

Practise, Apply, Solve 1.7, page 57

1. (a) i. geometric, ii. $r = \frac{12}{4} = 3$

(b) i. geometric, ii. $r = \frac{15}{-5} = -3$

(c) i. $r_1 = \frac{24}{17}, r_2 = \frac{31}{24}, r_1 \neq r_2$, not geometric

(d) i. geometric, ii. $r = \frac{8}{2} = 4$

(e) i. $r_1 = \frac{45}{56}, r_2 = \frac{34}{45}, r_1 \neq r_2$, not geometric

(f) i. geometric, ii. $r = \frac{1}{8} \div \frac{1}{4} = \frac{1}{2}$

2. (a) i. $t_n = 3^{n-1}$, geometric, ii. $r = 3$

(b) i. $t_n = (n+3)(n-5)$, not geometric

(c) i. $t_n = -2n^2 - 5n + 1$, not geometric

(d) i. $t_n = 2(6)^{n+1}$, geometric, ii. $r = 6$

(e) i. $t_n = \frac{1}{3n-2}$, not geometric

(f) i. $t_n = 5(-1)^n$, geometric, i. $r = -1$

3. (a) i. $r = \frac{15}{3} = 5$ ii. $t_n = 3(5)^{n-1}$

iii. $t_8 = 3(5)^{8-1} = 3(5)^7 = 234\,375$

(b) i. $r = \frac{-144}{-12} = 12$ ii. $t_n = (-12)12^{n-1} = -12^n$

iii. $t_8 = -12^8 = -429\,981\,696$

(c) i. $r = \frac{2}{4} = \frac{1}{2}$ ii. $t_n = 4\left(\frac{1}{2}\right)^{n-1} = (2^2)(2)^{-n+1} = 2^{-n+3}$

iii. $t_8 = 2^{-8+3} = 2^{-5} = 0.031\,25$

(d) i. $r = \frac{-12}{6} = -2$ ii. $t_n = 6(-2)^{n-1}$

iii. $t_8 = 6(-2)^{8-1} = 6(-2)^7 = -768$

(e) i. $r = \frac{0.02}{0.2} = 0.1$ ii. $t_n = 0.2(0.1)^{n-1}$

iii. $t_8 = 0.2(0.1)^{8-1} = 0.2(0.1)^7 = 0.000\,000\,02 = 2 \times 10^{-8}$

(f) i. $r = \frac{5}{5} = 1$ ii. $t_n = (5)(1)^{n-1} = 5$ iii. $t_8 = 5(1)^{8-1} = 5$

4. (a) $t_1 = 3, t_n = 5(t_{n-1})$

(b) $t_1 = -12, t_n = 12(t_{n-1})$

(c) $t_1 = 4, t_n = \frac{1}{2}(t_{n-1})$

(d) $t_1 = 6, t_n = -2(t_{n-1})$

(e) $t_1 = 0.2, t_n = 0.1(t_{n-1})$

(f) $t_1 = 5, t_n = t_{n-1}$

5. (a) $a = 3, r = 7, t_n = 3(7)^{n-1}$

(b) $a = -4, r = \frac{-1}{4},$

$$t_n = (-4)\left(\frac{-1}{4}\right)^{n-1} = (-1)(4)(-1)^{n-1}(4^{-1})^{n-1} = (-1)^n(4)^{-n+2}$$

(c) $a = 125, r = -0.2,$

$$t_n = 125(-0.2)^{n-1} = (5^3)(-1)^{n-1}(5^{-1})^{n-1} = (-1)^{n-1}(5)^{-n+4}$$

(d) $a = -6, r = 3, t_n = -6(3)^{n-1} = -2(3)(3)^{n-1} = -2(3)^n$

6. (a) i. $\{30, 40, 50, 60, \dots\}$, arithmetic, ii. $a = 30, d = 10,$
 $t_n = 30 + (n-1)(10) = 10n + 20$

(b) i. $\{4, 16, 64, 256, \dots\}$, geometric, ii. $a = 4, r = 4,$
 $t_n = 4(4)^{n-1} = 4^n$

