

Chapter 3 Review Test, page 297

1. (a) i. $f(5) = 0$

ii. $D = \{x \mid -5 \leq x \leq 5, x \in \mathbf{R}\}$, $R = \{y \mid -5 \leq y \leq 5, y \in \mathbf{R}\}$

iii. This is not a function: it fails the vertical line test.

(b) i. $f(5) = 8$

ii. $D = \{1, 3, 5, 7, 9, 13\}$, $R = \{3, 6, 8, 9, 12\}$

iii. This is a function: each domain value corresponds to one value of the range.

(c) i. $f(5) = -2$

ii. $D = \{1, 4, 5, 8\}$, $R = \{-2, 2, 5, 7, 10\}$

iii. This is not a function: 1 maps to both 2 and 5.

(d) i. $f(5) = 3(5) - 7 = 15 - 7 = 8$

ii. $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \in \mathbf{R}\}$

iii. This is a function: it passes the vertical line test.

(e) i. $f(5) = \sqrt{5-3} = \sqrt{2}$

ii. $D = \{x \mid x \geq 3, x \in \mathbf{R}\}$, $R = \{y \mid y \geq 0, y \in \mathbf{R}\}$

iii. This is a function: it passes the vertical line test.

(f) i. $f(5) = (5+3)^2 - 5 = 8^2 - 5 = 64 - 5 = 59$

ii. $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \geq -5, y \in \mathbf{R}\}$

iii. This is a function: it passes the vertical line test.

2. (a) i.

$$-3(3x-2) + 4x \geq -2x + 5(2x-4)$$

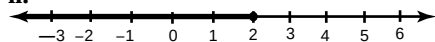
$$-9x + 6 + 4x \geq -2x + 10x - 20$$

$$-5x + 6 \geq 8x - 20$$

$$-13x \geq -26$$

$$x \leq 2$$

ii.



(b) i.

$$\frac{2x}{3} + \frac{3(x-5)}{2} \leq x$$

$$6\left(\frac{2x}{3} + \frac{3(x-5)}{2}\right) \leq x(6)$$

$$4x + 9(x-5) \leq 6x$$

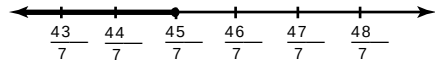
$$4x + 9x - 45 \leq 6x$$

$$4x + 9x - 6x - 45 \leq 0$$

$$7x \leq 45$$

$$x \leq \frac{45}{7}$$

ii.



3. (a)

$$y = 5x + 6$$

$$x = 5y + 6$$

$$5y = x - 6$$

$$y = \frac{x-6}{5}$$

$$f^{-1}(x) = \frac{x-6}{5}$$

(b)

$$y = \sqrt{x+4}, \quad x \geq -4$$

$$x = \sqrt{y+4}$$

$$y+4 = x^2$$

$$y = x^2 - 4$$

$$f^{-1}(x) = x^2 - 4, \quad y \geq -4$$

(c)

$$y = x^2 - 5$$

$$x = y^2 - 5$$

$$y^2 = x + 5$$

$$y = \pm\sqrt{x+5}$$

$$f^{-1}(x) = \pm\sqrt{x+5}$$

4. The inverse of a given function will not necessarily be a function. For example: if each domain value corresponds to one unique range value, then when you interchange the domain and range values to find the inverse, each domain value still corresponds to a unique range value. But, if you have a function where more than one domain value corresponds to the same value of the range, the inverse will not be a function. This happens because when you interchange the domain and range values to find the inverse, you now have a relationship where one domain value corresponds to more than one value of the range.

Example:

$$f = \{(1, 4), (2, 5), (3, 6)\} \text{ - function}$$

$$f^{-1} = \{(4, 1), (5, 2), (6, 3)\} \text{ - function}$$

But,

$$f = \{(1, 5), (2, 4), (3, 4)\} \text{ - function}$$

$$f^{-1} = \{(4, 2), (4, 3), (5, 1)\} \text{ - not a function}$$

5. If you draw a vertical line through a relation and they intersect at more than one point, then the relation is not a function. If the vertical line and the relation intersect at only one point, then the relation is a function. This vertical line test is a good test of whether or not a relation is a function because it tells us the relationship between the domain and range values. If there is more than one point of intersection between the line and the relation this means that for a certain domain value there is more than one corresponding range value. Thus, the vertical line test fails and the relation is not a function. If there is only one point of intersection between the line and the relation this means that each domain value corresponds to one unique range value. Thus, the vertical line test passes and the relation is a function.

