

24. 8, a , b , 36

Since the first three form an arithmetic sequence, then

$$b - a = a - 8, 2a - b = 8 \quad \textcircled{1}$$

Since the last three form a geometric sequence, then

$$\frac{36}{b} = \frac{b}{a}, 36a = b^2 \quad \textcircled{2}$$

$$\textcircled{1} \times 18: 36a - 18b = 144, 36a = 144 + 18b$$

$$\text{By } \textcircled{2}: 144 + 18b = b^2$$

$$b^2 - 18b - 144 = 0$$

$$(b + 6)(b - 24) = 0$$

$$b = -6 \text{ or } b = 24$$

If $b = -6$, $a = 1$. If $b = 24$, $a = 16$.

$\therefore (a, b) = (1, -6) \text{ or } (a, b) = (16, 24)$.

$$25. \frac{a^2}{b}, -a, b, \frac{-b^2}{a}, \dots$$

$$r = \frac{-a}{\frac{a^2}{b}} = \frac{-b}{a} = \frac{-b^2}{a^2}, t_n = \frac{a^2}{b} \left(\frac{-b}{a} \right)^{n-1}$$

$$t_{10} = \frac{a^2}{b} \left(\frac{(-1)^9 b^9}{a^9} \right) = \frac{-b^8}{a^7}$$

Practise, Apply, Solve 1.8, page 70

1. (a) i. $i = 0.05$ ii. $n = 5$

(b) i. $i = \frac{8\%}{2} = 0.04$

ii. $n = \frac{2 \text{ conversions}}{\text{year}} \times 9 \text{ years} = 18$

(c) i. $i = \frac{4.4\%}{4} = 0.011$

ii. $n = \frac{4 \text{ conversions}}{\text{year}} \times 7 \text{ years} = 28$

(d) i. $i = \frac{6\%}{12} = 0.005$

ii. $n = \frac{12 \text{ conversions}}{\text{year}} \times 3 \text{ years} = 36$

(e) i. $i = 0.06$ ii. $n = 5$

(f) i. $i = \frac{8.2\%}{4} = 0.0205$

ii. $n = \frac{4 \text{ conversions}}{12 \text{ months}} \times 16 \text{ months} = \frac{16}{3}$

(g) i. $i = \frac{4.5\%}{2} = 0.0225$

ii. $n = \frac{42 \text{ months}}{12 \text{ months}} \times \frac{2 \text{ conversions}}{\text{year}} = 7$

(h) i. $i = \frac{52\%}{52} = 0.001$

ii. $n = \frac{52 \text{ weeks}}{\text{year}} \times \frac{1 \text{ conversion}}{\text{week}} \times 2 \text{ years} = 104$

(i) i. $i = \frac{5.475\%}{365} = 0.00015$

ii. $n = \frac{365 \text{ days}}{\text{year}} \times \frac{1 \text{ conversion}}{\text{day}} \times 3 \text{ years} = 1095$

$$2. \text{ (a) } (1.1)^{12} = 3.1384$$

$$\text{ (b) } (1.05)^{-20} = 0.3769$$

$$\text{ (c) } (1.025)^{36} = 2.4325$$

$$\text{ (d) } (1.002)^{-45} = 0.9140$$

$$\text{ (e) } 200(1.04)^{24} = 512.6608$$

$$\text{ (f) } 5000(1.12)^{-5} = 2837.1343$$

$$\text{ (g) } 6525(1.03)^{-22} = 3405.3486$$

$$\text{ (h) } 10\,000(1.006)^{36} = 12\,403.0161$$

3. (a) A = amount of investment

(b) P = principle amount invested

(c) i = interest rate for each conversion period

(d) n = number of conversion periods

$$4. \text{ (a) i. } A = \$4000(1.03)^4 = \$4502.04$$

$$\text{ ii. } I = A - P = \$4502.04 - \$4000 = \$502.04$$

$$\text{ (b) i. } A = \$7500 \left[1 + \frac{0.06}{12} \right]^{6 \times 12} = \$10\,740.33$$

$$\text{ ii. } I = A - P = \$10\,740.33 - \$7500 = \$3240.33$$

$$\text{ (c) i. } A = \$15\,000 \left[1 + \frac{0.024}{4} \right]^{4 \times 5} = \$16\,906.39$$