(c) i. $\{2, 6, 7, 21, 22, \dots\}, d_1 = 6 - 2 = 4, d_2 = 7 - 6 = 1,$
 $d_1 \neq d_2, \therefore$ not arithmetic, $r_1 = \frac{6}{2} = 3, r_2 = \frac{7}{6}, r_1 \neq r_2, \therefore$ not
 geometric, $\therefore$ sequence is neither

(d) i. $\{30, 6, 1.2, 0.24, \dots\}$, geometric, ii. $a = 30, r = \frac{1}{5},$

$$t_n = 30\left(\frac{1}{5}\right)^{n-1}$$

(e) i. $\{145, 130, 115, 100, \dots\}$, arithmetic, ii. $a = 145,$
 $d = -15, t_n = 145 + (n-1)(-15) = -15n + 160$

(f) i. $\{1.21, 1.44, 1.69, 1.96, \dots\}, d_1 = 1.44 - 1.21 = 0.23,$
 $d_2 = 1.69 - 1.44 = 0.25, d_1 \neq d_2, \therefore$ not arithmetic,

$$r_1 = \frac{1.44}{1.21} = 1.19, r_2 = \frac{1.69}{1.44} = 1.17, r_1 \neq r_2, \therefore$$
 not geometric,
 $\therefore$ sequence is neither

7. (i) $a = 1, r = 2$ (a) $t_n = 2^{n-1}$ (b) $t_{10} = 2^9 = 512$

(ii) $a = -40, r = \frac{1}{2}$ (a) $t_n = -40\left(\frac{1}{2}\right)^{n-1} = -5(2^{-n+4})$

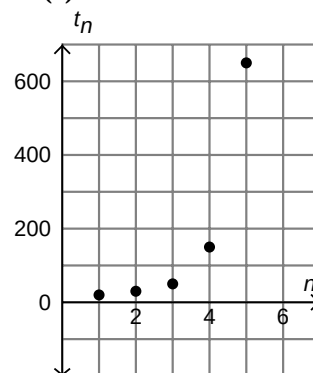
(b) $t_{10} = -5(2^{-6}) = \frac{-5}{64}$

(iii) $a = 4, r = -1$ (a) $t_n = 4(-1)^{n+1}$ (b) $t_{10} = 4(-1)^{10+1} = -4$

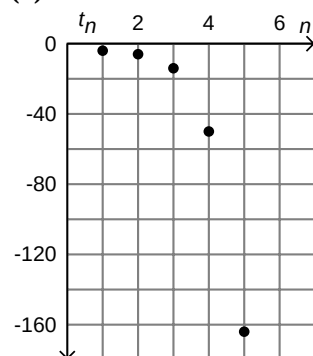
(iv) $a = 8, r = \frac{1}{2}$ (a) $t_n = 8\left(\frac{1}{2}\right)^{n-1} = 2^3(2)^{-n+1} = 2^{-n+4}$

