

Chapter 4

Practise, Apply, Solve 4.1, page 306

1. (a) $2x^2 - 6x = 2(x^2 - 3x)$

(b) $3x^2 + 2x = 3\left(x^2 + \frac{2}{3}x\right)$

(c) $-4x^2 + 2x = -4\left(x^2 + \frac{2}{-4}x\right)$
 $= -4\left(x^2 - \frac{1}{2}x\right)$

(d) $\frac{1}{2}x^2 - 5x = \frac{1}{2}\left(x^2 - \frac{5}{\frac{1}{2}}x\right)$
 $= \frac{1}{2}(x^2 - 10x)$

(e) $-1.2x^2 - 6.6x = -1.2\left(x^2 - \frac{6.6}{-1.2}x\right)$
 $= -1.2(x^2 + 5.5x)$

(f) $\frac{2}{3}x^2 + \frac{5}{6}x = \frac{2}{3}\left(x^2 + \frac{\frac{5}{6}}{\frac{2}{3}}x\right)$
 $= \frac{2}{3}\left[x^2 + \left(\frac{5}{6} \times \frac{3}{2}\right)x\right]$
 $= \frac{2}{3}\left(x^2 + \frac{5}{4}x\right)$

(g) $-\frac{4}{3}x^2 - 2x = -\frac{4}{3}\left(x^2 - \frac{2}{-\frac{4}{3}}x\right)$
 $= -\frac{4}{3}\left[x^2 + \left(\frac{2}{1} \times \frac{3}{4}\right)x\right]$
 $= -\frac{4}{3}\left(x^2 + \frac{3}{2}x\right)$

(h) $2.7x^2 - 0.9x = 2.7\left(x^2 - \frac{0.9}{2.7}x\right)$
 $= 2.7\left(x^2 - \frac{1}{3}x\right)$

(i) $-6x^2 + \frac{3}{5}x = -6\left(x^2 + \frac{\frac{3}{5}}{-6}x\right)$
 $= -6\left[x^2 - \left(\frac{3}{5} \times \frac{1}{6}\right)x\right]$
 $= -6\left(x^2 - \frac{1}{10}x\right)$

2. (a) $d = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

(b) $d = \left(-\frac{7}{2}\right)^2 = \frac{49}{4}$

(c) $d = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

(d) $d = \left(-\frac{3}{10}\right)^2 = \frac{9}{100}$

(e) $d = \left(-\frac{2.5}{2}\right)^2 = (-1.25)^2 = 1.5625$

(f) $d = \left(\frac{0.45}{2}\right)^2 = (0.225)^2 = 0.050625$

(g) $d = \left(-\frac{22.4}{2}\right)^2 = (-11.2)^2 = 125.44$

(h) $d = \left(-\frac{5}{18}\right)^2 = \frac{25}{324}$

(i) $d = \left(\frac{1}{200}\right)^2 = \frac{1}{40000}$

3. (a)

$$\begin{aligned} f(x) &= (x^2 - 4x) + 3 \\ &= (x^2 - 4x + 4 - 4) + 3 \\ &= (x^2 - 4x + 4) - 4 + 3 \\ &= (x - 2)^2 - 1 \end{aligned}$$

(b)

$$\begin{aligned} f(x) &= (x^2 + 6x) - 8 \\ &= (x^2 + 6x + 9 - 9) - 8 \\ &= (x^2 + 6x + 9) - 9 - 8 \\ &= (x + 3)^2 - 17 \end{aligned}$$

(c)

$$\begin{aligned} f(x) &= 3x^2 - 6x + 1 \\ &= 3(x^2 - 2x) + 1 \\ &= 3(x^2 - 2x + 1 - 1) + 1 \\ &= 3(x^2 - 2x + 1) - 3 + 1 \\ &= 3(x - 1)^2 - 2 \end{aligned}$$

(d)

$$\begin{aligned}
 f(x) &= x^2 - x \\
 &= x^2 - x + \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2 \\
 &= \left(x^2 - x + \frac{1}{4}\right) - \frac{1}{4} \\
 &= \left(x - \frac{1}{2}\right)^2 - \frac{1}{4}
 \end{aligned}$$

(e)

$$\begin{aligned}
 f(x) &= x^2 + 3x + 5 \\
 &= \left[x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] + 5 \\
 &= \left(x^2 + 3x + \frac{9}{4}\right) - \frac{9}{4} + 5 \\
 &= \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{20}{4} \\
 &= \left(x + \frac{3}{2}\right)^2 + \frac{11}{4}
 \end{aligned}$$

$$(\text{or } f(x) = (x + 1.5)^2 + 2.75)$$

(f)

