

Let the other sides of the triangle have length x cm and y cm, as shown. If the area is 15 cm^2 , then

$$\frac{xy}{2} = 15$$

$$xy = 30$$

Also, by the Pythagorean Theorem, $x^2 + y^2 = 9^2$
 $x^2 + y^2 = 81$

$$\begin{aligned}\text{Now } (x+y)^2 &= x^2 + 2xy + y^2 \\ &= x^2 + y^2 + 2xy \\ &= 81 + 60 \\ &= 141\end{aligned}$$

$$\begin{aligned}\text{and } (x-y)^2 &= x^2 - 2xy + y^2 \\ &= x^2 + y^2 - 2xy \\ &= 81 - 60 \\ &= 21\end{aligned}$$

$$x + y = \sqrt{141} \text{ -----(1)}$$

$$x - y = \pm\sqrt{21} \text{ -----(2)}$$

We know x and y are positive since they represent lengths.

Therefore, $x + y$ is also positive, but $x - y$ might be positive or negative.

Add the two equations:

$$(1) + (2): \quad 2x = \sqrt{141} \pm \sqrt{21}$$

$$x \doteq 8.228 \text{ or } 3.646 \text{ (to 3 decimal places)}$$

Corresponding values of y may be obtained by subtracting the equations (1) – (2) or by using equation (1).

$$(1) - (2): \quad 2y = \sqrt{141} \pm \sqrt{21}$$

$$y \doteq 3.646 \text{ or } 8.228 \text{ (to 3 decimal places)}$$

$$\begin{aligned}\text{or (1): } y &= \sqrt{141} - x \\ &= \sqrt{141} - 8.228 \\ &\doteq 3.646\end{aligned}$$

$$\begin{aligned}\text{or } y &= \sqrt{141} - 3.646 \\ &\doteq 8.228\end{aligned}$$

The length of the other two sides are 3.65 cm and 8.23 cm (to 2 decimal places).

Practise, Apply, Solve 4.4, page 331

1. (a) Integer, rational, real, and complex
- (b) Rational, real, and complex
- (c) Complex
- (d) Irrational, real, and complex
- (e) Rational, real, and complex
- (f) Rational, real, and complex
- (g) Rational, real, and complex
- (h) Natural, whole, integer, rational, real, and complex

$$2. \text{ (a) } \sqrt{-16} = \sqrt{16i^2} = 4i$$

$$\text{(b) } \sqrt{-121} = \sqrt{121i^2} = 11i$$

$$\text{(c) } -\sqrt{-9} = -\sqrt{9i^2} = -3i$$

$$\text{(d) } \sqrt{-18} = \sqrt{18i^2} = 3\sqrt{2}i$$

$$\text{(e) } -\sqrt{-28} = -\sqrt{28i^2} = -2\sqrt{7}i$$

$$\text{(f) } \pm\sqrt{-50} = \pm\sqrt{50i^2} = \pm5\sqrt{2}i$$

3. In each case, simply change the sign of the “ i ” term.

$$\text{(a) } 1 - i$$

$$\text{(b) } 2 + 3i$$

$$\text{(c) } -3 + 4i$$

$$\text{(d) } -7 - 5i$$

$$\text{(e) } 11 + 11i$$

$$\text{(f) } 2 - \sqrt{2}i$$

4. Check each possible solution by substituting into the given equation and examining the values of the left side and right side.

$$\text{(a) } x^2 = -16$$

LS

$$x^2 = (4i)^2$$

$$= 16i^2$$

$$= 16(-1)$$

$$= -16$$

RS

$$-16$$

Yes, $4i$ is a solution.

$$\text{(b) } x^2 = -14$$

LS

$$x^2 = (7i)^2$$

$$= 49i^2$$

$$= 49(-1)$$

$$= -49$$

RS

$$-14$$

No, $7i$ is not a solution.

$$\text{(c) } x^2 + 6 = 5$$

LS

$$x^2 + 6 = (i)^2 + 6$$

$$= -1 + 6$$

$$= 5$$

RS

$$5$$

Yes, i is a solution.

$$\text{(d) } x^2 + 10 = 16$$

LS

$$x^2 + 10 = (3i)^2 + 10$$

$$= 9i^2 + 10$$

$$= 9(-1) + 10$$

$$= 1$$

RS

$$16$$

No, $3i$ is not a solution.

