

16. If $a + bi$ is a root of $x^2 + cx + d = 0$, then $a - bi$ is the other root. $\therefore$ another way to write the equation is

$$(x - (a + bi))(x - (a - bi)) = 0$$

$$(x - a - bi)(x - a + bi) = 0$$

$$(x - a)^2 - b^2 i^2 = 0$$

$$x^2 - 2ax + a^2 + b^2 = 0$$

But the equation is $x^2 + cx + d = 0$

$$\therefore -2a = c \text{ and } a^2 + b^2 = d$$

$$0 = 2a + c \quad \text{and} \quad a^2 + b^2 = d$$

Practise, Apply, Solve 4.6, page 345

1. (a) In table 1, $xy = 48$ for each row of the table. So

$$y = \frac{48}{x} \text{ and } x = \frac{48}{y}. \text{ This is an example of inverse variation.}$$

(b) In table 2, $xy = 3600$ for each row of the table. So

$$y = \frac{3600}{x} \text{ and } x = \frac{3600}{y}. \text{ This is an example of inverse}$$

variation.

(c) In table 3, y increases as x increases so this is not an inverse variation. In fact, $y = x^2$ describes the relationship between x and y . This is a quadratic relationship.

2. (a) $xy = 48$, or $y = \frac{48}{x}$

(b) $xy = 3600$, or $y = \frac{3600}{x}$

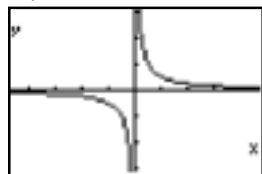
(c) $y = x^2$

3. (a) $f(x) = 2x$

i. $y = \frac{1}{f(x)} = \frac{1}{2x}$

ii. Vertical asymptotes occur where $2x = 0$. $\therefore$ equation of vertical asymptote is $x = 0$.

iii.



(b) $f(x) = x + 5$

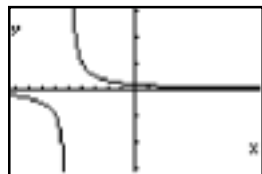
i. $y = \frac{1}{f(x)} = \frac{1}{x + 5}$

ii. $x + 5 = 0$

$$x = -5$$

Equation of vertical asymptote is $x = -5$.

iii.



(c) $f(x) = x - 4$

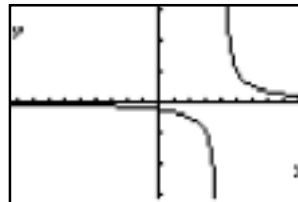
i. $y = \frac{1}{f(x)} = \frac{1}{x - 4}$

ii. $x - 4 = 0$

$$x = 4$$

Equation of vertical asymptote is $x = 4$.

iii.



(d) $f(x) = 2x + 5$

i. $y = \frac{1}{f(x)} = \frac{1}{2x + 5}$

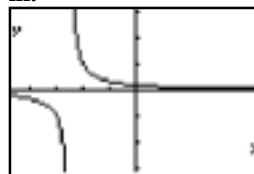
ii. $2x + 5 = 0$

$$2x = -5$$

$$x = -2.5$$

Equation of vertical asymptote is $x = -2.5$.

iii.



(e) $f(x) = -3x + 6$

i. $y = \frac{1}{f(x)} = \frac{1}{-3x + 6}$

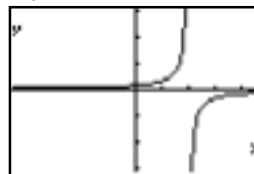
ii. $-3x + 6 = 0$

$$-3x = -6$$

$$x = 2$$

Equation of vertical asymptote is $x = 2$.

iii.



(f) $f(x) = (x - 3)^2$

i.

$$y = \frac{1}{f(x)}$$

$$y = \frac{1}{(x - 3)^2}$$

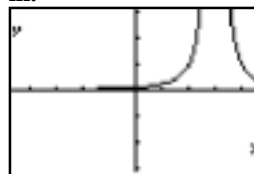
ii. $(x - 3)^2 = 0$

$$x - 3 = 0$$

$$x = 3$$

Equation of vertical asymptote is $x = 3$.

iii.



(g) $f(x) = x^2 - 3x - 10$

i. $y = \frac{1}{f(x)} = \frac{1}{x^2 - 3x - 10}$

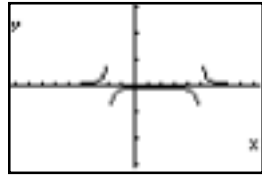
ii. $x^2 - 3x - 10 = 0$

$(x - 5)(x + 2) = 0$

$x = 5$ or $x = -2$

Equations of vertical asymptotes are $x = 5$ and $x = -2$.

iii.