6. (a)

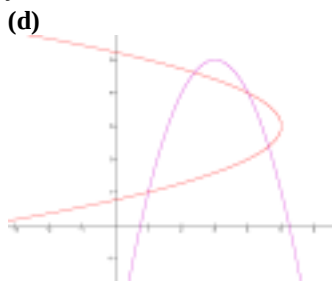


$$\begin{aligned} \text{(b)} \quad y &= -x^2 + 6x - 4 \\ &= -(x^2 - 6x) - 4 \\ &= -(x^2 - 6x + 9 - 9) - 4 \\ &= -(x^2 - 6x + 9) + 9 - 4 \\ &= -(x - 3)^2 + 5 \end{aligned}$$

This is a parabola with its vertex at (3, 5) and opening downward.

$$D = \{x \mid x \in \mathbf{R}\}, R = \{y \mid y \leq 5, y \in \mathbf{R}\}$$

$$\begin{aligned} \text{(c)} \quad y &= -(x - 3)^2 + 5 \\ x &= -(y - 3)^2 + 5 \\ (y - 3)^2 &= -x + 5 \\ y - 3 &= \pm\sqrt{-x + 5} \\ y &= \pm\sqrt{-x + 5} + 3 \end{aligned}$$



$$\text{(e)} \quad g^{-1}(x) = \pm\sqrt{-x + 5} + 3$$

This is a parabola with its vertex at (5, 3) and opening to the left.

$$D = \{x \mid x \leq 5, x \in \mathbf{R}\}, R = \{y \mid y \in \mathbf{R}\}$$

(f) $g(x)$ is a parabola that opens downward. It is a function because it passes the vertical line test. $g^{-1}(x)$ is a parabola that opens to the left. It is not a function because it fails the vertical line test.

7. Let $I(x)$ represent the vendor's total income for one game. Let $R(x)$ represent the total revenue generated. Let $P(x)$ represent the profit made and let x be the number of bags of peanuts sold.

$$\text{(a)} \quad I(x) = \$50 + \$0.05x$$

$$\text{(b)} \quad R(x) = \$2.50x$$

(c) Since $R(x)$ represents the revenue generated for the baseball club and $I(x)$ represents the income generated for the vendor, if the company is to make a profit then:

$$R(x) - I(x) > 0.$$

$$\$2.50x - (\$50 + \$0.05x) > 0$$

$$2.45x - 50 > 0$$

$$2.45x > 50$$

$$x > 20.4$$

$$x \geq 21$$

Thus, the vendor must sell at least 21 bags of peanuts before the baseball club makes a profit from his efforts.

$$\text{8. (a)} \quad y = f(x) \rightarrow y = -2f(-2x - 2) + 3$$

$$y = -2f[-2(x + 1)] + 3$$

We have an equation of the form: $y = af[k(x - p)] + q$, where $a = -2$, $k = -2$, $p = -1$ and $q = 3$.

Thus, the following transformations must be applied in order to produce the graph of $y = -2f(-2x - 2) + 3$:

(1) vertical stretch by a factor of 2

(2) reflection in the x-axis

(3) vertical translation up 3 units

(4) horizontal stretch by a factor of $\frac{1}{2}$

(5) reflection in y-axis

(6) horizontal translation left 1 unit

(b) To graph, we put the transformations in proper order:

(1) vertical stretch by factor of 2

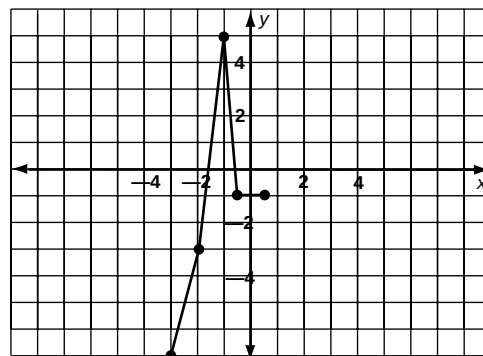
(2) reflection in x-axis

(3) vertical translation up 3 units

(4) horizontal stretch by factor of $\frac{1}{2}$

(5) reflect in y-axis

(6) horizontal translation left 1 unit



9. There are 0, 1, 2 or 4 points of intersection between a function and its inverse:

(a) **a linear function and its inverse:** If slope of $f(x)$ is not equal to 1, then the function and its inverse intersect at a point on $y = x$; otherwise, no intersection. There is a possibility of 0 or 1 point(s) of intersection.

(b) **a quadratic function and its inverse:** Parabolas opening upward: if y-intercept of $f(x)$ is less than zero, then two of the intersection points lie on line $y = x$; otherwise, no intersection. Parabolas opening downward: if y-intercept of $f(x)$ is greater than zero, then two of the intersection points lie on line $y = x$; otherwise, no intersection. There is a possibility of 0, 1, 2 or 4 point(s) of intersection.

(c) **a root function and its inverse:** When $f(x)$ and its inverse intersect, the point(s) of intersection lie(s) on line $y = x$. There is a possibility of 0, 1 or 2 point(s) of intersection.