5. (a)

$$x^2 + 4 = 0$$

$$x^2 = -4$$

$$= 4(-1)$$

$$= 4i^2$$

$$x = \pm\sqrt{4i^2}$$

$$= \pm 2i$$

(b)

$$x^2 + 64 = 0$$

$$x^2 = -64$$

$$= 64(-1)$$

$$= 64i^2$$

$$x = \pm\sqrt{64i^2}$$

$$= \pm 8i$$

(c)

$$3x^2 + 27 = 0$$

$$3x^2 = -27$$

$$x^2 = -9$$

$$= 9i^2$$

$$x = \pm 3i$$

(d)

$$-2x^2 = 98$$

$$x^2 = -49$$

$$= 49i^2$$

$$x = \pm 7i$$

(e)

$$x^2 + 32 = 0$$

$$x^2 = -32$$

$$x^2 = 32i^2$$

$$x = \pm \sqrt{32i^2}$$

$$x = 4\sqrt{2}i$$

(f)

$$-3x^2 - 150 = 0$$

$$-3x^2 = 150$$

$$x^2 = -50$$

$$= 50i^2$$

$$x = \pm \sqrt{50i^2}$$

$$= 5\sqrt{2}i$$

6. Use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(a) $x^2 - 4x + 5 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{4 \pm \sqrt{-4}}{2}$$

$$= \frac{4 \pm \sqrt{4i^2}}{2}$$

$$= \frac{4 \pm 2i}{2}$$

$$= 2 \pm i$$

Roots of the equation are $2 + i$ and $2 - i$.

(b) $x^2 - 2x + 17 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(17)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 - 68}}{2}$$

$$= \frac{2 \pm \sqrt{-64}}{2}$$

$$= \frac{2 \pm \sqrt{64i^2}}{2}$$

$$= \frac{2 \pm 8i}{2}$$

$$= 1 \pm 4i$$

Roots are $1 + 4i$ and $1 - 4i$.

(c) $-x^2 + 4x = 13$

$$-x^2 + 4x - 13 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-13)}}{2(-1)}$$

$$= \frac{-4 \pm \sqrt{16 - 52}}{-2}$$

$$= \frac{-4 \pm \sqrt{-36}}{-2}$$

$$= \frac{-4 \pm \sqrt{36i^2}}{-2}$$

$$= \frac{-4 \pm 6i}{-2}$$

$$= 2 \pm 3i$$

Roots are $2 + 3i$ and $2 - 3i$.

(d) $2x^2 + 2x + 1 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{-2 \pm \sqrt{4 - 8}}{4}$$

$$= \frac{-2 \pm \sqrt{-4}}{4}$$

$$= \frac{-2 \pm \sqrt{4i^2}}{4}$$

$$= \frac{-2 \pm 2i}{4}$$

$$= -\frac{1}{2} \pm \frac{1}{2}i$$

Roots are $-\frac{1}{2} + \frac{1}{2}i$ and $-\frac{1}{2} - \frac{1}{2}i$.

$$(e) \quad 2x^2 = x - 1$$

$$2x^2 - x + 1 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{1-8}}{4}$$

$$= \frac{1 \pm \sqrt{-7}}{4}$$

$$= \frac{1 \pm \sqrt{7i^2}}{4}$$

$$= \frac{1 \pm 2.646i}{4}$$

$$= 0.25 \pm 0.66i$$

Roots are $0.25 + 0.66i$ and $0.25 - 0.66i$.