(h) $f(x) = 3x^2 - 4x - 4$

i.

$y = \frac{1}{f(x)}$

$y = \frac{1}{3x^2 - 4x - 4}$

ii.

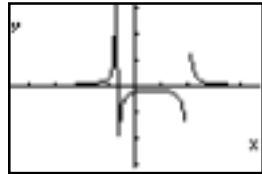
$3x^2 - 4x - 4 = 0$

$(3x + 2)(x - 2) = 0$

$x = -\frac{2}{3}$ or $x = 2$

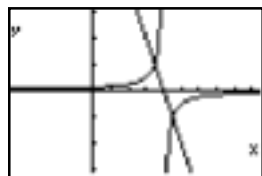
Equations of vertical asymptotes are $x = -\frac{2}{3}$ and $x = 2$.

iii.



4.

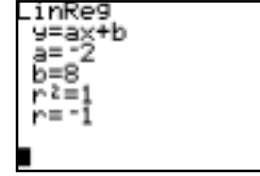
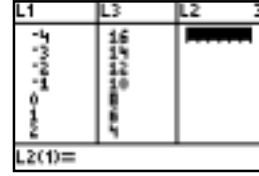
x	$f(x)$	$\frac{1}{f(x)}$
-4	16	0.0625
-3	14	0.0714
-2	12	0.0833
-1	10	0.1
0	8	0.125
1	6	0.1667
2	4	0.25
3	2	0.5
4	0	undefined
5	-2	-0.5
6	-4	-0.25
7	-6	-0.1667



The first differences for $f(x)$ are the same so we know $f(x)$ is linear. From the table, the intercepts are $(0, 8)$ and $(4, 0)$ and so the slope is $\frac{0-8}{4-0} = -2$. Equation is $f(x) = -2x + 8$.

Equation for $\frac{1}{f(x)}$ is $y = \frac{1}{-2x + 8}$.

Or enter data into a graphing calculator and do a linear regression.



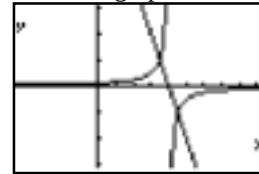
Then enter $\frac{1}{Y_1}$ into Y_2 , and press 2nd TABLE to view the

values for $\frac{1}{f(x)}$.

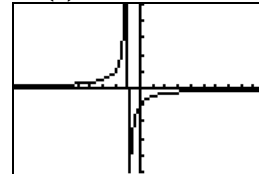


X	Y1	Y2
-4	16	.0625
-3	14	.0714
-2	12	.0833
-1	10	.1
0	8	.125
1	6	.1667
2	4	.25

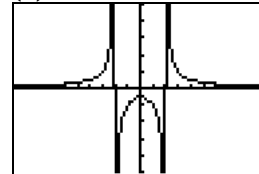
View the graphs.



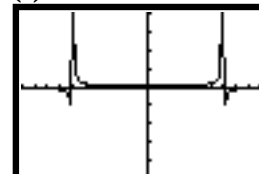
5. (a)



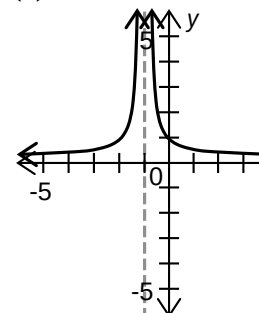
(b)



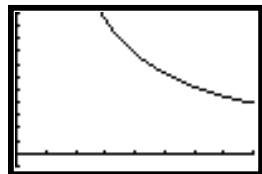
(c)



(d)



6.



The point (4, 87.5) is incorrect. Since $xy = 315$ for all other ordered pairs in the table, the correct value can be found

using the equation $y = \frac{315}{x}$. The correct value is

$$\frac{315}{4} = 78.75.$$

7. The equation for an inverse variation has the form $xy = k$, or $y = \frac{k}{x}$. In this case, the equation will take the form $at = k$,

where a represents the acceleration in m/s^2 and t represents the time to reach the required speed. k is a constant.

Since $t = 6$ when $a = 10$, $10 \times 6 = k$. Equation is $at = 60$, or

$t = \frac{60}{a}$. Time to reach required speed when acceleration is 12

m/s^2 is $t = \frac{60}{12} = 5$ s.

It takes 5 seconds to reach the same speed, when the acceleration is 12 m/s^2 .

