f)

x	----->	$\boxed{\times 5}$	----->	$\boxed{g}$	----->	$\boxed{+3}$	----->	y
0								3
1								1
2								4
3								-1

The mapping diagram for f is:



(g) $f(x) = g(5x) + 3$

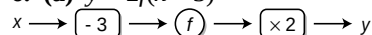
5. **(a)** No stretches or flips are needed. But, a translation left 2 units and up 3 units is needed to obtain g from f .

(b)



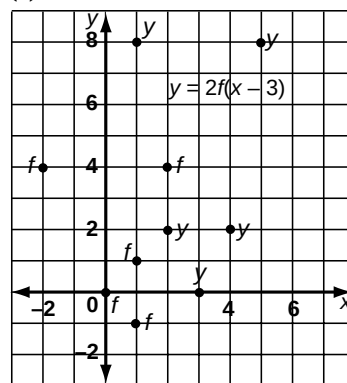
(c) $g(x) = f(x + 2) + 3$

6. **(a)** $y = 2f(x - 3)$



(b) Stretch $y = f(x)$ vertically with a factor of 2 and translate 3 units right, in any order.

(c)



7. **(a)** f has been stretched horizontally by a factor of 2. (The x-coordinates have been multiplied by 2.) This corresponds to: **ii.** $y = f(0.5x)$.

(b) f has been reflected in the y-axis. (The x-coordinates have been multiplied by -1.) This corresponds to: **iv.** $y = f(-x)$.

(c) f has been translated vertically 3 units down (3 has been subtracted from the y-coordinates.) This corresponds to: **x.** $y = f(x) - 3$.

(d) f has been stretched vertically by a factor of 0.5. (The y-coordinates have been multiplied by 0.5.) This corresponds to: **viii.** $y = 0.5f(x)$.

(e) f has been reflected in the x-axis. (The y-coordinates have been multiplied by -1.) This corresponds to: **iii.** $y = -f(x)$.

(f) f has been translated 3 units right. (3 has been added to the x-coordinates.) This corresponds to: **i.** $y = f(x - 3)$.

8. **(a)** f has been translated 2 units right and 3 units down. (2 has been added to the x-coordinates and 3 has been subtracted from the y-coordinates.) This corresponds to: **ix.** $y = f(x - 2) - 3$.

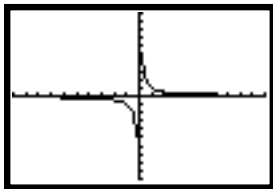
(b) f has been reflected in the y-axis and then translated down 3 units. (The x-coordinates have been multiplied by -1 and 3 has been subtracted from the y-coordinates.) This corresponds to: **xii.** $y = f(-x) - 3$.

(c) f has been stretched vertically by a factor of 0.5 and stretched horizontally by a factor of 2. (The x-coordinates have been multiplied by 2 and the y-coordinates have been multiplied by 0.5.) This corresponds to: **iii.** $y = 0.5f(0.5x)$.

(d) f has been stretched vertically by a factor of 0.5, stretched horizontally by a factor of $\frac{1}{3}$, then translated horizontally 2 units right and down 5 units. (The x-coordinates have been multiplied by $\frac{1}{3}$ and the y-coordinates have been multiplied

by 0.5. Then, 2 was added to the new x -coordinates, and 5 was subtracted from the new y -coordinates.) This corresponds to: **xvii.** $y = 0.5f(3(x - 2)) - 5$.

9. i.



ii. (a) f is translated 2 units right

(b) f is translated 1 unit left

(c) f is translated up 2 units

(d) f is translated down 6 units

(e) f is stretched vertically by a factor of 0.5

(f) f is stretched vertically by a factor of 2

(g) f is stretched horizontally by a factor of 2

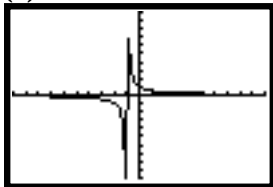
(h) f is stretched horizontally by a factor of 0.5

(i) f is reflected in the x -axis

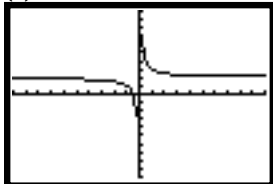
(iii) (a)