$$(f) \quad 3x(x+2) = -18$$

Divide by 3

$$x(x+2) = -6$$

$$x^2 + 2x + 6 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(6)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4-24}}{2}$$

$$= \frac{-2 \pm \sqrt{-20}}{2}$$

$$= \frac{-2 \pm \sqrt{20i^2}}{2}$$

$$= \frac{-2 \pm 2\sqrt{5}i}{2}$$

$$= -1 \pm \sqrt{5}i$$

Roots are $-1 + \sqrt{5}$ and $-1 - \sqrt{5}$.

$$7. (a) \quad x^2 + 2x + 5 = 0$$

$$b^2 - 4ac = 2^2 - 4(1)(5)$$

$$= 4 - 20$$

$$= -16$$

Since $b^2 - 4ac < 0$, the equation has two complex conjugate roots.

$$(b) \quad x^2 - 2x - 3 = 0$$

$$b^2 - 4ac = (-2)^2 - 4(1)(-3)$$

$$= 4 + 12$$

$$= 16$$

Since $b^2 - 4ac > 0$, the equation has two real roots.

$$(c) \quad 5x^2 - 4x + 2 = 0$$

$$b^2 - 4ac = (-4)^2 - 4(5)(2)$$

$$= 16 - 40$$

$$= -24$$

Since $b^2 - 4ac < 0$, the equation has two complex conjugate roots.

$$(d) \quad 3x^2 - 24x + 48 = 0$$

$$b^2 - 4ac = (-24)^2 - 4(3)(48)$$

$$= 576 - 576$$

$$= 0$$

Since $b^2 - 4ac = 0$, the equation has one real root.

8. Use the results from question 7.

$$(a) \quad x^2 + 2x + 5 = 0$$

$$x = \frac{-2 \pm \sqrt{-16}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{16i^2}}{2}$$

$$= \frac{-2 \pm 4i}{2}$$

$$= -1 \pm 2i$$

Solutions are $x = -1 + 2i$ and $x = -1 - 2i$.

$$(b) \quad x^2 - 2x - 3 = 0$$

This factors easily:

$$(x-3)(x+1) = 0$$

$x = 3$ or $x = -1$ are the solutions. Alternatively, use the quadratic formula:

$$x = \frac{2 \pm \sqrt{16}}{2}$$

$$= \frac{2 \pm 4}{2}$$

$$= 1 \pm 2$$

$$= 3 \text{ or } -1$$

$$(c) \quad 5x^2 - 4x + 2 = 0$$

$$x = \frac{-(-4) \pm \sqrt{-24}}{2(5)}$$

$$= \frac{4 \pm \sqrt{24i^2}}{10}$$

$$= \frac{4 \pm 4\sqrt{6}i}{10}$$

$$= \frac{2}{5} \pm \frac{\sqrt{6}}{5}i$$

Solutions are $\frac{2}{5} + \frac{\sqrt{6}}{5}i$ and $\frac{2}{5} - \frac{\sqrt{6}}{5}i$.

$$(d) \quad 3x^2 - 24x + 48 = 0$$

Factor first:

$$3(x^2 - 8x + 16) = 0$$

$$3(x-4)^2 = 0$$

$x = 4$ is the solution.

$$9. (a) \quad x^2 + 5x - 24 = 0$$

This factors:

$$(x+8)(x-3) = 0$$

$$\therefore x = -8 \text{ or } x = 3$$

Roots are -8.00 and 3.00 .

$$(b) \quad 3(x-2)^2 - 15 = 0$$

Method A:

$$3(x-2)^2 = 15$$

$$(x-2)^2 = 5$$

$$x-2 = \pm\sqrt{5}$$

$$x = 2 \pm \sqrt{5}$$

Roots are $2 + \sqrt{5}$ and $2 - \sqrt{5}$.

Method B: Expand and use quadratic formula.

$$3(x^2 - 4x + 4) - 15 = 0$$

$$3x^2 - 12x + 12 - 15 = 0$$

$$3x^2 - 12x - 3 = 0$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(-3)}}{2(3)}$$

$$= \frac{12 \pm \sqrt{144 + 36}}{6}$$

$$= \frac{12 \pm \sqrt{180}}{6}$$

$$= \frac{12 \pm 6\sqrt{5}}{6}$$

$$= 2 \pm \sqrt{5}$$

Roots are $2 + \sqrt{5}$ and $2 - \sqrt{5}$ as before.