8. (a)

$$I = \frac{W}{V}$$

$$= \frac{60}{110}$$

Current $\doteq 0.545$ amps

(b)

$$IV = W$$

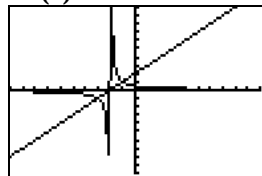
$$15 \times 110 = W$$

$$1650 = W$$

Total power that could be obtained is 1650 W, or 1.65 kW.

(c) If V is fixed, $W = VI$ represents a direct variation. (It is in the form $y = mx$.) The relationship between I and W is linear.

9. (a)



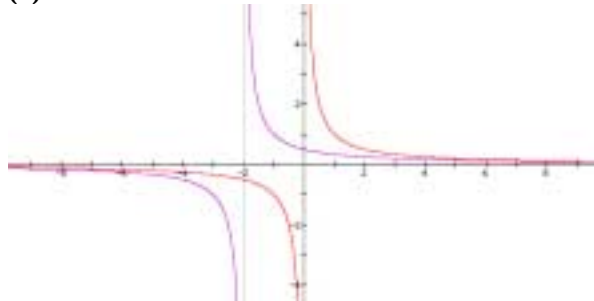
$f(x) = \frac{1}{x+2}$ has a vertical asymptote at the zero for

$f(x) = x + 2$. The equation of the asymptote is $x = -2$. Points

common to both $y = x + 2$ and $y = \frac{1}{x+2}$ are $(-1, 1)$ and

$(-3, -1)$.

(b)



The graph of $f(x) = \frac{1}{x}$ is shifted horizontally two units to the

left to obtain the graph of $f(x) = \frac{1}{x+2}$.

For $f(x) = \frac{1}{x+2}$ the domain is all real numbers except

$x = -2$ and the range is all real numbers except $y = 0$.

$$D = \{x \mid x \neq -2, x \in \mathbf{R}\} \quad R = \{y \mid y \neq 0, y \in \mathbf{R}\}$$

$$\text{Let } y = \frac{1}{x+2}$$

Exchange x and y

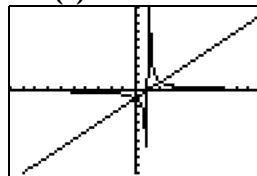
$$x = \frac{1}{y+2}$$

$$\frac{1}{x} = y + 2$$

$$\frac{1}{x} - 2 = y$$

Inverse function is $f^{-1}(x) = \frac{1}{x} - 2$.

10. (a)



For $f(x) = \frac{1}{x-1}$,

Domain = $\{x \mid x \neq 1, x \in \mathbf{R}\}$ Range = $\{y \mid y \neq 0, y \in \mathbf{R}\}$

To find the inverse, exchange x and y :

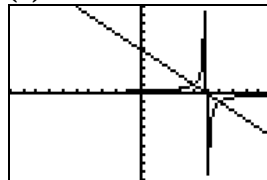
$$x = \frac{1}{y-1}$$

$$\frac{1}{x} = y - 1$$

$$\frac{1}{x} + 1 = y$$

Inverse function is $f^{-1}(x) = \frac{1}{x} + 1$.

(b)



For $f(x) = \frac{1}{5-x}$,

Domain = $\{x \mid x \neq 5, x \in \mathbf{R}\}$ Range = $\{y \mid y \neq 0, y \in \mathbf{R}\}$

To find the inverse, exchange x and y :

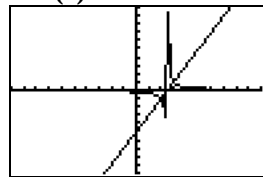
$$x = \frac{1}{5-y}$$

$$\frac{1}{x} = 5-y$$

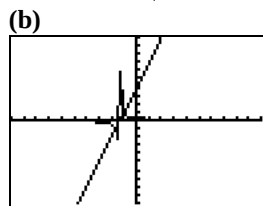
$$y = 5 - \frac{1}{x}$$

Inverse function is $f^{-1}(x) = 5 - \frac{1}{x}$.

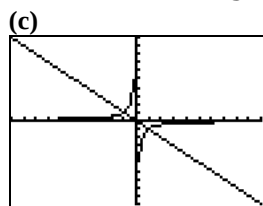
11. (a)



Domain = $\{x \mid x \neq 2.5, x \in \mathbf{R}\}$ Range = $\{y \mid y \neq 0, y \in \mathbf{R}\}$



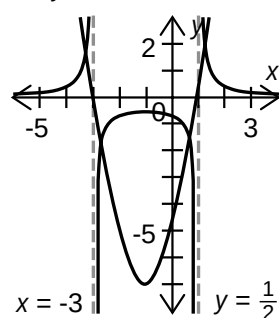
Domain = $\{x \mid x \neq -\frac{4}{3}, x \in \mathbf{R}\}$ Range = $\{y \mid y \neq 0, y \in \mathbf{R}\}$



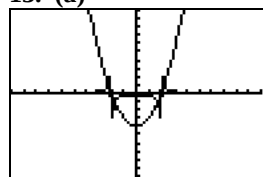
Domain = $\{x \mid x \neq 0, x \in \mathbf{R}\}$ Range = $\{y \mid y \neq 0, y \in \mathbf{R}\}$

12.