$$\begin{aligned}
 f(x) &= -x^2 + 5x - 2 \\
 &= -(x^2 - 5x) - 2 \\
 &= -\left[x^2 - 5x + \left(-\frac{5}{2}\right)^2 - \left(-\frac{5}{2}\right)^2\right] - 2 \\
 &= -\left(x^2 - 5x + \frac{25}{4} - \frac{25}{4}\right) - 2 \\
 &= -\left(x^2 - 5x + \frac{25}{4}\right) + \frac{25}{4} - 2 \\
 &= -\left(x - \frac{5}{2}\right)^2 + \frac{25}{4} - \frac{8}{4} \\
 &= -\left(x - \frac{5}{2}\right)^2 + \frac{17}{4}
 \end{aligned}$$

$$(\text{or } f(x) = -(x - 2.5)^2 + 4.25)$$

(g)

$$\begin{aligned}
 f(x) &= x^2 - 2.6x - 2.7 \\
 &= (x^2 - 2.6x + 1.3^2 - 1.3^2) - 2.7 \\
 &= (x^2 - 2.6x + 1.69) - 1.69 - 2.7 \\
 &= (x - 1.3)^2 - 4.39
 \end{aligned}$$

(h)

$$\begin{aligned}
 f(x) &= x^2 + 5.4x - 1.8 \\
 &= (x^2 + 5.4x + 2.7^2 - 2.7^2) - 1.8 \\
 &= (x^2 + 5.4x + 7.29) - 7.29 - 1.8 \\
 &= (x + 2.7)^2 - 9.09
 \end{aligned}$$

(i)

$$\begin{aligned}
 f(x) &= x^2 - 3.4x - 2.8 \\
 &= (x^2 - 3.4x + (-1.7)^2 - (-1.7)^2) - 2.8 \\
 &= (x^2 - 3.4x + 2.89) - 2.89 - 2.8 \\
 &= (x - 1.7)^2 - 5.69
 \end{aligned}$$

$$4. \quad g(x) = 2x^2 + 5x + 4$$

Step 1: Factor 2 out of first two terms:

$$g(x) = 2\left(x^2 + \frac{5}{2}x\right) + 4$$

Step 2: Add and subtract the square of half of $\frac{5}{2}$:

$$g(x) = 2\left[x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 4$$

Step 3: Move $-\left(\frac{5}{4}\right)^2$ outside bracket and remember to

$$\text{multiply by 2: } g(x) = 2\left(x^2 + \frac{5}{2}x + \frac{25}{16}\right) - 2\left(\frac{25}{16}\right) + 4$$

Step 4: Factor the perfect square and simplify the last term:

$$\begin{aligned}
 g(x) &= 2\left(x + \frac{5}{4}\right)^2 - \frac{25}{8} + \frac{32}{8} \\
 &= 2\left(x + \frac{5}{4}\right)^2 + \frac{7}{8}
 \end{aligned}$$

$$(\text{or } g(x) = 2(x + 1.25)^2 + 0.875)$$

5. (a)

$$\begin{aligned}
 f(x) &= x^2 - 3x + 2 \\
 &= \left[x^2 - 3x + \left(-\frac{3}{2}\right)^2 - \left(-\frac{3}{2}\right)^2\right] + 2 \\
 &= \left(x^2 - 3x + \frac{9}{4}\right) - \frac{9}{4} + \frac{8}{4} \\
 &= \left(x - \frac{3}{2}\right)^2 - \frac{1}{4}
 \end{aligned}$$

(b)

$$\begin{aligned}
 f(x) &= 2x^2 + 5x - 4 \\
 &= 2\left(x^2 + \frac{5}{2}x\right) - 4 \\
 &= 2\left[x^2 + 5x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] - 4 \\
 &= 2\left(x^2 + \frac{5}{2}x + \frac{25}{16}\right) - 2\left(\frac{25}{16}\right) - 4 \\
 &= 2\left(x + \frac{5}{4}\right)^2 - \frac{25}{8} - \frac{32}{8} \\
 &= 2\left(x + \frac{5}{4}\right)^2 - \frac{57}{8}
 \end{aligned}$$

$$(\text{or } f(x) = 2(x + 1.25)^2 - 7.125)$$

(c)

$$\begin{aligned}
f(x) &= 3x^2 - 4x + 1 \\
&= 3\left(x^2 - \frac{4}{3}x\right) + 1 \\
&= 3\left[x^2 - \frac{4}{3}x + \left(-\frac{2}{3}\right)^2 - \left(-\frac{2}{3}\right)^2\right] + 1 \\
&= 3\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) - 3\left(\frac{4}{9}\right) + 1 \\
&= 3\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) - \frac{4}{3} + 1 \\
&= 3\left(x - \frac{2}{3}\right)^2 - \frac{1}{3}
\end{aligned}$$