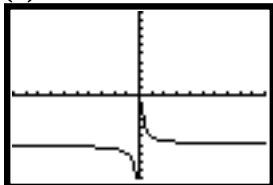
(b)



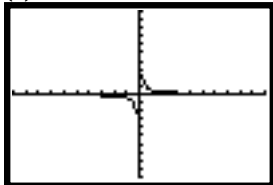
(c)



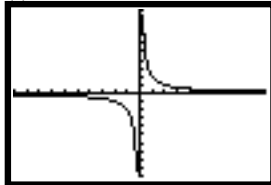
(d)



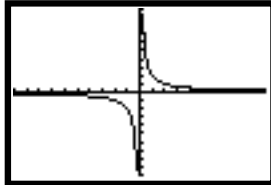
(e)



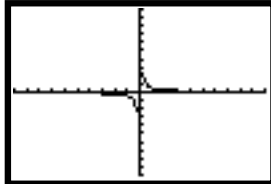
(f)



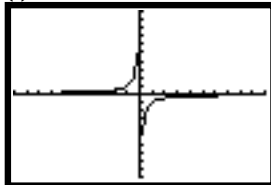
(g)



(h)



(i)



(iv) $f: D = \{x \mid x \neq 0, x \in \mathbf{R}\}$

$R = \{y \mid y \neq 0, y \in \mathbf{R}\}$

(a) $D = \{x \mid x \neq 2, x \in \mathbf{R}\}$

$R = \{y \mid y \neq 0, y \in \mathbf{R}\}$

(b) $D = \{x \mid x \neq -1, x \in \mathbf{R}\}$

$R = \{y \mid y \neq 0, y \in \mathbf{R}\}$

(c) $D = \{x \mid x \neq 0, x \in \mathbf{R}\}$

$R = \{y \mid y \neq 2, y \in \mathbf{R}\}$

(d) $D = \{x \mid x \neq 0, x \in \mathbf{R}\}$

$R = \{y \mid y \neq -6, y \in \mathbf{R}\}$

(e) $D = \{x \mid x \neq 0, x \in \mathbf{R}\}$

$R = \{y \mid y \neq 0, y \in \mathbf{R}\}$

(f) $D = \{x \mid x \neq 0, x \in \mathbf{R}\}$

$R = \{y \mid y \neq 0, y \in \mathbf{R}\}$

(g) $D = \{x \mid x \neq 0, x \in \mathbf{R}\}$

$R = \{y \mid y \neq 0, y \in \mathbf{R}\}$

(h) $D = \{x \mid x \neq 0, x \in \mathbf{R}\}$

$R = \{y \mid y \neq 0, y \in \mathbf{R}\}$

(i) $D = \{x \mid x \neq 0, x \in \mathbf{R}\}$

$R = \{y \mid y \neq 0, y \in \mathbf{R}\}$

(v) Equation **(f)**:

$$\begin{aligned} y &= \frac{1}{0.5x} \\ &= \left(\frac{1}{0.5}\right)\left(\frac{1}{x}\right) \\ &= (2)\left(\frac{1}{x}\right) \\ &= \frac{2}{x} \end{aligned}$$

Equation (g):

$$y = \frac{2}{x}$$

Thus, the graphs are identical because the equations are identical.

10. $g(x) = x^2$

(a) $y = -3g(x-2) + 1$

$$x \rightarrow \boxed{-2} \rightarrow \boxed{9} \rightarrow \boxed{x(-3)} \rightarrow \boxed{+1} \rightarrow y$$

(b)

$$x \leftarrow +2 \leftarrow g \rightarrow \boxed{\times(-3)} \rightarrow +1 \rightarrow y$$

$$6 \leftarrow \dots 4 \quad 16 \rightarrow \dots -48 \rightarrow \dots -47$$

Thus, (4, 16) corresponds to (6, -47) on the new graph.

(c) Add 2 to x-coordinates: horizontal translation 2 units right; multiply y-coordinates by -3: vertical stretch by a factor of 3 and reflection in x-axis; add 1 to the y-coordinates: vertical translation up 1 unit. The horizontal translation can take place at any time, but the vertical stretch and reflection must happen before the vertical translation.

