

$$x - 3.11 \div 0.053(y - 0.87)^2$$

$$\frac{x - 3.11}{0.053} \div (y - 0.87)^2$$

$$y - 0.87 = \pm \sqrt{\frac{x - 3.11}{0.053}}$$

$$y = \pm \sqrt{\frac{x - 3.11}{0.053}} + 0.87$$

If we ignore the lower branch and simplify we get:

$$y \div \frac{\sqrt{x - 3.11}}{\sqrt{0.053}} + 0.87$$

$$y \div \frac{\sqrt{x - 3.11}}{0.230} + 0.87$$

$$y \div 4.34\sqrt{x - 3.11} + 0.87$$

(c) The equations are very similar. The second equation involves a vertical translation which made it closer to the points on the left of the scatter plot.

24. (a) (0, 0) → (3, -1)

(2, 4) → (5, 2)

A translation of 3 units right and 1 unit down is needed after any necessary stretches and reflections.

Before the translation the parabola had a vertex at (0, 0) and the point corresponding to (5, 2) was at (2, 4).

A vertical stretch by a factor of $\frac{3}{4}$ would transform (2, 4) into (2, 3).

Apply the following sequence of transformations: stretch

vertically by a factor of $\frac{3}{4}$, translate right 3 units and

translate down 1 unit.

$$x \leftarrow \boxed{+3} \leftarrow \boxed{-1} \rightarrow \boxed{\times 3/4} \rightarrow \boxed{-1} \rightarrow y$$

$$x \rightarrow \boxed{-3} \rightarrow (x - 3) \rightarrow \boxed{1} \rightarrow (x - 3)^2 \rightarrow \boxed{\times 3/4} \rightarrow$$

$$\left(\frac{3}{4}(x - 3)^2\right) \rightarrow \boxed{-1} \rightarrow \left(\frac{3}{4}(x - 3)^2 - 1\right) \rightarrow y$$

The quadratic equation is $y = \frac{3}{4}(x - 3)^2 - 1$.

(b) (0, 0) → (3, -1)

(1, 1) → (5, 2)

As in part (a), before the required translation, the vertex was at (0, 0) and the point corresponding to (5, 2) was (1, 1).

To stretch (1, 1) to (5, 2), we could stretch vertically by a factor of 3 and then horizontally by a factor of 2.

Apply the following sequence of transformations: stretch horizontally by a factor of 2, stretch vertically by a factor of 3, translate down 1 unit and translate right 3 units.

$$x \leftarrow \boxed{\times 2} \leftarrow \boxed{-1} \rightarrow \boxed{\times 3} \rightarrow \boxed{-1} \rightarrow y$$

$$x \rightarrow \boxed{-3} \rightarrow (x - 3) \rightarrow \boxed{\times 1/2} \rightarrow \frac{1}{2}(x - 3) \rightarrow \boxed{1} \rightarrow$$

$$\left(\frac{1}{2}(x - 3)\right)^2 \rightarrow \boxed{\times 3} \rightarrow 3\left(\frac{1}{2}(x - 3)\right)^2 \rightarrow \boxed{-1} \rightarrow$$

$$\left(3\left(\frac{1}{2}(x - 3)\right)^2 - 1\right) \rightarrow y$$

Thus, the required quadratic equation is:

$$y = 3\left(\frac{1}{2}(x - 3)\right)^2 - 1$$

Part 2: Functions

(c) If we take the answer to part (b) and simplify:

$$y = 3\left(\frac{1}{4}(x - 3)^2\right) - 1$$

$$= \frac{3}{4}(x - 3)^2 - 1$$

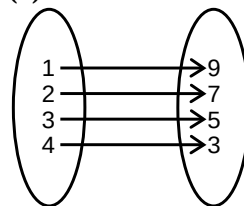
we get the same equation as part(a).

This implies that any two parabolas with the same vertex can be stretched into each other using any two points(not the vertices) to find a correspondence.