(c) $3x^2 - 6x + 1 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(1)}}{2(3)}$$

$$= \frac{6 \pm \sqrt{36 - 12}}{6}$$

$$= \frac{6 \pm \sqrt{24}}{6}$$

$$= \frac{6 \pm 2\sqrt{6}}{6}$$

$$= 1 \pm \frac{2}{3}\sqrt{6}$$

Roots are $1 + \frac{2}{3}\sqrt{6}$ and $1 - \frac{2}{3}\sqrt{6}$.

(d) $x^2 - 2x + 5 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 - 20}}{2}$$

$$= \frac{2 \pm \sqrt{16i^2}}{2}$$

$$= \frac{2 \pm 4i}{2}$$

$$= 1 \pm 2i$$

Roots are $1 + 2i$ and $1 - 2i$.

10. When a quadratic equation has complex roots, the roots form a complex conjugate pair.

(a) If $2i$ is one root, $-2i$ is the other root. Equation is

$$(x - 2i)(x - (-2i)) = 0$$

$$\text{ie. } (x - 2i)(x + 2i) = 0$$

(b) If $2 - i$ is one root, $2 + i$ is the other root. Equation is

$$(x - (2 - i))(x - (2 + i)) = 0$$

$$(x - 2 + i)(x - 2 - i) = 0$$

(c) Roots are $1 + 3i$ and $1 - 3i$. Equation is

$$(x - (1 + 3i))(x - (1 - 3i)) = 0$$

$$(x - 1 - 3i)(x - 1 + 3i) = 0$$

(d) Roots are $-1 - i$ and $-1 + i$. Equation is

$$(x - (-1 - i))(x - (-1 + i)) = 0$$

$$(x + 1 + i)(x + 1 - i) = 0$$

(e) Roots are $3 - 4i$ and $3 + 4i$. Equation is

$$(x - (3 - 4i))(x - (3 + 4i)) = 0$$

$$(x - 3 + 4i)(x - 3 - 4i) = 0$$

(f) Roots are $-12 + 5i$ and $-12 - 5i$. Equation is

$$(x - (-12 + 5i))(x - (-12 - 5i)) = 0$$

$$(x + 12 - 5i)(x + 12 + 5i) = 0$$

11. (a) $x^2 + 4x + 5$

Use the quadratic formula to find zeros.

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm \sqrt{4i^2}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$$= -2 \pm i$$

Factors are $(x - (-2 + i))(x - (-2 - i))$

ie. $(x + 2 - i)(x + 2 + i)$

(b) $x^2 + 4x + 13$

Zeros occur when

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{-36}}{2}$$

$$= \frac{-4 \pm \sqrt{36i^2}}{2}$$

$$= \frac{-4 \pm 6i}{2}$$

$$= -2 \pm 3i$$

Factors are $(x - (-2 + 3i))(x - (-2 - 3i))$

ie. $(x + 2 - 3i)(x + 2 + 3i)$

(c) Zeros of $x^2 - x + 1$ occur when

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{-3}}{2}$$

$$= \frac{1 \pm \sqrt{3i^2}}{2}$$

$$= \frac{1 \pm \sqrt{3}i}{2}$$

$$= \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

Factors are

$$\left(x - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\right)\left(x - \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\right)$$

$$\text{ie } \left(x - \frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\left(x - \frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

(d) Zeros of $x^2 - 2x + 26$ occur when

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(26)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-100}}{2}$$

$$= \frac{2 \pm \sqrt{100i^2}}{2}$$

$$= \frac{2 \pm 10i}{2}$$

$$= 1 \pm 5i$$

Factors are

$$(x - (1 + 5i))(x - (1 - 5i))$$

$$\text{ie } (x - 1 - 5i)(x - 1 + 5i)$$

12. (a) $3x^2 + 2x + 1 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(1)}}{2(3)}$$

$$= \frac{-2 \pm \sqrt{-8}}{6}$$

$$= \frac{-2 \pm \sqrt{8i^2}}{6}$$

$$= \frac{-2 \pm 2.828i}{6}$$

$$= -0.33 \pm 0.47i$$

Solutions are $x = -0.33 + 0.47i$ and $x = -0.33 - 0.47i$.