(d) $g: D = \{x \mid x \in \mathbf{R}\}$

$$R = \{y \mid y \geq 0, y \in \mathbf{R}\}$$

Only horizontal transformations affect the domain, and only vertical transformations affect the range.

The horizontal translation of 2 units does not affect the domain since it is $\mathbf{R}$. Thus, the new domain is $\mathbf{R}$.

The reflection in the x-axis changes the range to: $\{y \mid y \leq 0, y \in \mathbf{R}\}$. The vertical stretch has no effect but the vertical translation of 1 unit transforms the range to: $\{y \mid y \leq 1, y \in \mathbf{R}\}$.

The new function is a parabola that opens downward and has a vertex at (2, 1). It's domain is: $\{x \mid x \in \mathbf{R}\}$ and the range is: $\{y \mid y \leq 1, y \in \mathbf{R}\}$.

11. (a)

$$4 \rightarrow \boxed{-3} \rightarrow (1) \rightarrow x^3 \rightarrow (1) \rightarrow \boxed{\times 2} \rightarrow (2) \rightarrow \boxed{-4} \rightarrow -2$$

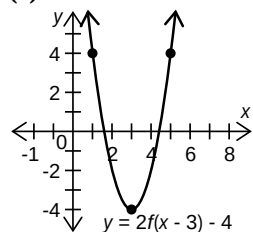
Thus, $g(4) = -2$.

(b) $x \leftarrow \boxed{+3} \leftarrow \boxed{7} \rightarrow \boxed{\times 2} \rightarrow \boxed{-4} \rightarrow y$

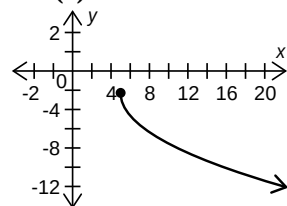
$$5 \leftarrow \boxed{+3} \leftarrow (2) \leftarrow \boxed{7} \rightarrow (0) \rightarrow \boxed{\times 2} \rightarrow (0) \rightarrow \boxed{-4} \rightarrow -4$$

If $(2, 0) \in f$, then $(5, -4) \in g$.

(c)



12. (a)



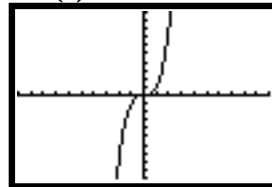
(b) $x \rightarrow \boxed{-5} \rightarrow (x-5) \rightarrow \boxed{\times -1} \rightarrow -(x-5) \rightarrow \boxed{\sqrt{\quad}}$

$$\rightarrow (\sqrt{-(x-5)}) \rightarrow \boxed{\times 3} \rightarrow (3\sqrt{-(x-5)}) \rightarrow \boxed{-2}$$

$$\rightarrow (3\sqrt{-(x-5)} - 2) \rightarrow y$$

Thus, the new equation is: $y = 3\sqrt{-(x-5)} - 2$.

13. (a)

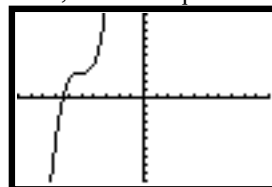


(b) 5 is subtracted from the x-coordinates, and 3 is added to the y-coordinates of f .

$$x \leftarrow \boxed{-5} \leftarrow \boxed{f(x) = x^3} \rightarrow \boxed{+3} \rightarrow y$$

$$x \rightarrow \boxed{+5} \rightarrow (x+5) \rightarrow \boxed{f} \rightarrow [(x+5)^3] \rightarrow \boxed{+3} \rightarrow [(x+5)^3 + 3] \rightarrow y$$

Thus, the new equation is: $y = (x+5)^3 + 3$.



14. (a) f is reflected in the y-axis, so the x-coordinates are multiplied by -1. f is stretched vertically by a factor of 3, so the y-coordinates are multiplied by 3. f is translated up 2 units, so add 2 to the y-coordinates. f is also translated left 1 unit, so subtract 1 from the x-coordinates.

$$x \leftarrow \boxed{-1} \leftarrow \boxed{x(-1)} \leftarrow (f) \rightarrow \boxed{\times 3} \rightarrow \boxed{+2} \rightarrow y$$

(b) $x \rightarrow \boxed{+1} \rightarrow (x+1) \rightarrow \boxed{x(-1)} \rightarrow (-x-1) \rightarrow \boxed{7}$

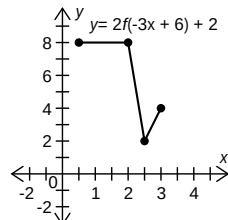
$$\rightarrow (f(-x-1)) \rightarrow \boxed{\times 3} \rightarrow (3f(-x-1)) \rightarrow \boxed{+2}$$

$$\rightarrow (3f(-x-1) + 2) \rightarrow y$$

Thus, the new equation is: $y = 3f(-x-1) + 2$.