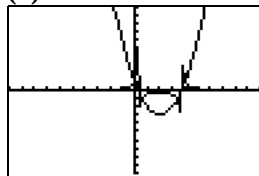
$$y = 2x^2 + 5x - 3$$



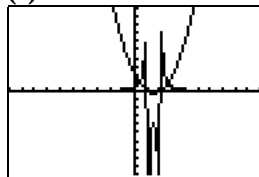
13. (a)



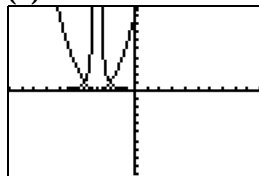
(b)



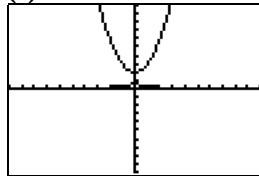
(c)



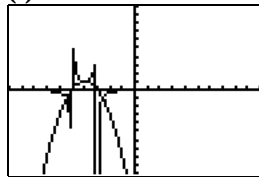
(d)



(e)

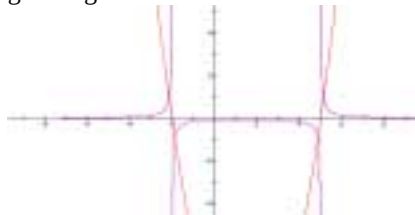


(f)

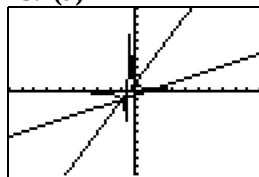


14. At each zero of $y = x^2 - 3x - 10$, draw a vertical asymptote for the graph of $y = \frac{1}{x^2 - 3x - 10}$. Mark the points

where $y = 1$, and where $y = -1$. These are common to both graphs. Find the reciprocal of the y -coordinate of the vertex and mark this point on the grid. Complete the graph using the fact that as $y = x^2 - 3x - 10$ gets smaller/larger, the reciprocal gets larger/smaller.



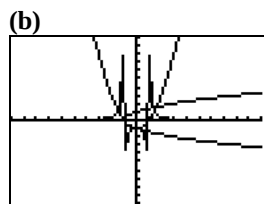
15. (a)



$D(f) = \{x \mid x \in \mathbf{R}\}$ $R(f) = \{y \mid y \in \mathbf{R}\}$

$D\left(\frac{1}{f}\right) = \{x \mid x \neq -\frac{1}{2}, x \in \mathbf{R}\}$ $R\left(\frac{1}{f}\right) = \{y \mid y \neq 0, y \in \mathbf{R}\}$

$D(f^{-1}) = \{x \mid x \in \mathbf{R}\}$ $R(f^{-1}) = \{y \mid y \in \mathbf{R}\}$



$$D(f) = \{x \mid x \in \mathbf{R}\} \quad R(f) = \{y \mid y \geq -1, y \in \mathbf{R}\}$$

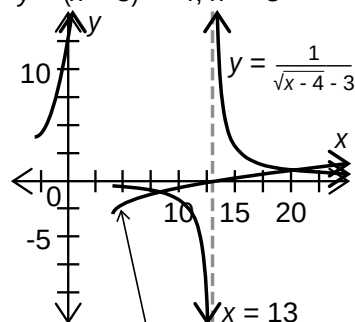
$$D\left(\frac{1}{f}\right) = \{x \mid x \neq \pm 1, x \in \mathbf{R}\}$$

$$R\left(\frac{1}{f}\right) = \{y \mid y \leq -1 \text{ or } y > 0, y \in \mathbf{R}\}$$

$$D(f^{-1}) = \{x \mid x \geq -1, x \in \mathbf{R}\} \quad R(f^{-1}) = \{y \mid y \in \mathbf{R}\}$$

(c)

$$y = (x + 3)^2 + 4, x > -3$$



$$y = \sqrt{x - 4} - 3$$

$$D(f) = \{x \mid x \geq 4, x \in \mathbf{R}\} \quad R(f) = \{y \mid y \geq -3, y \in \mathbf{R}\}$$

$$D\left(\frac{1}{f}\right) = \{x \mid x \neq 13, x \in \mathbf{R}\} \quad R\left(\frac{1}{f}\right) = \{y \mid y \neq 0, y \in \mathbf{R}\}$$

$$D(f^{-1}) = \{x \mid x \geq -3, x \in \mathbf{R}\} \quad R(f^{-1}) = \{y \mid y \geq 4, y \in \mathbf{R}\}$$

16.