(b) $2x^2 - 3x + 7 = 0$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(7)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{-47}}{4}$$

$$= \frac{3 \pm \sqrt{47i^2}}{4}$$

$$= \frac{3 \pm 6.856i}{4}$$

$$= 0.75 \pm 1.71i$$

Solutions are $x = 0.75 + 1.71i$ and $x = 0.75 - 1.71i$.

(c) $3x^2 - 2x + 10 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(10)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{-116}}{6}$$

$$= \frac{2 \pm \sqrt{116i^2}}{6}$$

$$= \frac{2 \pm 10.770i}{6}$$

$$= 0.33 \pm 1.80i$$

Solutions are $x = 0.33 + 1.80i$ and $x = 0.33 - 1.80i$.

(d) $5x^2 + x + 7 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(5)(7)}}{2(5)}$$

$$= \frac{-1 \pm \sqrt{-139}}{10}$$

$$= \frac{-1 \pm \sqrt{139i^2}}{10}$$

$$= \frac{-1 \pm 11.790i}{10}$$

$$= -0.10 \pm 1.18i$$

Solutions are $x = -0.10 + 1.18i$ and $x = -0.10 - 1.18i$.

13. Let one number be x and the other number by y .

$$x + y = 4 \text{ -----(1)}$$

$$xy = 13 \text{ -----(2)}$$

Rearrange equation (1): $y = 4 - x$

Substitute in (2):

$$x(4 - x) = 13$$

$$4x - x^2 = 13$$

$$0 = x^2 - 4x + 13$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$= \frac{4 \pm \sqrt{36i^2}}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$$= 2 \pm 3i$$

The two numbers are $2 + 3i$ and $2 - 3i$.

14. Let one number be x and the other number be y .

$$x + y = 5 \text{ -----(1)}$$

$$xy = 7 \text{ -----(2)}$$

From (1): $y = 5 - x$

Substitute in (2):

$$x(5 - x) = 7$$

$$5x - x^2 = 7$$

$$0 = x^2 - 5x + 7$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{-3}}{2}$$

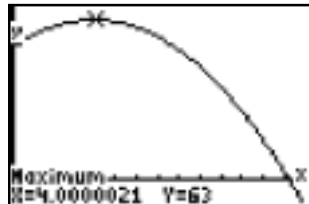
$$= \frac{5 \pm \sqrt{3}i^2}{2}$$

$$= \frac{5 \pm \sqrt{3}i}{2}$$

$$\doteq 2.50 \pm 0.87i$$

The two numbers are $2.50 + 0.87i$ and $2.50 - 0.87i$.

15. If the function is entered in a graphing calculator and the CALC Maximum function is used, then, according to the model, the maximum value for n is 63, which represents 63 000 units.



Algebraically, if $n(t) = 65$

$$65 = -0.75t^2 + 6t + 51$$

$$0 = -0.75t^2 + 6t - 14$$

$$t = \frac{-6 \pm \sqrt{6^2 - 4(-0.75)(-14)}}{-1.5}$$

$$= \frac{-6 \pm \sqrt{-6}}{1.5}$$

The equation does not have real roots. The company cannot expect monthly sales to reach 65 000 units in any month over the next year.

16. The number systems we use today have evolved over time to enable the solution of different types of equations. We began with the counting numbers: 1, 2, 3, ..., called the set of Natural numbers. When the number zero was added to the list, the set of whole numbers was created.