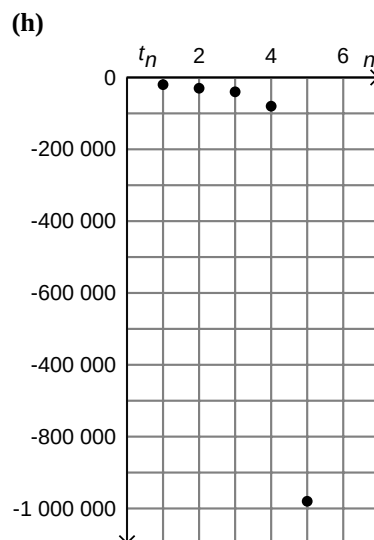
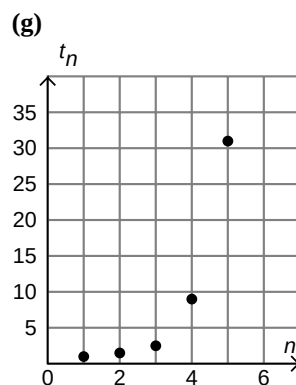
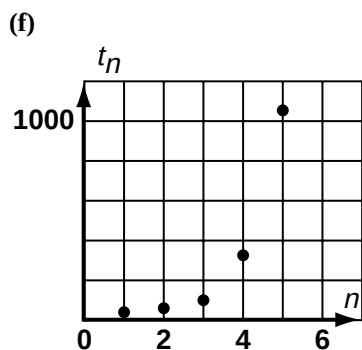
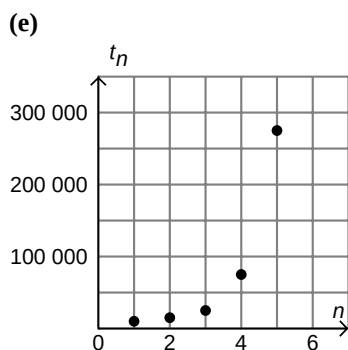
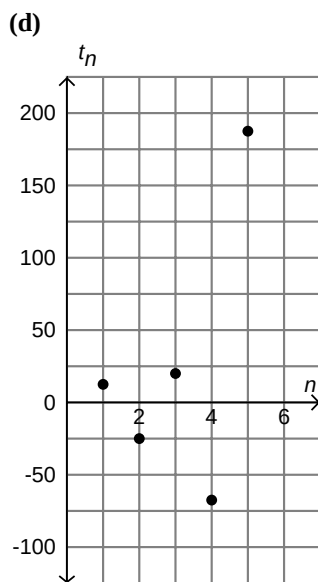
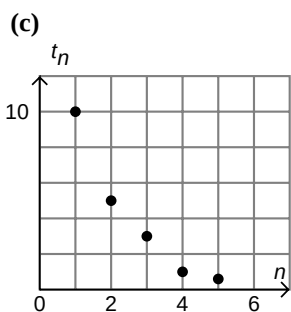
(b) $t_{10} = 2^{-10+4} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$

8. (a)



(b)





9. (a) $a = 1, r = \frac{-2}{3}, t_n = \left(\frac{-2}{3}\right)^{n-1}$

(b) $t_8 = \left(\frac{-2}{3}\right)^7 = \frac{-128}{2187}$

10. (a) \$28 500, \$25 080, \$22 070.40, \$19 421.95, \$17 091.32, \$15 040.36

(b) $a = \$28\,500, r = 88\% = 0.88, t_n = 28\,500(0.88)^{n-1}$

(c) $t_9 = \$28\,500(0.88)^8 = \$10\,249.58$

11. (a) \$34 000, \$34 850, \$35 721.25, \$36 614.28, \$37 529.64

(b) $a = \$34\,000, r = 1.025, t_n = \$34\,000(1.025)^{n-1}$

(c) $t_{10} = \$34\,000(1.025)^9 = \$42\,461.34$

12. $a = \$500, r = 6\% = 1.06, t_n = \$500(1.06)^{n-1998}$

For 2010, $t_{2010} = \$500(1.06)^{2010-1998}$

$$= \$500(1.06)^{12}$$

$$= \$1006.10$$

∴ the coin has a value of \$1006.10 in 2010.

$$13. a = 500, r = \frac{1}{2}, t_n = 500\left(\frac{1}{2}\right)^n$$

$$\text{In 8 hours, } n = 8 \text{ h} \times \frac{60 \text{ min/h}}{10 \text{ min}}$$

$$= 8 \text{ h} \times \frac{60 \text{ min}}{\text{h}} \times \frac{1}{10 \text{ min}}$$

$$= 48$$

$$t_{48} = 500\left(\frac{1}{2}\right)^{48} = 1.776 \times 10^{-12} \text{ g}$$

$\therefore 1.776 \times 10^{-12} \text{ g}$ of the substance remains after 8 h.