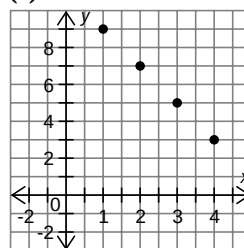
Chapter 3, Review and Practice, page 293

1. (a) $D = \{1, 2, 3, 4\}$, $R = \{3, 5, 7, 9\}$

(b)



(c)



(d) The slope of the line through these points is -2:

$$m = \frac{7 - 9}{2 - 1} = -2$$

Extrapolating we would have a y-intercept of 11. Thus, the equation is: $y = -2x + 11$, $\{x \leq 4, x \in \mathbf{N}\}$

(e) Mapping Rule: $x \rightarrow -2x + 11$, $\{x \leq 4, x \in \mathbf{N}\}$

If $f(x)$ is the function, then $f(x) = -2x + 11$, $\{x \leq 4, x \in \mathbf{N}\}$.

(f) This is a function because each element of the domain corresponds to a unique element of the range. It also passes the vertical line test.

(g) Choose any x-coordinate (1, 2, 3 or 4) with a different y-coordinate than the one given. This x value now corresponds to more than one value of the range, so that $f(x)$ is no longer a function. E.g. (1, 1)

2. (a) $D = \{-3, -2, -1, 0, 1, 2\}$ or $\{x \mid -3 \leq x \leq 2, x \in \mathbf{I}\}$,

$R = \{-1, 0, 1, 2, 3, 4, 5\}$ or $\{y \mid -1 \leq y \leq 5, y \in \mathbf{I}\}$

This is not a function because -2 maps onto 2 and 5.

(b) $D = \{x \mid -3 \leq x \leq 4, x \in \mathbf{R}\}$, $R = \{y \mid -1 \leq y \leq 5, y \in \mathbf{R}\}$

This is not a function because 0 maps onto 5 and -1.

(c) $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \geq -1, y \in \mathbf{R}\}$

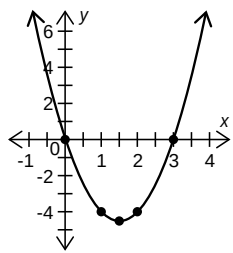
This is a function. It passes the vertical line test.

3. $f(x) = 2x^2 - 6x$

(a) $f(-2) = 2(-2)^2 - 6(-2) = 2(4) + 12 = 20$

(b) $f(x) = 2x(x - 3)$

zeros at 0 and 3



(c) f is a function. It passes the vertical line test.

(d) $(5, 20) \in f$. Thus, $f(5) = 20$

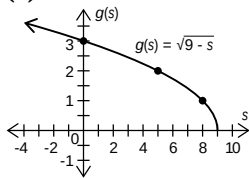
(e) If $f(x) = 0$, then $2x(x - 3) = 0$.

Thus, $2x = 0$ and $x - 3 = 0$.

So, $x = 0$ or $x = 3$.

4. $g(s) = \sqrt{9 - s}$

(a)



(b) $D = \{s \mid s \leq 9, s \in \mathbf{R}\}$, $R = \{y \mid y \geq 0, y \in \mathbf{R}\}$

5. (a) The independent variable is her weekly sales and the dependent variable is her weekly earnings. It should be a function because the same value for her sales should produce the same total earnings. It is important that the employee's pay be reliable and predictable.

(b) Her sales are most likely greater than 0. But there is the possibility that she might not sell anything and have a total weekly sales of 0. Theoretically, her sales could be infinitely large but it is unlikely that she would sell more than \$20 000 worth of goods in one week. Thus, a reasonable domain is: $\{x \mid 0 \leq x \leq 20\,000, x \in \mathbf{R}\}$. Her weekly earnings will be at least \$85 and if she sells \$20 000:

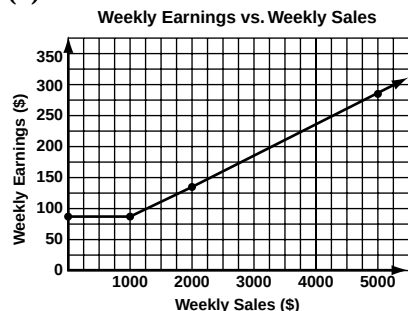
$$\text{earnings} = 85 + 0.05(19\,000) = \$1035.$$

Thus, a reasonable range is: $\{y \mid 85 \leq y \leq 1035, y \in \mathbf{R}\}$.