$$\text{ ii. } I = A - P = \$16\,906.39 - \$15\,000 = \$1906.39$$

$$\text{ (d) i. } A = \$28\,200 \left[1 + \frac{0.055}{2} \right]^{20} = \$48\,516.08$$

$$\text{ ii. } I = A - P = \$48\,516.08 - \$28\,200 = \$20\,316.08$$

$$\text{ (e) i. } A = \$850 \left[1 + \frac{0.0365}{365} \right]^{365} = \$881.60$$

$$\text{ ii. } I = A - P = \$881.60 - \$850 = \$31.60$$

$$\text{ (f) i. } A = \$2225 \left[1 + \frac{0.052}{52} \right]^{47} = \$2332.02$$

$$\text{ ii. } I = A - P = \$2332.02 - \$2225 = \$107.02$$

$$5. \text{ (a) i. } P = \frac{\$5000}{(1.04)^3} = \$4444.98$$

$$\text{ ii. } I = A - P = \$5000 - \$4444.98 = \$555.02$$

$$\text{ (b) i. } P = \frac{\$13\,500}{\left[1 + \frac{0.06}{12} \right]^{48}} = \$10\,625.83$$

$$\text{ ii. } I = A - P = \$13\,500 - \$10\,625.83 = \$2874.17$$

$$\text{ (c) i. } P = \frac{\$11\,200}{\left[1 + \frac{0.044}{4} \right]^{20}} = \$8999.01$$

$$\text{ ii. } I = A - P = \$11\,200 - \$8999.01 = \$2200.99$$

$$\text{ (d) i. } P = \frac{\$128\,500}{\left[1 + \frac{0.065}{2} \right]^{16}} = \$77\,030.40$$

$$\text{ ii. } I = A - P = \$128\,500 - \$77\,030.40 = \$51\,469.60$$

$$\text{ (e) i. } P = \frac{\$850}{\left[1 + \frac{0.0584}{365} \right]^{400}} = \$797.31$$

$$\text{ ii. } I = A - P = \$850 - \$797.31 = \$52.69$$

$$\text{ (f) i. } P = \frac{\$6225}{\left[1 + \frac{0.13}{52} \right]^{100}} = \$4849.55$$

$$\text{ ii. } I = A - P = \$6225 - \$4849.55 = \$1375.45$$

6. The amount is the future value of the principal amount after the principal amount earns interest. The present value is

the principal amount that must be invested now to grow to a specific amount in the future.

Example #1: \$5000 is invested at 5.5%/a interest, compounded quarterly, for a term of 3 years. Determine the amount of the investment at the end of the 3 years.

In this example, the present value of the investment is given as \$5000. The future value of the investment can be found by

$$A = \$5000 \left[1 + \frac{0.055}{4} \right]^{12} = \$5890.34. \text{ This represents the}$$

amount of the investment after 3 years under the given conditions.

Example #2: Determine the present value of an investment that will be worth \$10 000 in 4 years. The interest rate is 4.9%/a compounded semi-annually.

In this example, the amount (future value of principal amount) is given as \$10 000. The present value can be found by

$$P = \frac{\$10\,000}{\left[1 + \frac{0.049}{2} \right]^8} = \$8239.57. \text{ This represents the amount of}$$

money that must be invested under the given conditions in order to grow to \$10 000.

7. Plan A: The total Margaret would have to pay is

$$A = \$949.99(1.05)^2 = \$1047.36$$

Plan B: The total Margaret would have to pay is

$$A = \$949.99 \left[1 + \frac{0.05}{4} \right]^8 = \$1049.25.$$

∴ Margaret should choose Plan A because it costs less.

$$\mathbf{8. (a)} \quad A_{10} = \$1000(1.05)^{10} = \$1628.89$$

$$\mathbf{(b)} \quad A_{20} = \$1000(1.05)^{20} = A_{10}(1.05)^{10} = \$2653.30$$

$$\mathbf{(c)} \quad A_{30} = \$1000(1.05)^{30} = A_{20}(1.05)^{10} = \$4321.94$$

9. To determine when $A = \$2000$, solve the equation

$$\$2000 = \$1000(1.05)^n \text{ for } n.$$

$$\$2000 = \$1000(1.05)^n$$

$$2 = (1.05)^n$$

$$\ln(2) = \ln(1.05)^n$$

$$\ln(2) = n \ln(1.05)$$

$$n = \frac{\ln(2)}{\ln(1.05)}$$

$$n \doteq 14.21$$

Thus, the amount will be twice the original principal after approximately 14 years.