$$14. a = 20, r = 2, t_n = 20(2)^n$$

$$\text{After 4 hours, } n = 4 \text{ h} \times \frac{60 \text{ min}}{\text{h}} \times \frac{1}{20 \text{ min}} = 12$$

$$t_{12} = 20(2)^{12} = 81\,920$$

$\therefore 81\,920$ bacteria exist after 4 h.

$$15. (a) a = 1.65 \text{ billion} = 1.65 \times 10^9, r = 1.35\% = 1.0135$$

$$t_n = (1.65 \times 10^9)(1.0135)^{n-1900}$$

$$(b) t_{2010} = (1.65 \times 10^9)(1.0135)^{2010-1900}$$

$$= (1.65 \times 10^9)(1.0135)^{110}$$

$$= 7.21 \times 10^9$$

16. The value is multiplied by a factor of 0.75 each year, which is the definition of a geometric sequence.

$$17. (a) 1, 3, 9, 27$$

$$(b) a = 1, r = 3, t_n = (1)(3)^{n-1}, t_8 = 3^7 = 2187$$

$$(c) 3, \frac{9}{2}, \frac{27}{4}, \frac{81}{8}$$

$$(d) a = 3, r = \frac{3}{2}, P_n = 3\left(\frac{3}{2}\right)^{n-1} = \frac{3^n}{2^{n-1}},$$

$$P_7 = \frac{3^7}{2^6} = \frac{2187}{64} \doteq 34.17 \text{ units}$$

$$18. t_1 = a, t_n = rt_{n-1}$$

$$19. a = \$45\,000, r = 100\% - 8\% = 92\% = 0.92,$$

$$t_n = \$45\,000(0.92)^n$$

$$t_n < 0.40(45\,000)$$

$$45\,000(0.92)^n < 0.40(45\,000)$$

$$0.92^n < 0.40$$

$$n \ln(0.92) < \ln(0.40)$$

$$n > \frac{\ln(0.40)}{\ln(0.92)}$$

$$n \doteq 10.98$$

$$n \geq 11$$

$\therefore$ the sports utility vehicle is worth less than 40% of its original value after 11 years.

$$20. (a) t_6 = -4, t_7 = -20, r = \frac{t_7}{t_6} = \frac{-20}{-4} = 5,$$

$$t_6 = a(5)^5$$

$$-4 = a(3125)$$

$$a = -0.001\,28$$

$$\therefore t_n = -0.00128(5)^{n-1}$$

$$(b) t_2 = 8, t_4 = 32, \frac{t_4}{t_2} = \frac{ar^3}{ar} = r^2 = \frac{32}{8} = 4, r = \pm 2$$

$$t_2 = 8$$

$$t_2 = 8$$

$$a(-2) = 8$$

$$a(2) = 8$$

$$a = -4$$

$$a = 4$$

OR

$$t_n = 4(2)^{n-1}$$

$$t_n = (-4)(-2)^{n-1}$$

$$t_n = 2^{n+1}$$

$$t_n = (-1)(-1)^{n-1} 2^2 2^{n-1}$$

$$t_n = (-1)^n 2^{n+1}$$

$$(c) t_4 = 54, t_7 = 1458$$

$$\frac{t_7}{t_4} = \frac{1458}{54} = 27 = r^3, \therefore r = 3$$

$$t_4 = a(3)^3 = 54, \therefore a = 2, t_n = 2(3)^{n-1}$$

$$21. (a) a = 4, r = \frac{28}{4} = 7, t_n = 4(7)^{n-1},$$

$$t_{10} = 4(7)^9 = 161\,414\,428$$

$$(b) t_3 = 12, t_4 = 4, r = \frac{t_4}{t_3} = \frac{4}{12} = \frac{1}{3}, t_3 = 3 \times t_4,$$

$$t_2 = 3 \times t_3 = 3 \times 12 = 36, t_1 = 3 \times t_2 = 3 \times 36 = 108$$

Going in reverse, multiply by 3 since $r = \frac{1}{3}$. Going forward,

multiply by $\frac{1}{3}$ to get the next term.