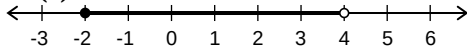
(c) Earnings = \$85 + 5% sales over \$1000

$$g(5000) = 85 + 0.05(4000) = 85 + 200 = 285$$

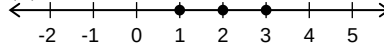
(d)



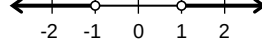
6. (a)



(b)



(c)



7. (a) $\{x \mid x \geq -2, x \in \mathbf{I}\}$

(b) $\{x \mid -5 < x \leq 3, x \in \mathbf{R}\}$

8. (a) If $f(x) > 2$ then:

$$\frac{4 - 2x}{5} > 2$$

$$5\left(\frac{4 - 2x}{5}\right) > 5(2)$$

$$4 - 2x > 10$$

$$-2x > 6$$

$$x < -3$$

Thus, $D = \{x \mid x < -3, x \in \mathbf{R}\}$.

(b) $-8 \leq x < 2$

then:

$$16 \geq -2x > -4$$

$$20 \geq 4 - 2x > 0$$

$$4 \geq \frac{4 - 2x}{5} > 0$$

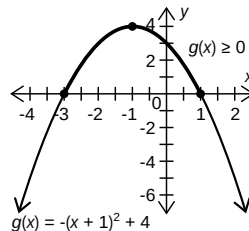
Thus, $R = \{y \mid 0 < y \leq 4, y \in \mathbf{R}\}$.

9. (a) vertex at $(-1, 4)$, opens down

$$g(0) = g(-2) = 3,$$

$$g(1) = g(-3) = 0$$

$$g(2) = g(-4) = -5$$



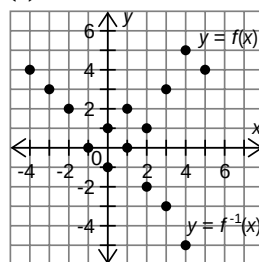
(b) If $g(x) > 0$, then $D = \{x \mid -3 \leq x \leq 1, x \in \mathbf{R}\}$

10. (a) $D = \{-5, -3, -2, -1, 0, 1, 2, 3, 4\}$,

$R = \{-1, 0, 1, 2, 3, 4, 5\}$

(b) $f^{-1} = \{(-1, 0), (0, -1), (0, 1), (1, 2), (2, -2), (3, -3), (3, 3), (4, -5), (5, 4)\}$

(c)



From the graph in part(c):

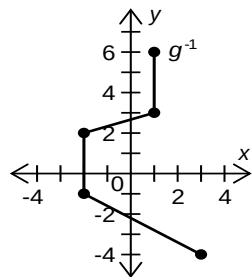
(d) $f(1) = 0$

(e) $f^{-1}(1) = 2$

(f) $f^{-1}(0) = \pm 1$

(g) $f(-4)$ is undefined.

11.



12. $f(x) = \frac{x+3}{2}$

Method 1(mapping diagrams):

$f: x \rightarrow \boxed{+3} \rightarrow \boxed{\div 2} \rightarrow y$

$f^{-1}: y \leftarrow \boxed{(2x-3)} \leftarrow \boxed{-3} \leftarrow (2x) \leftarrow \boxed{\times 2} \leftarrow x$

Method 2(algebraic):

$$y = \frac{x+3}{2}$$

$$x = \frac{y+3}{2}$$

$$2x = y+3$$

$$2x-3 = y$$

$$f^{-1}(x) = 2x-3$$

13. (a) Let x represent the number of shirts needed.

Total charge = \$100 + (\$11.50)(x)

Thus, $g(x) = 100 + 11.5x$, $x \in \mathbf{W}$

(b) $g: x \rightarrow \boxed{\times 11.5} \rightarrow \boxed{+100} \rightarrow g(x)$

$g^{-1}(x) \leftarrow \left(\frac{x-100}{11.5} \right) \leftarrow \boxed{\div 11.5} \leftarrow (x-100) \leftarrow \boxed{-100} \leftarrow x$

Thus, $g^{-1}(x) = \frac{x-100}{11.5}$, $g^{-1}(x) \in \mathbf{W}$

(c) Take the total charge, subtract \$100, and divide by \$11.50 to get the number of shirts printed.

(d) The size of printing orders may be huge or very small.

We know that the number of shirts must be a whole number, so the range of g^{-1} is $\mathbf{W}$. The total cost is \$100 + \$11.50 k (where k is the number of shirts, $k \in \mathbf{W}$).

Thus, $D = \{x \mid x = 100 + 11.5s, s \in \mathbf{W}\}$.