15. $y = 2f(-3x+6) + 2 = 2f[-3(x-2)] + 2$

x	y
$\frac{1}{3}$	8
2	8
$\frac{8}{3}$	2
$\frac{10}{3}$	4



16. $f: D = \{x \mid x \geq -4, x \in \mathbf{R}\}, R = \{y \mid y < -1, y \in \mathbf{R}\}$

(a) Multiply each y-coordinate by 2.

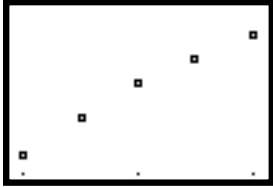
$D = \{x \mid x \geq -4, x \in \mathbf{R}\}, R = \{y \mid y < -2, y \in \mathbf{R}\}$

(b) Multiply each x -coordinate by -1 . (Change the sign of the inequality.)
 $D = \{x \mid x \leq 4, x \in \mathbf{R}\}$, $R = \{y \mid y < -1, y \in \mathbf{R}\}$

(c) Add -1 to each x -coordinate. Multiply each y -coordinate by 3 and then add 4 .
 $D = \{x \mid x \geq -5, x \in \mathbf{R}\}$, $R = \{y \mid y < 1, y \in \mathbf{R}\}$

(d) Multiply each x -coordinate by -1 (change the sign of the inequality) and add 5 . Multiply each y -coordinate by -1 (change the sign of the inequality), then multiply by 2 and then add 1 .
 $D = \{x \mid x \leq 9, x \in \mathbf{R}\}$, $R = \{y \mid y > 3, y \in \mathbf{R}\}$

17. (a)



(b) $y = 0.55\sqrt{x}$ (Answers may vary)

(c) The points on the left are above the curve in **(b)** while the points on the right are below the curve in **(b)**. It appears that a translation is needed.

(d) $y = 0.46\sqrt{x} + 0.14$ (Answers may vary)

18. **(a)** The order of the two vertical transformations is important. The vertical stretch must be performed before the translation down. If $(0, 1)$ is stretched first to $(0, 2)$ and then translated down, the image is $(0, -2)$; if $(0, 1)$ is translated first to $(0, -3)$ and then stretched, the image is $(0, -6)$. The translation right can be applied any time since it has no impact on the y -coordinates.

(b) Yes, the horizontal translation could happen at any time. For example: translation right 3 units, vertical stretch by a factor of 2, translation 4 units down **or** vertical stretch by a factor of 2, translation 4 units down, translation right 3 units.

19. $y = 3f(2x - 4)$

(a) $x \rightarrow \boxed{\times 2} \rightarrow \boxed{-4} \rightarrow \boxed{f} \rightarrow \boxed{\times 3} \rightarrow y$

$x \leftarrow \boxed{\times 1/2} \leftarrow \boxed{+4} \leftarrow \boxed{f} \rightarrow \boxed{\times 3} \rightarrow y$

Correct sequence: stretch vertically by a factor of 3, stretch horizontally by a factor of $\frac{1}{2}$, and translate 2 units right.

(b) The vertical stretch can occur at any time, giving two other possibilities: horizontal stretch by a factor of $\frac{1}{2}$, translation right 2 units, vertical stretch by a factor of 3 **or** horizontal stretch by a factor of $\frac{1}{2}$, vertical stretch by a factor of 3, translation right 2 units.

20. **(a)** Let H_1 represent the first horizontal transformation: a stretch by a factor of $\frac{1}{2}$.

Let H_2 represent the second horizontal transformation: translation left 1 unit.

Let V_1 represent the first vertical transformation: reflection in the x -axis.

Let V_2 represent the second vertical transformation: translation 3 units up.