(d)

$$\begin{aligned}
f(x) &= 5x^2 - x - 5 \\
&= 5\left(x^2 - \frac{1}{5}x\right) - 5 \\
&= 5\left[x^2 - \frac{1}{5}x + \left(\frac{1}{10}\right)^2 - \left(\frac{1}{10}\right)^2\right] - 5 \\
&= 5\left(x^2 - \frac{1}{5}x + \frac{1}{100}\right) - 5\left(\frac{1}{100}\right) - 5 \\
&= 5\left(x - \frac{1}{10}\right)^2 - \frac{5}{100} - \frac{500}{100} \\
&= 5\left(x - \frac{1}{10}\right)^2 - \frac{505}{100} \\
&= 5\left(x - \frac{1}{10}\right)^2 - \frac{101}{20}
\end{aligned}$$

(or $f(x) = 5(x + 0.1)^2 + 5.05$)**(e)**

$$\begin{aligned}
f(x) &= -2x^2 + 3x + 5 \\
&= -2\left(x^2 - \frac{3}{2}x\right) + 5 \\
&= -2\left[x^2 - \frac{3}{2}x + \left(-\frac{3}{4}\right)^2 - \left(-\frac{3}{4}\right)^2\right] + 5 \\
&= -2\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) - (-2)\left(\frac{9}{16}\right) + 5 \\
&= -2\left(x - \frac{3}{4}\right)^2 + \frac{9}{8} + \frac{40}{8} \\
&= -2\left(x - \frac{3}{4}\right)^2 + \frac{49}{8}
\end{aligned}$$

(or $f(x) = -2(x + 0.75)^2 + 6.125$)**(f)**

$$\begin{aligned}
f(x) &= -4x^2 - 7x - 2 \\
&= -4\left(x^2 + \frac{7}{4}x\right) - 2 \\
&= -4\left[x^2 + \frac{7}{4}x + \left(\frac{7}{8}\right)^2 - \left(\frac{7}{8}\right)^2\right] - 2 \\
&= -4\left(x^2 + \frac{7}{4}x + \frac{49}{64}\right) - (-4)\left(\frac{7}{8}\right)^2 - 2 \\
&= -4\left(x + \frac{7}{8}\right)^2 + \frac{49}{16} - \frac{32}{16} \\
&= -4\left(x + \frac{7}{8}\right)^2 + \frac{17}{16}
\end{aligned}$$

(or $f(x) = -4(x + 0.875)^2 + 1.0625$)**(g)**

$$\begin{aligned}
f(x) &= 1.4x^2 - 4.9x - 2.7 \\
&= 1.4(x^2 - 3.5x) - 2.7 \\
&= 1.4(x^2 - 3.5x + 1.75^2 - 1.75^2) - 2.7 \\
&= 1.4(x^2 - 3.5x + 3.0625) - 1.4(3.0625) - 2.7 \\
&= 1.4(x - 1.75)^2 - 4.2875 - 2.7 \\
&= 1.4(x - 1.75)^2 - 6.9875
\end{aligned}$$

(h)

$$\begin{aligned}
f(x) &= -3.6x^2 + 5.4x - 1.67 \\
&= -3.6(x^2 - 1.5x) - 1.67 \\
&= -3.6\left(x^2 - 1.5x + 0.75^2 - 0.75^2\right) - 1.67 \\
&= -3.6(x^2 - 1.5x + 0.75^2) - (-3.6)(0.75)^2 - 1.67 \\
&= -3.6(x - 0.75)^2 + 2.025 - 1.67 \\
&= -3.6(x - 0.75)^2 + 0.355
\end{aligned}$$

(i)

$$\begin{aligned}
f(x) &= \frac{1}{2}x^2 - 4x + 1 \\
&= \frac{1}{2}(x^2 - 8x) + 1 \\
&= \frac{1}{2}(x^2 - 8x + 16 - 16) + 1 \\
&= \frac{1}{2}(x^2 - 8x + 16) - \left(\frac{1}{2}\right)(16) + 1 \\
&= \frac{1}{2}(x - 4)^2 - 8 + 1 \\
&= \frac{1}{2}(x - 4)^2 - 7
\end{aligned}$$

(j)

$$\begin{aligned}
 f(x) &= \frac{2}{3}x^2 - 5x - 2 \\
 &= \frac{2}{3}\left(x^2 - \frac{15}{2}x\right) - 2 \\
 &= \frac{2}{3}\left[x^2 - \frac{15}{2}x + \left(\frac{15}{4}\right)^2 - \left(\frac{15}{4}\right)^2\right] - 2 \\
 &= \frac{2}{3}\left(x^2 - \frac{15}{2}x + \frac{225}{16}\right) - \left(\frac{2}{3}\right)\left(\frac{225}{16}\right) - 2 \\
 &= \frac{2}{3}\left(x - \frac{15}{4}\right)^2 - \frac{75}{8} - \frac{16}{8} \\
 &= \frac{2}{3}\left(x - \frac{15}{4}\right)^2 - \frac{91}{8}
 \end{aligned}$$