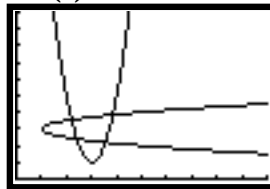
(e) Example: A school council ordered shirts from a printing company and received a bill for \$1000 (excluding tax). If the company charges \$100 + \$11.50/per shirt, determine the number of shirts ordered.

14. $y = x^2$ defines a parabola that opens up and has a vertex at (0, 0). $y = \sqrt{x}$ defines the upper branch of a parabola that opens to the right and has a vertex at (0, 0). The inverse of $y = x^2$ is a parabola that contains the branch defined by $y = \sqrt{x}$ and the branch defined by $y = -\sqrt{x}$. The inverse of $y = \sqrt{x}$ is the right branch of the parabola defined by $y = x^2$ (where $x \geq 0$). When $y = x^2$ is reflected in the line $y = x$, the result is the parabola that contains the branch defined by

$y = \sqrt{x}$. When $y = \sqrt{x}$ is reflected in the line $y = x$, the result is the right branch of the parabola defined by $y = x^2$.

15. $f^{-1} = \{(-1, 0), (0, -1), (0, 1), (1, 2), (2, -2), (3, -3), (3, 3), (4, -5), (5, 4)\}$. One of (0, -1) and (0, 1) and one of (3, -3) and (3, 3) must be eliminated from f^{-1} to make it a function. e.g. if $f^{-1} = \{(-1, 0), (0, 1), (1, 2), (2, -2), (3, 3), (4, -5), (5, 5)\}$, then it is a function because each x -coordinate corresponds to a unique value of y . This subset has the same domain as the original: $D = \{x \mid -1 \leq x \leq 5, x \in \mathbf{I}\}$.

16. (a)



(b) From the calculator: (2.2, 3.55), (2.4, 2.4), (3.55, 2.2), and (3.84, 3.84) are the approximate points of intersection.

(c) $g^{-1}: x = 4(y-3)^2 + 1$

(d)

$$x = 4(y-3)^2 + 1$$

$$x-1 = 4(y-3)^2$$

$$\frac{x-1}{4} = (y-3)^2$$

$$(y-3) = \pm \sqrt{\frac{x-1}{4}}$$

$$y = \pm \sqrt{\frac{x-1}{4}} + 3$$

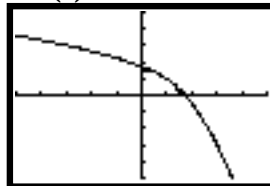
$$g^{-1}(x) = \pm \sqrt{\frac{x-1}{4}} + 3$$

(e) The vertex of g is at (3, 1) and the parabola opens upward. Its inverse is a parabola with vertex (1, 3) opening to the right. If we restrict g so that $x \geq 3$, then g^{-1} will be one branch of the parabola. (or $x \leq 3$)

(f) See the graph in part (a). If domain of g is $\{x \mid 2 \leq x \leq 5, x \in \mathbf{R}\}$, then $g(2) = 5$, $g(5) = 17$.

In this case, the inverse would not be a function. Note that $g^{-1}(5)$ would be 2 or 4. (It does not pass the vertical line test.)

17. (a)



(b) $f: y = \sqrt{3-2x}$, $x \leq \frac{3}{2}$, $f^{-1}: x = \sqrt{3-2y}$, $y \leq \frac{3}{2}$

(c)

$$y = \sqrt{3-2x}, \quad x \leq \frac{3}{2}$$

$$x = \sqrt{3-2y}, \quad y \leq \frac{3}{2}$$

$$x^2 = (\sqrt{3-2y})^2$$

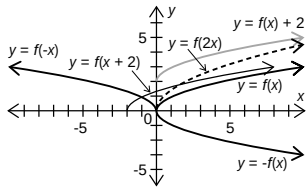
$$x^2 = 3-2y$$

$$2y = 3-x^2$$

$$y = \frac{3-x^2}{2}$$

$$f^{-1}(x) = \frac{3-x^2}{2}, \quad y \leq \frac{3}{2}$$

18.