$$22. \text{Example: } \{5, 5, 5, 5, \dots\}, a = 5, r = 1, d = 0$$

$$\{a, a, a, a, \dots\}, a = a, r = 1, d = 0$$

23. The sequence looks as follows:

$$\{6 \triangle_{6, 6\sqrt{2}}, \frac{6}{2} \triangle_{\frac{6}{2}\sqrt{2}, \frac{6}{2}\sqrt{2}\sqrt{2}}, \dots\}$$

$$= \{6 \triangle_{6, 6\sqrt{2}}, 3\sqrt{2} \triangle_{3\sqrt{2}, 6}, 3 \triangle_{3, 3\sqrt{2}}, \dots\}$$

$$\text{A leg of an } n^{\text{th}} \text{ level triangle, } L(n) = 6\left(\frac{\sqrt{2}}{2}\right)^{n-1} = 6 \times 2^{\frac{1-n}{2}}.$$

The area of an n^{th} level triangle is

$$A(n) = \frac{1}{2} L^2(n) = \frac{1}{2} (36) 2^{1-n} = (18) 2^{1-n}.$$

$$\text{Total Area} = A(1) + A(2) + \dots + A(6)$$

$$= 18 \left[1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{32} \right]$$

$$= 35 \frac{7}{16} \text{ units}^2$$

$\therefore$ the total area of the shaded region at this point is approximately 35.44 units^2 .

24. 8, a , b , 36

Since the first three form an arithmetic sequence, then

$$b - a = a - 8, 2a - b = 8 \quad \textcircled{1}$$

Since the last three form a geometric sequence, then

$$\frac{36}{b} = \frac{b}{a}, 36a = b^2 \quad \textcircled{2}$$

$$\textcircled{1} \times 18: 36a - 18b = 144, 36a = 144 + 18b$$

$$\text{By } \textcircled{2}: 144 + 18b = b^2$$

$$b^2 - 18b - 144 = 0$$

$$(b + 6)(b - 24) = 0$$

$$b = -6 \text{ or } b = 24$$

If $b = -6$, $a = 1$. If $b = 24$, $a = 16$.

$\therefore (a, b) = (1, -6) \text{ or } (a, b) = (16, 24)$.

$$25. \frac{a^2}{b}, -a, b, \frac{-b^2}{a}, \dots$$

$$r = \frac{-a}{\frac{a^2}{b}} = \frac{-b}{a} = \frac{-b^2}{a^2}, t_n = \frac{a^2}{b} \left(\frac{-b}{a} \right)^{n-1}$$

$$t_{10} = \frac{a^2}{b} \left(\frac{(-1)^9 b^9}{a^9} \right) = \frac{-b^8}{a^7}$$

Practise, Apply, Solve 1.8, page 70

1. (a) i. $i = 0.05$ ii. $n = 5$

(b) i. $i = \frac{8\%}{2} = 0.04$

ii. $n = \frac{2 \text{ conversions}}{\text{year}} \times 9 \text{ years} = 18$

(c) i. $i = \frac{4.4\%}{4} = 0.011$

ii. $n = \frac{4 \text{ conversions}}{\text{year}} \times 7 \text{ years} = 28$

(d) i. $i = \frac{6\%}{12} = 0.005$

ii. $n = \frac{12 \text{ conversions}}{\text{year}} \times 3 \text{ years} = 36$

(e) i. $i = 0.06$ ii. $n = 5$

(f) i. $i = \frac{8.2\%}{4} = 0.0205$

ii. $n = \frac{4 \text{ conversions}}{12 \text{ months}} \times 16 \text{ months} = \frac{16}{3}$

(g) i. $i = \frac{4.5\%}{2} = 0.0225$

ii. $n = \frac{42 \text{ months}}{12 \text{ months}} \times \frac{2 \text{ conversions}}{\text{year}} = 7$

(h) i. $i = \frac{52\%}{52} = 0.001$

ii. $n = \frac{52 \text{ weeks}}{\text{year}} \times \frac{1 \text{ conversion}}{\text{week}} \times 2 \text{ years} = 104$