(k)

$$\begin{aligned}
 f(x) &= -\frac{3}{2}x^2 - \frac{7}{8}x + \frac{1}{2} \\
 &= -\frac{3}{2}\left(x^2 + \frac{7}{12}x\right) + \frac{1}{2} \\
 &= -\frac{3}{2}\left[x^2 + \frac{7}{12}x + \left(\frac{7}{24}\right)^2 - \left(\frac{7}{24}\right)^2\right] + \frac{1}{2} \\
 &= -\frac{3}{2}\left(x^2 + \frac{7}{12}x + \frac{49}{576}\right) - \left(-\frac{3}{2}\right)\left(\frac{49}{576}\right) + \frac{1}{2} \\
 &= -\frac{3}{2}\left(x + \frac{7}{24}\right)^2 + \frac{49}{384} + \frac{192}{384} \\
 &= -\frac{3}{2}\left(x + \frac{7}{24}\right)^2 + \frac{241}{384}
 \end{aligned}$$

(l)

$$\begin{aligned}
 f(x) &= -\frac{4}{5}x^2 + \frac{2}{3}x - \frac{3}{10} \\
 &= -\frac{4}{5}\left(x^2 - \frac{5}{6}x\right) - \frac{3}{10} \\
 &= -\frac{4}{5}\left[x^2 - \frac{5}{6}x + \left(-\frac{5}{12}\right)^2 - \left(-\frac{5}{12}\right)^2\right] - \frac{3}{10} \\
 &= -\frac{4}{5}\left(x^2 - \frac{5}{6}x + \frac{25}{144}\right) - \left(-\frac{4}{5}\right)\left(\frac{25}{144}\right) - \frac{3}{10} \\
 &= -\frac{4}{5}\left(x - \frac{5}{12}\right)^2 + \frac{5}{36} - \frac{3}{10} \\
 &= -\frac{4}{5}\left(x - \frac{5}{12}\right)^2 + \frac{25}{180} - \frac{54}{180} \\
 &= -\frac{4}{5}\left(x - \frac{5}{12}\right)^2 - \frac{29}{180}
 \end{aligned}$$

6. (a) i.

$$\begin{aligned}
 f(x) &= x^2 + 10x + 24 \\
 &= (x^2 + 10x + 25 - 25) + 24 \\
 &= (x^2 + 10x + 25) - 25 + 24 \\
 &= (x + 5)^2 - 1
 \end{aligned}$$

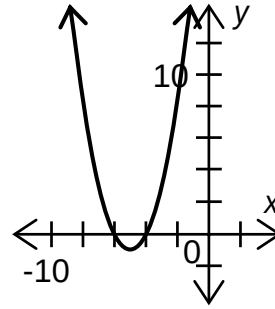
Vertex is at $(-5, -1)$.

ii. Domain is $\{x \mid x \in \mathbf{R}\}$

Range is $\{y \mid y \geq -1, y \in \mathbf{R}\}$

iii.

$$y = x^2 + 10x + 24$$



(b) i.

$$\begin{aligned}
 f(x) &= 4x^2 + 8x + 7 \\
 &= 4(x^2 + 2x) + 7 \\
 &= 4(x^2 + 2x + 1 - 1) + 7 \\
 &= 4(x^2 + 2x + 1) - 4 + 7 \\
 &= 4(x + 1)^2 + 3
 \end{aligned}$$

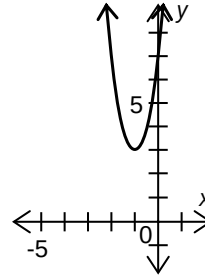
Vertex is at $(-1, 3)$.

ii. Domain is $\{x \mid x \in \mathbf{R}\}$

Range is $\{y \mid y \geq 3, y \in \mathbf{R}\}$

iii.

$$y = 4x^2 + 8x + 7$$



(c) i.

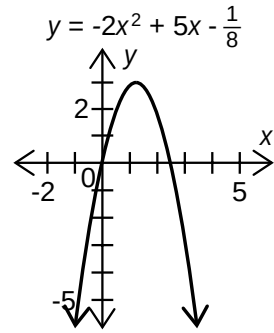
$$\begin{aligned}
 f(x) &= -2x^2 + 5x - \frac{1}{8} \\
 &= -2\left(x^2 - \frac{5}{2}x\right) - \frac{1}{8} \\
 &= -2\left[x^2 - \frac{5}{2}x + \left(-\frac{5}{4}\right)^2 - \left(-\frac{5}{4}\right)^2\right] - \frac{1}{8} \\
 &= -2\left(x^2 - \frac{5}{2}x + \frac{25}{16}\right) - (-2)\left(\frac{25}{16}\right) - \frac{1}{8} \\
 &= -2\left(x - \frac{5}{4}\right)^2 + \frac{25}{8} - \frac{1}{8} \\
 &= -2\left(x - \frac{5}{4}\right)^2 + 3
 \end{aligned}$$

Vertex is at $\left(\frac{5}{4}, 3\right)$.

ii. Domain is $\{x \mid x \in \mathbf{R}\}$

Range is $\{y \mid y \leq 3, y \in \mathbf{R}\}$

iii.