Any sequence that keeps H_1 before H_2 and V_1 before V_2 will work. All possible sequences are:

H_1, H_2, V_1, V_2

H_1, V_1, H_2, V_2

H_1, V_1, V_2, H_2

V_1, H_1, H_2, V_2

V_1, H_1, V_2, H_2

V_1, V_2, H_1, H_2

(b) Yes, translate horizontally 2 units left, stretch

horizontally with factor of $\frac{1}{2}$, translate vertically 3 units

down, and then reflect in x -axis.

21. When a function is expressed in terms of another base function, use order of operations to draw a mapping diagram. Operations to the right of the base function correspond to the vertical transformations. Multiplication/division yields a stretch and addition/subtraction gives a translation.

Multiplication by -1 is a reflection. The order of operations on the right, matches the order in which the transformations need to be applied. Operations to the left of the base function correspond to the horizontal transformations. You must reverse the operations and their sequence to find the correct transformations to apply to the base function to graph the required function.

22. **(a)** The vertex has moved from $(0, 0)$ to $(-6, 2)$. If the parabola was stretched first and then translated, translations of 6 left and 2 up would be needed. Before the translations, it would have a vertex at $(0, 0)$, open down and be slightly wider than $y = x^2$. A reflection in the x -axis is also needed. The point $(0, -7)$ is on the translated parabola. Before the translation, it would have been at $(6, -9)$. If $x = 6$, $x^2 = 36$, so $(6, 36)$ is on $y = x^2$. This would require a reflection in the x -axis and a vertical stretch by a factor of $\frac{1}{4}$.

Thus a sequence of transformations that would work is: a reflection in the x -axis, stretch vertically by a factor of $\frac{1}{4}$, translation of 6 units left, and a translation of 2 units up.

(b) $y = -\frac{1}{4}(x+6)^2 + 2$

23. **(a)** $y = 5\sqrt{x-3.1}$

(b) The inverse relation has a quadratic of best fit with approximate equation $y = 0.053x^2 - 0.092x + 3.15$. Thus, the original function is: $x = 0.053y^2 - 0.092y + 3.15$.

Thus:

$$x \doteq 0.053(y^2 - 1.74y) + 3.15$$

$$= 0.053(y^2 - 1.74y + 0.7569 - 0.7569) + 3.15$$

$$\doteq 0.053(y - 0.87)^2 - 0.040 + 3.15$$

$$= 0.053(y - 0.87)^2 + 3.11$$

$$x - 3.11 \doteq 0.053(y - 0.87)^2$$

$$\frac{x - 3.11}{0.053} \doteq (y - 0.87)^2$$

$$y - 0.87 = \pm \sqrt{\frac{x - 3.11}{0.053}}$$

$$y = \pm \sqrt{\frac{x - 3.11}{0.053}} + 0.87$$

If we ignore the lower branch and simplify we get:

$$y \doteq \frac{\sqrt{x - 3.11}}{\sqrt{0.053}} + 0.87$$

$$y \doteq \frac{\sqrt{x - 3.11}}{0.230} + 0.87$$

$$y \doteq 4.34\sqrt{x - 3.11} + 0.87$$

(c) The equations are very similar. The second equation involves a vertical translation which made it closer to the points on the left of the scatter plot.

24. (a) (0, 0) $\rightarrow$ (3, -1)

(2, 4) $\rightarrow$ (5, 2)

A translation of 3 units right and 1 unit down is needed after any necessary stretches and reflections.

Before the translation the parabola had a vertex at (0, 0) and the point corresponding to (5, 2) was at (2, 4).

A vertical stretch by a factor of $\frac{3}{4}$ would transform (2, 4) into (2, 3).

Apply the following sequence of transformations: stretch

vertically by a factor of $\frac{3}{4}$, translate right 3 units and

translate down 1 unit.

$$x \leftarrow \boxed{+3} \leftarrow \boxed{f} \rightarrow \boxed{\times 3/4} \rightarrow \boxed{-1} \rightarrow y$$

$$x \rightarrow \boxed{-3} \rightarrow (x - 3) \rightarrow \boxed{f} \rightarrow (x - 3)^2 \rightarrow \boxed{\times 3/4} \rightarrow$$

$$\left(\frac{3}{4}(x - 3)^2\right) \rightarrow \boxed{-1} \rightarrow \left(\frac{3}{4}(x - 3)^2 - 1\right) \rightarrow y$$

The quadratic equation is $y = \frac{3}{4}(x - 3)^2 - 1$.