19. (a) $g(-1) = 3f(-(-1) + 1) + 7$

$$= 3f(2) + 7$$

$$= 3(7) + 7$$

$$= 28$$

(b) $g(1) = 3f(-1 + 1) + 7$

$$= 3f(0) + 7$$

$$= 3(5) + 7$$

$$= 22$$

(c) $g(3) = 3f(-3 + 1) + 7 = 3f(-2) + 7$

undefined

(d)

x	$\rightarrow \boxed{\times (-1)}$	$\rightarrow \boxed{+1}$	$\rightarrow f$	$\rightarrow \boxed{\times 3}$	$\rightarrow \boxed{+7} \rightarrow y$
x	$\leftarrow \boxed{\div (-1)}$	$\leftarrow \boxed{-1}$	$\leftarrow f$	$\rightarrow \boxed{\times 3}$	$\rightarrow \boxed{+7} \rightarrow y$
1		-1	0	5	15
0		0	1	4	12
-1		1	2	7	21
-2		2	3	14	42

$$g = \{(-2, 49), (-1, 28), (0, 19), (1, 22)\}$$

20. $g(x) = \frac{1}{2} f(3(x+2)) - 4$

(a)

$$x \rightarrow \boxed{+2} \rightarrow \boxed{\times 3} \rightarrow \textcircled{f} \rightarrow \boxed{\times \frac{1}{2}} \rightarrow \boxed{-4} \rightarrow y$$

(b) Perform the horizontal stretch by a factor of $\frac{1}{3}$ before the horizontal translation left 2 units. Perform the vertical stretch by a factor of $\frac{1}{2}$ before the vertical translation down 4 units. Otherwise, order does not matter.

For example: vertical stretch by a factor of $\frac{1}{2}$, horizontal

stretch by a factor of $\frac{1}{3}$, translation 2 units left, translation 4 units down.

(c) $f: D = \{x \mid 2 \leq x < 5, x \in \mathbf{R}\}$

After the horizontal stretch by a factor of $\frac{1}{3}$:

$$\left(\frac{1}{3}\right)2 \leq x < 5\left(\frac{1}{3}\right)$$

$$\frac{2}{3} \leq x < \frac{5}{3}$$

After the horizontal translation left 2 units:

$$\frac{2}{3} - 2 \leq x < \frac{5}{3} - 2$$

$$-\frac{4}{3} \leq x < -\frac{1}{3}$$

Thus, the domain of $g = \{x \mid -\frac{4}{3} \leq x < -\frac{1}{3}, x \in \mathbf{R}\}$.

$f: R = \{y \mid y \geq -2, y \in \mathbf{R}\}$

After the vertical stretch by factor of $\frac{1}{2}$:

$$y \geq -2\left(\frac{1}{2}\right)$$

$$y \geq -1$$

After the vertical translation down 4 units: $y \geq -1 - 4 = -5$

Thus, the range of $g = \{y \mid y \geq -5, y \in \mathbf{R}\}$.

21. $f(x) = -g\left(\frac{1}{2}x - 3\right) + 1$

(a) $x \rightarrow \boxed{\times 1/2} \rightarrow \boxed{-3} \rightarrow g \rightarrow \boxed{\times (-1)} \rightarrow \boxed{+1} \rightarrow y$

(b) $g: D = \{x \mid -4 \leq x \leq 6, x \in \mathbf{R}\}$

After translation right 3 units:

$$-4 + 3 \leq x \leq 6 + 3$$

$$-1 \leq x \leq 9$$

After horizontal stretch by factor of 2:

$$-1(2) \leq x \leq 9(2)$$

$$-2 \leq x \leq 18$$

Thus, the domain of f is: $\{x \mid -2 \leq x \leq 18, x \in \mathbf{R}\}$.

$g: R = \{y \mid -2 \leq y \leq 3, y \in \mathbf{R}\}$

After reflection in x-axis:

$$-2(-1) \leq y \leq 3(-1)$$

$$2 \geq y \geq -3$$

After vertical translation up 1 unit:

$$2 + 1 \geq y \geq -3 + 1$$

$$3 \geq y \geq -2$$

Thus, the range of f is: $\{y \mid -2 \leq y \leq 3, y \in \mathbf{R}\}$.

(c)

$$f(4) = -g\left(\frac{1}{2}(4) - 3\right) + 1$$

$$= -g(2 - 3) + 1$$

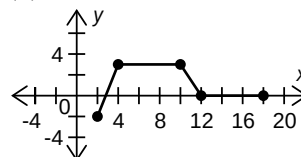
$$= -g(-1) + 1$$

$$= -(-2) + 1$$

$$= 2 + 1$$

$$= 3$$

(d)



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1. (a) i. $f(5) = 0$

ii. $D = \{x \mid -5 \leq x \leq 5, x \in \mathbf{R}\}$, $R = \{y \mid -5 \leq y \leq 5, y \in \mathbf{R}\}$

iii. This is not a function: it fails the vertical line test.