$$\mathbf{10.} \quad i = \frac{0.04}{2} = 0.02, n = 21 \times 2 = 42,$$

$$P = \frac{\$10\,000}{(1.02)^{42}} = \$4353.04$$

∴ Mike should invest \$4353.04 today in order for the investment to grow to \$10 000 in 21 years.

$$\mathbf{11.} \quad A = \$7500(1.06)^{10} = \$13\,431.36$$

\$7500 will be worth \$13 431.36 in 10 years.

$$\text{Check: } P = \frac{\$13\,431.36}{(1.06)^{10}} = \$7500$$

$$\mathbf{12.} \quad A = \$1000(1.05)^7 = \$1407.10$$

∴ the bond has a value of \$1407.10 when it matures.

13.

$$(P - \$10\,000)(1.031)^4 = \$7500$$

$$(P - \$10\,000)(1.129\,886) \doteq \$7500$$

$$P - \$10\,000 \doteq \$6637.84$$

$$P \doteq \$16\,637.84$$

∴ the purchase price of the boat is \$16 637.84.

$$\mathbf{14.} \quad \text{at } 10\%/a: i = \frac{0.10}{4} = 0.025, n = 4 \times 3 = 12$$

$$\text{at } 9\%/a: i = \frac{0.09}{2} = 0.045, n = 2 \times 2 = 4$$

$$A = [\$9000(1.025)^{12}](1.045)^4 = \$14\,434.24$$

∴ Tiffany's investment has a value of \$14 434.24 two years after the change in the interest rate.

$$\mathbf{15.} \quad i = \frac{0.09}{2} = 0.045$$

After 6 months, $\$2000(1.045) - \$800 = \$1290$ remains. After another 6 months, he should pay $\$1290(1.045) = \$1348.05$.

16. In order to determine the original amount invested, calculate the amount that Sigrid had before the change in conversion periods.

$$P_{(2 \text{ years ago})} = \frac{\$7424.83}{\left[1 + \frac{0.06}{12} \right]^{24}} = \$6587.20$$

$$P_{(\text{original})} = \frac{\$6587.20}{\left[1 + \frac{0.06}{2} \right]^8} = \$5200.00$$

∴ the original principal that Sigrid deposited was \$5200.

17. Compounded interest is similar to a geometric sequence because each amount is multiplied by $(1 + i)$ for each conversion period.

Example: \$5000 invested at 6%/a, compounded semi-annually, for 2 years. Thus, $a = \$5000$ and $r = (1 + \frac{0.06}{2})$.

$$t_1 = \$5000, t_2 = \$5000(1 + \frac{0.06}{2})^1 = \$5150,$$

$$t_3 = \$5000(1 + \frac{0.06}{2})^2 = \$5304.50,$$

$$t_4 = \$5000(1 + \frac{0.06}{2})^3 = \$5463.64$$

$$t_5 = \$5000(1 + \frac{0.06}{2})^4 = \$5627.54$$

These terms form a geometric sequence, with the final term representing the amount of the investment after the 2 years.

$$\mathbf{18.} \quad A = \$2000 \left(1 + \frac{0.06}{12} \right)^{60} \left(1 + \frac{0.08}{4} \right)^{26} = \$4514.38$$

∴ Anna's account has a balance of \$4514.38 on January 1, 2008.

Note: The money is moved June 1, 2001, and January 1, 2008, is $6 + 6(12) = 78 = 26(3)$ months later.

19. At the end of the first year, $i = \frac{0.04}{4} = 0.01$ and
 $A = \$4000(1.01)^4 = \$4162.42$.
 At the end of the second year, $i = \frac{0.04 + 0.002}{4} = 0.0105$ and
 $A = \$4162.42(1.0105)^4 = \$4340.01$.
 At the end of the third year, $i = \frac{0.04 + 2(0.002)}{4} = 0.011$ and
 $A = \$4340.01(1.011)^4 = \$4534.14$.

Alternate Solution:

$A = \$4000[(1.01)^4(1.0105)^4(1.011)^4] = \$4534.14$.
 $\therefore$ Bernie's account has a balance of \$4534.14 after three years.