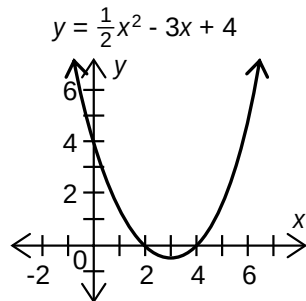
(d) i.

$$\begin{aligned} f(x) &= \frac{1}{2}x^2 - 3x + 4 \\ &= \frac{1}{2}(x^2 - 6x) + 4 \\ &= \frac{1}{2}(x^2 - 6x + 9 - 9) + 4 \\ &= \frac{1}{2}(x^2 - 6x + 9) - \frac{9}{2} + 4 \\ &= \frac{1}{2}(x - 3)^2 - \frac{1}{2} \end{aligned}$$

Vertex is at $(3, -\frac{1}{2})$.

ii. Domain is $\{x \mid x \in \mathbb{R}\}$ Range is $\{y \mid y \geq -\frac{1}{2}, y \in \mathbb{R}\}$

iii.



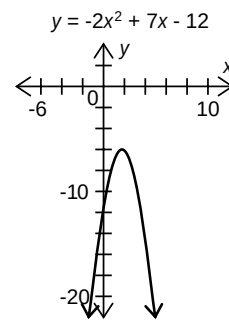
(e) i.

$$\begin{aligned} f(x) &= -2x^2 + 7x - 12 \\ &= -2\left(x^2 - \frac{7}{2}x\right) - 12 \\ &= -2\left[x^2 - \frac{7}{2}x + \left(-\frac{7}{4}\right)^2 - \left(-\frac{7}{4}\right)^2\right] - 12 \\ &= -2\left(x^2 - \frac{7}{2}x + \frac{49}{16}\right) - (-2)\left(\frac{49}{16}\right) - 12 \\ &= -2\left(x - \frac{7}{4}\right)^2 + \frac{49}{8} - \frac{96}{8} \\ &= -2\left(x - \frac{7}{4}\right)^2 - \frac{47}{8} \end{aligned}$$

Vertex is at $(\frac{7}{4}, -\frac{47}{8})$.

ii. Domain is $\{x \mid x \in \mathbb{R}\}$ Range is $\{y \mid y \leq -\frac{47}{8}, y \in \mathbb{R}\}$

iii.



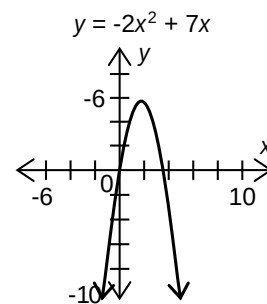
(f) i.

$$\begin{aligned} f(x) &= -2x^2 + 7x \\ &= -2\left(x^2 - \frac{7}{2}x\right) \\ &= -2\left(x^2 - \frac{7}{2}x + \frac{49}{16} - \frac{49}{16}\right) \\ &= -2\left(x^2 - \frac{7}{2}x + \frac{49}{16}\right) - (-2)\left(\frac{49}{16}\right) \\ &= -2\left(x - \frac{7}{4}\right)^2 + \frac{49}{8} \end{aligned}$$

Vertex is at $(\frac{7}{4}, \frac{49}{8})$.

ii. Domain is $\{x \mid x \in \mathbb{R}\}$ Range is $\{y \mid y \leq \frac{49}{8}, y \in \mathbb{R}\}$

iii.



Note that this curve is a vertical translation (12 units up) of the curve in 6(e).

7. (a)

$$\begin{aligned} d(x) &= 5x^2 - 3x + 1 \\ &= 5\left(x^2 - \frac{3}{5}x\right) + 1 \\ &= 5\left[x^2 - \frac{3}{5}x + \left(-\frac{3}{10}\right)^2 - \left(-\frac{3}{10}\right)^2\right] + 1 \\ &= 5\left(x^2 - \frac{3}{5}x + \frac{9}{100}\right) - 5\left(\frac{9}{100}\right) + 1 \\ &= 5\left(x - \frac{3}{10}\right)^2 + \frac{55}{100} \\ &= 5\left(x - \frac{3}{10}\right)^2 + \frac{11}{20} \\ &(\text{or } d(x) = 5(x - 0.3)^2 + 0.55) \end{aligned}$$

(b) Axis of symmetry is line $x = \frac{3}{10}$.

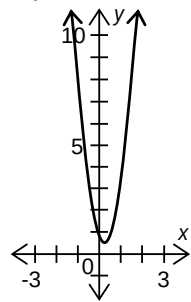
(c) Vertex is at $\left(\frac{3}{10}, \frac{11}{20}\right)$.

(d) Since $a > 0$, $d(x)$ has a minimum value. This minimum value is $\frac{55}{100}$ or 0.55.

(e) Domain is $\{x \mid x \in \mathbb{R}\}$.