(b) i. $f(5) = 8$

ii. $D = \{1, 3, 5, 7, 9, 13\}$, $R = \{3, 6, 8, 9, 12\}$

iii. This is a function: each domain value corresponds to one value of the range.

(c) i. $f(5) = -2$

ii. $D = \{1, 4, 5, 8\}$, $R = \{-2, 2, 5, 7, 10\}$

iii. This is not a function: 1 maps to both 2 and 5.

(d) i. $f(5) = 3(5) - 7 = 15 - 7 = 8$

ii. $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \in \mathbf{R}\}$

iii. This is a function: it passes the vertical line test.

(e) i. $f(5) = \sqrt{5-3} = \sqrt{2}$

ii. $D = \{x \mid x \geq 3, x \in \mathbf{R}\}$, $R = \{y \mid y \geq 0, y \in \mathbf{R}\}$

iii. This is a function: it passes the vertical line test.

(f) i. $f(5) = (5+3)^2 - 5 = 8^2 - 5 = 64 - 5 = 59$

ii. $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \geq -5, y \in \mathbf{R}\}$

iii. This is a function: it passes the vertical line test.

2. (a) i.

$$-3(3x-2) + 4x \geq -2x + 5(2x-4)$$

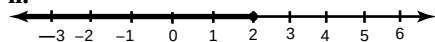
$$-9x + 6 + 4x \geq -2x + 10x - 20$$

$$-5x + 6 \geq 8x - 20$$

$$-13x \geq -26$$

$$x \leq 2$$

ii.



(b) i.

$$\frac{2x}{3} + \frac{3(x-5)}{2} \leq x$$

$$6\left(\frac{2x}{3} + \frac{3(x-5)}{2}\right) \leq x(6)$$

$$4x + 9(x-5) \leq 6x$$

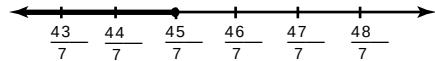
$$4x + 9x - 45 \leq 6x$$

$$4x + 9x - 6x - 45 \leq 0$$

$$7x \leq 45$$

$$x \leq \frac{45}{7}$$

ii.



3. (a)

$$y = 5x + 6$$

$$x = 5y + 6$$

$$5y = x - 6$$

$$y = \frac{x-6}{5}$$

$$f^{-1}(x) = \frac{x-6}{5}$$

(b)

$$y = \sqrt{x+4}, \quad x \geq -4$$

$$x = \sqrt{y+4}$$

$$y+4 = x^2$$

$$y = x^2 - 4$$

$$f^{-1}(x) = x^2 - 4, \quad y \geq -4$$

(c)

$$y = x^2 - 5$$

$$x = y^2 - 5$$

$$y^2 = x + 5$$

$$y = \pm\sqrt{x+5}$$

$$f^{-1}(x) = \pm\sqrt{x+5}$$

4. The inverse of a given function will not necessarily be a function. For example: if each domain value corresponds to one unique range value, then when you interchange the domain and range values to find the inverse, each domain value still corresponds to a unique range value. But, if you have a function where more than one domain value corresponds to the same value of the range, the inverse will not be a function. This happens because when you interchange the domain and range values to find the inverse, you now have a relationship where one domain value corresponds to more than one value of the range.

Example:

$$f = \{(1, 4), (2, 5), (3, 6)\} \text{ - function}$$

$$f^{-1} = \{(4, 1), (5, 2), (6, 3)\} \text{ - function}$$

But,

$$f = \{(1, 5), (2, 4), (3, 4)\} \text{ - function}$$

$$f^{-1} = \{(4, 2), (4, 3), (5, 1)\} \text{ - not a function}$$

5. If you draw a vertical line through a relation and they intersect at more than one point, then the relation is not a function. If the vertical line and the relation intersect at only one point, then the relation is a function. This vertical line test is a good test of whether or not a relation is a function because it tells us the relationship between the domain and range values. If there is more than one point of intersection between the line and the relation this means that for a certain domain value there is more than one corresponding range value. Thus, the vertical line test fails and the relation is not a function. If there is only one point of intersection between the line and the relation this means that each domain value corresponds to one unique range value. Thus, the vertical line test passes and the relation is a function.