(b) (0, 0) $\rightarrow$ (3, -1)

(1, 1) $\rightarrow$ (5, 2)

As in part (a), before the required translation, the vertex was at (0, 0) and the point corresponding to (5, 2) was (1, 1).

To stretch (1, 1) to (5, 2), we could stretch vertically by a factor of 3 and then horizontally by a factor of 2.

Apply the following sequence of transformations: stretch horizontally by a factor of 2, stretch vertically by a factor of 3, translate down 1 unit and translate right 3 units.

$$x \leftarrow \boxed{\times 2} \leftarrow \boxed{\times 3} \leftarrow \boxed{f} \rightarrow \boxed{-1} \rightarrow y$$

$$x \rightarrow \boxed{-3} \rightarrow (x - 3) \rightarrow \boxed{\times 1/2} \rightarrow \frac{1}{2}(x - 3) \rightarrow \boxed{f} \rightarrow$$

$$\left(\frac{1}{2}(x - 3)\right)^2 \rightarrow \boxed{\times 3} \rightarrow 3\left(\frac{1}{2}(x - 3)\right)^2 \rightarrow \boxed{-1} \rightarrow$$

$$\left(3\left(\frac{1}{2}(x - 3)\right)^2 - 1\right) \rightarrow y$$

Thus, the required quadratic equation is:

$$y = 3\left(\frac{1}{2}(x - 3)\right)^2 - 1$$

Part 2: Functions

(c) If we take the answer to part (b) and simplify:

$$y = 3\left(\frac{1}{4}(x - 3)^2\right) - 1$$

$$= \frac{3}{4}(x - 3)^2 - 1$$

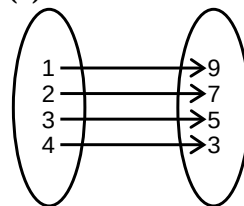
we get the same equation as part(a).

This implies that any two parabolas with the same vertex can be stretched into each other using any two points(not the vertices) to find a correspondence.

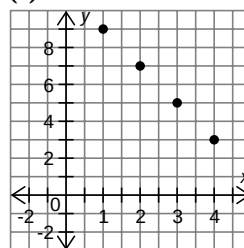
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1. (a) $D = \{1, 2, 3, 4\}$, $R = \{3, 5, 7, 9\}$

(b)



(c)



(d) The slope of the line through these points is -2:

$$m = \frac{7 - 9}{2 - 1} = -2$$

Extrapolating we would have a y-intercept of 11. Thus, the equation is: $y = -2x + 11$, $\{x \leq 4, x \in \mathbf{N}\}$

(e) Mapping Rule: $x \rightarrow -2x + 11$, $\{x \leq 4, x \in \mathbf{N}\}$

If $f(x)$ is the function, then $f(x) = -2x + 11$, $\{x \leq 4, x \in \mathbf{N}\}$.

(f) This is a function because each element of the domain corresponds to a unique element of the range. It also passes the vertical line test.

(g) Choose any x-coordinate (1, 2, 3 or 4) with a different y-coordinate than the one given. This x value now corresponds to more than one value of the range, so that $f(x)$ is no longer a function. E.g. (1, 1)

2. (a) $D = \{-3, -2, -1, 0, 1, 2\}$ or $\{x \mid -3 \leq x \leq 2, x \in \mathbf{I}\}$,

$R = \{-1, 0, 1, 2, 3, 4, 5\}$ or $\{y \mid -1 \leq y \leq 5, y \in \mathbf{I}\}$

This is not a function because -2 maps onto 2 and 5.

(b) $D = \{x \mid -3 \leq x \leq 4, x \in \mathbf{R}\}$, $R = \{y \mid -1 \leq y \leq 5, y \in \mathbf{R}\}$

This is not a function because 0 maps onto 5 and -1.

(c) $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \geq -1, y \in \mathbf{R}\}$

This is a function. It passes the vertical line test.

3. $f(x) = 2x^2 - 6x$

(a) $f(-2) = 2(-2)^2 - 6(-2) = 2(4) + 12 = 20$