(f) Range is $\{y \mid y \geq 0.55, y \in \mathbb{R}\}$

(g)



8. (a)

$$\begin{aligned} h(t) &= -4.9t^2 + 24.5t + 1 \\ &= -4.9\left(t^2 + \frac{24.5}{-4.9}t\right) + 1 \\ &= -4.9(t^2 - 5t) + 1 \\ &= -4.9(t^2 - 5t + 2.5^2 - 2.5^2) + 1 \\ &= -4.9(t^2 - 5t + 2.5^2) - (-4.9)(2.5)^2 + 1 \\ &= -4.9(t - 2.5)^2 + 30.625 + 1 \\ &= -4.9(t - 2.5)^2 + 31.625 \end{aligned}$$

Maximum height reached by the ball is approximately 31.63 m (to 2 decimal places).

(b) The ball reaches this maximum height when $t = 2.5$, ie. after 2.5 seconds.

9. (a)

$$\begin{aligned} f(x) &= 2x^2 - \frac{5}{6}x - \frac{1}{2} \\ &= 2\left(x^2 - \frac{5}{12}x\right) - \frac{1}{2} \\ &= 2\left[x^2 - \frac{5}{12}x + \left(-\frac{5}{24}\right)^2 - \left(-\frac{5}{24}\right)^2\right] - \frac{1}{2} \\ &= 2\left(x^2 - \frac{5}{12}x + \frac{25}{576}\right) - 2\left(\frac{25}{576}\right) - \frac{1}{2} \\ &= 2\left(x - \frac{5}{24}\right)^2 - \frac{25}{288} - \frac{144}{288} \\ &= 2\left(x - \frac{5}{24}\right)^2 - \frac{169}{288} \end{aligned}$$

Vertex has coordinates $\left(\frac{5}{24}, -\frac{169}{288}\right)$

(or approximately (0.2, -0.6))

(b)

$$\begin{aligned} f(x) &= -3x^2 - \frac{2}{3}x + \frac{3}{4} \\ &= -3\left(x^2 + \frac{2}{9}x\right) + \frac{3}{4} \\ &= -3\left[x^2 + \frac{2}{9}x + \left(\frac{1}{9}\right)^2 - \left(\frac{1}{9}\right)^2\right] + \frac{3}{4} \\ &= -3\left(x^2 + \frac{2}{9}x + \frac{1}{81}\right) - (-3)\left(\frac{1}{81}\right) + \frac{3}{4} \\ &= -3\left(x + \frac{1}{9}\right)^2 + \frac{1}{27} + \frac{3}{4} \\ &= -3\left(x + \frac{1}{9}\right)^2 + \frac{4}{108} + \frac{81}{108} \\ &= -3\left(x + \frac{1}{9}\right)^2 + \frac{85}{108} \end{aligned}$$

Vertex has coordinates $\left(-\frac{1}{9}, \frac{85}{108}\right)$.

(or approximately coordinates (-0.1, 0.8))

(c)

$$\begin{aligned} f(x) &= \frac{1}{2}x^2 + 4x - 8 \\ &= \frac{1}{2}(x^2 + 8x) - 8 \\ &= \frac{1}{2}(x^2 + 8x + 16 - 16) - 8 \\ &= \frac{1}{2}(x^2 + 8x + 16) - \frac{1}{2}(16) - 8 \\ &= \frac{1}{2}(x + 4)^2 - 16 \end{aligned}$$

Vertex has coordinates $(-4, -16)$.

(d)

$$\begin{aligned} f(x) &= -6x^2 + \frac{6}{7}x - \frac{3}{7} \\ &= -6\left[x^2 - \frac{6}{42}x + \left(-\frac{3}{42}\right)^2 - \left(-\frac{3}{42}\right)^2\right] - \frac{3}{7} \\ &= -6\left(x - \frac{3}{42}\right)^2 + 6\left(-\frac{3}{42}\right)^2 - \frac{3}{7} \\ &= -6\left(x - \frac{1}{14}\right)^2 + 6\left(\frac{1}{196}\right) - \frac{3}{7} \\ &= -6\left(x - \frac{1}{14}\right)^2 + \frac{3}{98} - \frac{42}{98} \\ &= -6\left(x - \frac{1}{14}\right)^2 - \frac{39}{98} \end{aligned}$$

Vertex has coordinates $\left(\frac{1}{14}, -\frac{39}{98}\right)$.