For many centuries, the difference $a - b$ was only defined for $a > b$, but then the notion of negative numbers was introduced and the set of integers was created. Rational numbers were introduced to enable us to solve equations like $3x = 2$ and then irrational numbers were needed for us to solve equations like $x^2 = 2$. The set of Rational numbers contains the set of integers which contains the set of Whole numbers, which contains the set of Natural numbers. The set of Rationals and the set of Irrationals are separate from each other but together make up the set of Real numbers.

Complex numbers are needed to enable us to solve equations like $x^2 = -2$, which have no solution in the Real number system. They are different from real numbers in that each

complex number has two parts, a real part and an imaginary part. The imaginary number, i , is the number such that $i^2 = -1$. With this number, the equation $x^2 = -2$ may be written $x^2 = 2i^2$ and may then be solved by writing $x = \pm\sqrt{2}i$. As complex numbers, these would be written $x = 0 + \sqrt{2}i$ and $x = 0 - \sqrt{2}i$. The 0's are the real parts and the $\pm\sqrt{2}$ are the imaginary parts.

Quadratic equations that have no real roots always have two complex roots. These are always in the form $a + bi$ and $a - bi$. They are said to form a conjugate pair.

17. Change MODE



The number i is above the decimal point on a TI-83 plus.

Addition:

$$\begin{array}{r} 2+3i+3-4i \\ -5+i+5+2i \\ 3-4i+3+4i \\ \hline 5-i \\ 3i \\ 6 \end{array}$$

Subtraction:

$$\begin{array}{r} 3+4i-(2+i) \\ -2-3i-(1-3i) \\ 4+i-(4-3i) \\ \hline 1+3i \\ -3 \\ 4i \end{array}$$

You must use brackets around the second complex number in a subtraction.

The sum of two complex numbers may be real or complex.

When two complex conjugates are added, the result is always a real number.

Multiplication:

$$\begin{array}{r} (3+i)(4-i) \\ (-5-2i)(2+i) \\ (3-4i)(3+4i) \\ \hline 13+i \\ -8-9i \\ 25 \end{array}$$

The product of two complex numbers may be real or complex. When two complex conjugate numbers are multiplied, the result is always a real number.

18. A complex number is a number which is the sum of a real number and an imaginary number. It is written in the form $a + bi$, where a and b are real numbers, and i is the number such that $i^2 = -1$. The number a is called the real part of the complex number, and the number b is called the imaginary part. The number i and multiples of i are imaginary numbers.

Quadratic equations that do not have real roots do have solutions in the set of complex numbers. The complex roots are always in conjugate pairs. If one root of a quadratic equation is $a + bi$, then the other root is $a - bi$. For example, consider the equation $2x^2 + 3x + 4 = 0$. The discriminant $b^2 - 4ac = 3^2 - 4(2)(4) = -23$.

Since this is less than zero, we know the equation has no roots in the Real number system. If we work in the Complex number system, we can use the fact that $i^2 = -1$ to solve the equation.

$$2x^2 + 3x + 4 = 0$$

$$x = \frac{-3 \pm \sqrt{-23}}{4}$$

$$= \frac{-3 \pm \sqrt{23i^2}}{4}$$

$$\doteq \frac{-3 \pm 4.796i}{4}$$

$$\doteq -0.75 \pm 1.20i \text{ (to 2 decimal places)}$$

The solutions to $2x^2 + 3x + 4 = 0$ are $-0.75 + 1.20i$ and $-0.75 - 1.20i$.

19. $x^4 - 81 = 0$ Factor the left side as much as possible

$$(x^2)^2 - 9^2 = 0 \text{ A difference of squares}$$

$$(x^2 - 9)(x^2 + 9) = 0$$

$$(x - 3)(x + 3)(x^2 + 9) = 0$$

$$\therefore x = 3, -3 \text{ or } x^2 + 9 = 0$$

$$x^2 = -9$$

$$x^2 = 9i^2$$

$$x = \pm 3i$$

Roots of the equation $x^4 - 81 = 0$ are $\pm 3, \pm 3i$.