(i) i. $i = \frac{5.475\%}{365} = 0.00015$

ii. $n = \frac{365 \text{ days}}{\text{year}} \times \frac{1 \text{ conversion}}{\text{day}} \times 3 \text{ years} = 1095$

$$2. \text{ (a) } (1.1)^{12} = 3.1384$$

$$\text{ (b) } (1.05)^{-20} = 0.3769$$

$$\text{ (c) } (1.025)^{36} = 2.4325$$

$$\text{ (d) } (1.002)^{-45} = 0.9140$$

$$\text{ (e) } 200(1.04)^{24} = 512.6608$$

$$\text{ (f) } 5000(1.12)^{-5} = 2837.1343$$

$$\text{ (g) } 6525(1.03)^{-22} = 3405.3486$$

$$\text{ (h) } 10\,000(1.006)^{36} = 12\,403.0161$$

3. (a) A = amount of investment

(b) P = principle amount invested

(c) i = interest rate for each conversion period

(d) n = number of conversion periods

$$4. \text{ (a) i. } A = \$4000(1.03)^4 = \$4502.04$$

$$\text{ ii. } I = A - P = \$4502.04 - \$4000 = \$502.04$$

$$\text{ (b) i. } A = \$7500 \left[1 + \frac{0.06}{12} \right]^{6 \times 12} = \$10\,740.33$$

$$\text{ ii. } I = A - P = \$10\,740.33 - \$7500 = \$3240.33$$

$$\text{ (c) i. } A = \$15\,000 \left[1 + \frac{0.024}{4} \right]^{4 \times 5} = \$16\,906.39$$

$$\text{ ii. } I = A - P = \$16\,906.39 - \$15\,000 = \$1906.39$$

$$\text{ (d) i. } A = \$28\,200 \left[1 + \frac{0.055}{2} \right]^{20} = \$48\,516.08$$

$$\text{ ii. } I = A - P = \$48\,516.08 - \$28\,200 = \$20\,316.08$$

$$\text{ (e) i. } A = \$850 \left[1 + \frac{0.0365}{365} \right]^{365} = \$881.60$$

$$\text{ ii. } I = A - P = \$881.60 - \$850 = \$31.60$$

$$\text{ (f) i. } A = \$2225 \left[1 + \frac{0.052}{52} \right]^{47} = \$2332.02$$

$$\text{ ii. } I = A - P = \$2332.02 - \$2225 = \$107.02$$

$$5. \text{ (a) i. } P = \frac{\$5000}{(1.04)^3} = \$4444.98$$

$$\text{ ii. } I = A - P = \$5000 - \$4444.98 = \$555.02$$

$$\text{ (b) i. } P = \frac{\$13\,500}{\left[1 + \frac{0.06}{12} \right]^{48}} = \$10\,625.83$$

$$\text{ ii. } I = A - P = \$13\,500 - \$10\,625.83 = \$2874.17$$

$$\text{ (c) i. } P = \frac{\$11\,200}{\left[1 + \frac{0.044}{4} \right]^{20}} = \$8999.01$$

$$\text{ ii. } I = A - P = \$11\,200 - \$8999.01 = \$2200.99$$

$$\text{ (d) i. } P = \frac{\$128\,500}{\left[1 + \frac{0.065}{2} \right]^{16}} = \$77\,030.40$$

$$\text{ ii. } I = A - P = \$128\,500 - \$77\,030.40 = \$51\,469.60$$

$$\text{ (e) i. } P = \frac{\$850}{\left[1 + \frac{0.0584}{365} \right]^{400}} = \$797.31$$

$$\text{ ii. } I = A - P = \$850 - \$797.31 = \$52.69$$

$$\text{ (f) i. } P = \frac{\$6225}{\left[1 + \frac{0.13}{52} \right]^{100}} = \$4849.55$$

$$\text{ ii. } I = A - P = \$6225 - \$4849.55 = \$1375.45$$

6. The amount is the future value of the principal amount after the principal amount earns interest. The present value is