(or approximately (0.1, -0.4))

10. $y = 0.4x^2 - 2x + 3.5$

Step 1: Factor 0.4 from the first two terms

$$y = 0.4(x^2 - 5x) + 3.5$$

Step 2: Add and subtract the square of half of -5 inside the bracket.

$$y = 0.4[x^2 - 5x + (-2.5)^2 - (-2.5)^2] + 3.5$$

Step 3: Group the terms that form the complete square, and move the subtracted value outside the brackets, remembering to multiply by 0.4.

$$y = 0.4[x^2 - 5x + (-2.5)^2] - 0.4(-2.5)^2 + 3.5$$

Step 4: Factor the completed square and simplify the last term.

$$y = 0.4(x - 2.5)^2 - 2.5 + 3.5$$

$$y = 0.4(x - 2.5)^2 + 1$$

The transformations which have been applied to $y = x^2$ to obtain this equation are:

(i) a vertical stretch (compression) with factor 0.4

(ii) a horizontal shift 2.5 units to the right

(iii) a vertical shift 1 unit up

11.

$$y = -446x^2 + 3345x - 1478$$

$$= -446\left(x^2 - \frac{3345}{446}x\right) - 1478$$

$$= -446\left(x^2 - 7.5x\right) - 1478$$

$$= -446\left(x^2 - 7.5x + 3.75^2 - 3.75^2\right) - 1478$$

$$= -446\left(x^2 - 7.5x + 3.75^2\right) - (-446)(3.75)^2 - 1478$$

$$= -446(x - 3.75)^2 + 6271.875 - 1478$$

$$= -446(x - 3.75)^2 + 4793.875$$

According to this model, the yield (represented by y) has a maximum value when $x = 3.75$. $\therefore$ 3.75 metres of water / hectare will produce the greatest yield.

12. If $t^2 < 12t - 20$

$$\text{then } t^2 - 12t + 20 < 0$$

$$(t^2 - 12t + 36 - 36) + 20 < 0$$

$$(t^2 - 12t + 36) - 36 + 20 < 0$$

$$(t - 6)^2 - 16 < 0$$

$$(t - 6)^2 < 16$$

$$-4 < t - 6 < 4$$

$$2 < t < 10$$

$\therefore$ t can be any number between 2 and 10.

13. (a)

$$g(x) = x^2 + 3x - 4$$

$$= x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 4$$

$$= \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - 4$$

$$= \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{16}{4}$$

$$= \left(x + \frac{3}{2}\right)^2 - \frac{25}{4}$$

$$\therefore D = \{x \mid x \in \mathbf{R}\} \quad R = \{y \mid y \geq -\frac{25}{4}, y \in \mathbf{R}\}$$

(b)

$$h(x) = -2x^2 + 5x - 1$$

$$= -2\left(x^2 - \frac{5}{2}x\right) - 1$$

$$= -2\left[x^2 - \frac{5}{2}x + \left(-\frac{5}{4}\right)^2 - \left(-\frac{5}{4}\right)^2\right] - 1$$

$$= -2\left(x - \frac{5}{4}\right)^2 + 2\left(\frac{25}{16}\right) - 1$$

$$= -2\left(x - \frac{5}{4}\right)^2 + \frac{25}{8} - 1$$

$$= -2\left(x - \frac{5}{4}\right)^2 + \frac{17}{8}$$

$$\therefore D = \{x \mid x \in \mathbf{R}\} \quad R = \{y \mid y \leq \frac{17}{8}, y \in \mathbf{R}\}$$

(c)

$$f(x) = \frac{1}{2}x^2 - \frac{2}{3}x + \frac{3}{4}$$

$$= \frac{1}{2}\left(x^2 - \frac{4}{3}x\right) + \frac{3}{4}$$

$$= \frac{1}{2}\left[x^2 - \frac{4}{3}x + \left(-\frac{2}{3}\right)^2 - \left(-\frac{2}{3}\right)^2\right] + \frac{3}{4}$$

$$= \frac{1}{2}\left(x - \frac{2}{3}\right)^2 - \frac{1}{2}\left(\frac{4}{9}\right) + \frac{3}{4}$$

$$= \frac{1}{2}\left(x - \frac{2}{3}\right)^2 - \frac{2}{9} + \frac{3}{4}$$

$$= \frac{1}{2}\left(x - \frac{2}{3}\right)^2 - \frac{8}{36} + \frac{27}{36}$$

$$= \frac{1}{2}\left(x - \frac{2}{3}\right)^2 + \frac{19}{36}$$

$$\therefore D = \{x \mid x \in \mathbf{R}\} \quad R = \{y \mid y \geq \frac{19}{36}, y \in \mathbf{R}\}$$

(d)

$$k(x) = 0.1x^2 - 0.5x + 0.3$$

$$= 0.1\left(x^2 - 5x\right) + 0.3$$

$$= 0.1\left[x^2 - 5x + \left(-\frac{5}{2}\right)^2 - \left(-\frac{5}{2}\right)^2\right] + 0.3$$

$$= 0.1\left(x - \frac{5}{2}\right)^2 - 0.1\left(\frac{25}{4}\right) + 0.3$$

$$= 0.1\left(x - \frac{5}{2}\right)^2 - 0.625 + 0.3$$

$$= 0.1(x - 2.5)^2 - 0.325$$

$$\therefore D = \{x \mid x \in \mathbf{R}\} \quad R = \{y \mid y \geq -0.325, y \in \mathbf{R}\}$$

14. Each of the various forms of the equation of a quadratic function provides a different set of information about the function. All quadratic functions have parabolic graphs and the general shape and even the direction of opening is clear from any of the equation forms. Further to this, the standard

form, $f(x) = ax^2 + bx + c$, provides the y -intercept $(0, c)$. However, if information about the maximum or minimum value of a quadratic function is required, the vertex form of its equation will provide this. In vertex form, $f(x) = a(x - p) + q$, the coordinates of the vertex (p, q) locate the position of the maximum or minimum point on the parabola. The equation of a quadratic function in standard form may be transformed to vertex form by the method of “completing the square.” The steps are as follows.