20. No, you cannot put complex numbers in order. Although the real parts of a complex number could be put in increasing or decreasing order, the multiples of i cannot. To do this, we would need to set $i > 0$ or $i < 0$.

But if $i > 0$ and we multiply both sides by i we get $i^2 > 0$, which is not true ($i^2 < 0$). If $i < 0$ and we multiply both sides by i we get $i^2 > 0$ again, since multiplying by a negative number reverses the inequality sign.

21. Using the distributive property of multiplication over addition, $(x + 1)(x^2 - x + 1) = x^3 - x^2 + x + x^2 - x + 1$

$$= x^3 + 1$$

$$x^3 + 1 = 0$$

$$(x + 1)(x^2 - x + 1) = 0$$

$$x = -1 \text{ or } x^2 - x + 1 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{-3}}{2}$$

$$= \frac{1 \pm \sqrt{3i^2}}{2}$$

$$= \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

The roots of equation $x^3 + 1 = 0$ are $-1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$
(or $-1.00, 0.50 + 0.87i, 0.50 - 0.87i$).

Practise, Apply, Solve 4.5U, page 336

$$1. \text{ (a) } 2 + 6i + 5 - i = (2 + 5) + (6 - 1)i \\ = 7 + 5i$$

$$\text{ (b) } 4 - 3i + 1 + i = (4 + 1) + (-3 + 1)i \\ = 5 - 2i$$

$$\text{ (c) } 3 + 7i + (-4) - 9i = (3 - 4) + (7 - 9)i \\ = -1 - 2i$$

$$\text{ (d) } -1 + 3i + 2 - 13i = (-1 + 2) + (3 - 13)i \\ = 1 - 10i$$

$$\text{ (e) } 5i + 1 - 2i = (0 + 1) + (5 - 2)i \\ = 1 + 3i$$

$$\text{ (f) } 3 + \sqrt{2}i + \sqrt{2}i = (3 + 0) + (\sqrt{2} + \sqrt{2})i \\ = 3 + 2\sqrt{2}i$$

$$2. \text{ (a) } 2 + 6i - (5 - i) = (2 - 5) + (6 + 1)i \\ = -3 + 7i$$

$$\text{ (b) } 4 - 3i - (1 + i) = (4 - 1) + (-3 - 1)i \\ = 3 - 4i$$

$$\text{ (c) } 3 + 7i - (-4 - 9i) = (3 + 4) + (7 + 9)i \\ = 7 + 16i$$

$$\text{ (d) } -1 + 3i - (2 - 13i) = (-1 - 2) + (3 + 13)i \\ = -3 + 16i$$

$$\text{ (e) } 5i - (1 - 2i) = (0 - 1) + (5 + 2)i \\ = -1 + 7i$$

$$\text{ (f) } 3 + \sqrt{2}i - \sqrt{2}i = (3 + 0) + (\sqrt{2} - \sqrt{2})i \\ = 3 + 0i \\ = 3$$

$$3. \text{ (a) } (2 + i)(3 + 5i) = 6 + 10i + 3i + 5i^2 \\ = 6 + 13i - 5 \\ = 1 + 13i$$

$$\text{ (b) } (5 - 3i)(7 + i) = 35 + 5i - 21i - 3i^2 \\ = 35 - 16i + 3 \\ = 38 - 16i$$

$$\text{ (c) } (4 - 3i)(-2 - i) = -8 - 4i + 6i + 3i^2 \\ = -8 + 2i - 3 \\ = -11 + 2i$$

$$\text{ (d) } i(5 + i) = 5i + i^2 = 5i - 1 \\ = -1 + 5i$$

$$\text{ (e) } (2 + i)^2 = 4 + 4i + i^2 \\ = 4 + 4i - 1 \\ = 3 + 4i$$

$$\text{ (f) } (2 + 3i)(2 - 3i) = 4 - 9i^2 \\ = 4 + 9 \\ = 13 \text{ (or } 13 + 0i)$$