20. For 5 years, $n = 5 \times 12 = 60$ and for 3 years
 $n = 3 \times 12 = 36$.

$$i = \frac{0.048}{12} = 0.004$$

After Sarah's 5th birthday,

$$A = \$500(1.004)^{60} + \$500 = \$1135.32.$$

After Sarah's 10th birthday,

$$A = \$1135.32(1.004)^{60} + \$500 = \$1942.58.$$

After Sarah's 15th birthday,

$$A = \$1942.58(1.004)^{60} + \$500 = \$2968.33.$$

After Sarah's 18th birthday,

$$A = \$2968.33(1.004)^{36} = \$3427.09$$

$\therefore$ the account balance is \$3427.09 on Sarah's 18th birthday.

21. (a) $1 + \frac{0.04}{12} = \frac{301}{300}$, $P_0 = \$100$,

$$P_{n+1} = (P_n) \left(\frac{301}{300} \right) + \$100$$

$$P_1 = \$200.33, P_2 = \$301.00, P_3 = \$402.00, P_4 = \$503.34,$$

$$P_5 = \$605.02, P_6 = \$707.04, P_7 = \$809.40, P_8 = \$912.09,$$

$$P_9 = \$1015.13, P_{10} = \$1118.52, P_{11} = \$1222.25,$$

$$P_{12} = \$1326.32$$

Assuming that he hasn't yet invested \$100 for the first month after the end of the first year, there is

$\$1326.32 - \$100 = \$1226.32$ in Josh's bank account.

(b) $P = \frac{\$1226.32}{\left(\frac{301}{300}\right)^{12}} = \$1178.31$.

22. Matt: $i_1 = \frac{0.08}{2} = 0.04$, $n_1 = 5 \times 2 = 10$,

$$A_1 = \$500(1.04)^{10} = \$740.12$$

Justin: $i_2 = \frac{0.08}{4} = 0.02$, $n_2 = 5 \times 4 = 20$,

$$A_2 = \$500(1.02)^{20} = \$742.97$$

$\therefore$ Justin will have more money after five years (\$2.85 more) because there are more compounding periods associated with his investment.

23.

$$2P = P(1+i)^7$$

$$2 = (1+i)^7$$

$$i = 2^{\frac{1}{7}} - 1$$

$$i = 0.104$$

$$i = 10.4\%$$

$\therefore$ an interest rate of 10.4% would result in a doubled investment in seven years.

24.

$$\$500(1.08)^n = \$600(1.06)^n$$

$$5(1.08)^n = 6(1.06)^n$$

$$\frac{5}{6} = \left(\frac{1.06}{1.08} \right)^n$$

$$n \ln \left(\frac{1.06}{1.08} \right) = \ln \left(\frac{5}{6} \right)$$

$$n \doteq 9.75 \text{ years}$$

$\therefore$ the amounts in both accounts would be equal after approximately 9.75 years.

25. Does $(1 + 2i)^n = 2(1 + i)^n$? No, since powers are involved. Consider $(1 + 0.02)^5 = 2(1 + 0.01)^5$.

$$\text{LHS} = 1.1041, \text{RHS} = 2.1020.$$

Practise, Apply, Solve 1.9, page 79

1. (a) $4^3 = 64$

(b) $(-6)^2 = 36$

(c) $-3^2 = -9$

(d) $(-9)^3 = -729$

(e) $15^0 = 1$

(f) $\left(\frac{1}{2} \right)^3 = \frac{1}{8}$

(g) $\left(\frac{2}{3} \right)^4 = \frac{16}{81}$

(h) $\left(\frac{-2}{3} \right)^3 = \frac{-8}{27}$

(i) $\left(\frac{3}{4} \right)^{-2} = \left(\frac{4}{3} \right)^2 = \frac{16}{9} = 1\frac{7}{9}$

(j) $\left(\frac{-1}{4} \right)^{-3} = \left(\frac{4}{-1} \right)^3 = -64$

2. (a) $\sqrt{121} = 11$

(b) $\sqrt[3]{8} = 2$

(c) $\sqrt[4]{16} = 2$

(d) $\sqrt[3]{125} = 5$

(e) $\sqrt[5]{1024} = 4$

3. (a) $4^{\frac{1}{2}} = 2$

(b) $(-8)^{\frac{1}{3}} = -2$