Given $f(x) = ax^2 + bx + c$:

- Factor ‘a’ out of the first two terms.

$$f(x) = a\left(x^2 + \frac{b}{a}x\right) + c$$

- Add and subtract the square of half $\frac{b}{a}$ inside the bracket.

$$f(x) = a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c$$

- Group the perfect square terms and move the subtracted value outside the brackets, remembering to multiply by ‘a’.

$$f(x) = a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right] - a\left(\frac{b}{2a}\right)^2 + c$$

- Factor the perfect square.

$$f(x) = a\left(x + \frac{b}{2a}\right)^2 - a\left(\frac{b}{2a}\right)^2 + c$$

Practise, Apply, Solve 4.2, page 314

- (a) i.

$$f(x) = x^2 - 5x - 8$$

$$= \left(x^2 - 5x + \frac{25}{4} - \frac{25}{4}\right) - 8$$

$$= \left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{4} - \frac{32}{4}$$

$$= \left(x - \frac{5}{2}\right)^2 - \frac{57}{4}$$

Vertex has coordinates $\left(\frac{5}{2}, -\frac{57}{4}\right)$.

- Function has a minimum value since $a > 0$.

- $x = \frac{5}{2}$ produces this minimum value.

- The minimum value is $-\frac{57}{4}$ (or -14.25).

- (b) i.

$$f(x) = -2x^2 + 16x - 17$$

$$= -2(x^2 - 8x) - 17$$

$$= -2(x^2 - 8x + 16 - 16) - 17$$

$$= -2(x^2 - 8x + 16) - (-2)(-16) - 17$$

$$= -2(x - 4)^2 + 15$$

Vertex has coordinates $(4, 15)$.

- Function has a maximum value since $a < 0$.

- Maximum occurs when $x = 4$.

- Maximum value is 15.

- (c) i.

$$f(x) = -4x^2 - 6x - 5$$

$$= -4\left(x^2 + \frac{3}{2}x\right) - 5$$

$$= -4\left(x^2 + \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}\right) - 5$$

$$= -4\left(x^2 + \frac{3}{2}x + \frac{9}{16}\right) - (-4)\left(\frac{9}{16}\right) - 5$$

$$= -4\left(x + \frac{3}{4}\right)^2 - \frac{11}{4}$$

Vertex is at $\left(-\frac{3}{4}, -\frac{11}{4}\right)$ (or $(-0.75, -2.75)$).

- ii The function has a maximum value since $a < 0$.

- iii. Maximum value occurs where $x = -\frac{3}{4}$ (or -0.75).

- iv. Maximum value of the function is $-\frac{11}{4}$ (or -2.75).

- (d) i.

$$f(x) = -\frac{1}{2}x^2 + 2x - 9$$

$$= -\frac{1}{2}(x^2 - 4x) - 9$$

$$= -\frac{1}{2}(x^2 - 4x + 4 - 4) - 9$$

$$= -\frac{1}{2}(x^2 - 4x + 4) - \left(-\frac{1}{2}\right)(4) - 9$$

$$= -\frac{1}{2}(x - 2)^2 - 7$$

Vertex is at $(2, -7)$.

- ii. Function has a maximum value since $a < 0$.

- iii. Maximum value occurs where $x = 2$.

- iv. Maximum value of the function is -7 .

2. (a) $f(x) = (x - 3)(x - 8)$ has x -intercepts at $(3, 0)$ and $(8, 0)$.

The vertex is halfway between these intercepts and so its

x -coordinate is $\frac{3+8}{2}$ or 5.5 . The corresponding y -coordinate

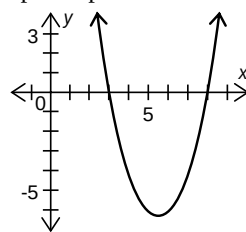
is found by substituting 5.5 for x in the function equation.

$$f(5.5) = (5.5 - 3)(5.5 - 8)$$

$$= (2.5)(-2.5)$$

$$= -6.25$$

Vertex has coordinates $(5.5, -6.25)$. Since $a > 0$, the parabola opens up.



- (b) $f(x) = -2(x - 1)(x + 2)$ has x -intercepts at $(1, 0)$ and

$(-2, 0)$. The x -coordinate of the vertex is $\frac{1+(-2)}{2}$ or -0.5 .

The y -coordinate is $f(-0.5)$.