- The function $f(x) = \frac{1}{x}$ is the simplest function that displays inverse variation. As x increases, the value of $f(x)$ decreases and as x decreases, the value of $f(x)$ increases.
- Because x appears in the denominator, x may not have the value zero. The domain is restricted to all real numbers except 0. Similarly, the range is all real numbers except zero.
- The graph of $f(x) = \frac{1}{x}$ is a rectangular hyperbola with asymptotes which are the x and y -axes. The hyperbola has two congruent parts, one in the first quadrant and one in the third.
- The function $f(x) = \frac{1}{x}$ is a "self-inverse," ie. $f^{-1}(x) = \frac{1}{x}$ also. The graph of $f(x) = \frac{1}{x}$ has the line $y = x$ as a line of symmetry.
- In common with the graph of $f(x) = x$, the graph of $f(x) = \frac{1}{x}$ passes through the points $(1, 1)$ and $(-1, -1)$.
- In real world situations, only one branch of the graph of $f(x) = \frac{k}{x}$ are useful to describe:
 - the relationship between speed and time for a fixed distance traveled

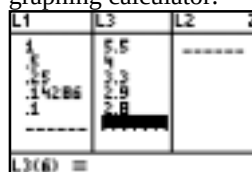
- the relationship between pressure and time for a gas at a given temperature
- the relationship between time on the job and hourly rate for a lump sum job
- and so on

Given some data, you could check to see if $xy = k$ for each ordered pair of data. You could also enter the data in a graphing calculator and do a power regression to see if the relationship could be modelled by a function in the form $y = ax^b$ with b close to -1 .

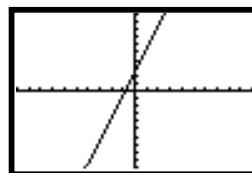
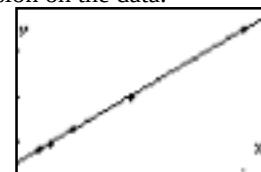
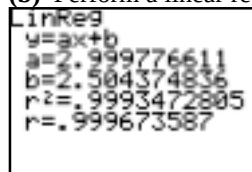
17. (a)

x	1	2	4	7	10
$\frac{1}{x}$	1	0.5	0.25	0.142 857	0.1
y	5.5	4	3.3	2.9	2.8

Enter the values for $\frac{1}{x}$ and corresponding y values in a graphing calculator.



(b) Perform a linear regression on the data.



(c) Slope is 3 and y -intercept is 2.5. Relationship between x and y has equation

$$y = 3\left(\frac{1}{x}\right) + 2.5$$

$$\text{ie. } y = \frac{3}{x} + 2.5$$

18. Counting squares and dividing by 16 (since 16 small squares make 1 unit²), the chart reads:

x	$A(x)$
1	0
2	0.69
3	1.10
4	1.39
5	1.61
6	1.79
7	1.95
8	2.08
9	2.20
10	2.30
11	2.40
12	2.48

$$A(3) \doteq 17 \text{ small squares}$$

$$\therefore 16 \text{ small squares} = 1 \text{ unit}$$

$$\therefore 17 \text{ small squares} = \frac{17}{16} \text{ units}$$

$$\doteq 1.1$$

(a)

$$A(3) + A(4) = 1.10 + 1.39$$

$$= 2.49$$

$$\doteq A(12)$$

(b) $A(2) + A(4) = 0.69 + 1.39$

$$= 2.08$$

$$= A(8)$$

(c) $A(10) - A(2) = 2.30 - 0.69$

$$= 1.61$$

$$= A(5)$$

(d)

$$A(9) = 2.20$$

$$= 2 \times 1.10$$

$$= 2 \times A(3)$$

(i) $A(18) = A(3) + A(6)$

$$= 1.10 + 1.79$$

$$= 2.89$$

or $A(18) = A(2) + A(9)$

$$= 0.69 + 2.20$$

$$= 2.89$$

(ii)

$$A(25) = 2 \times A(5)$$

$$= 2 \times 1.61$$

$$= 3.22$$

(iii) $A(2.5) = A(5) - A(2)$

$$= 1.61 - 0.69$$

$$= 0.92$$

(iv) $A(4.5) = A(9) - A(2)$

$$= 2.20 - 0.69$$

$$= 1.51$$

19. The graph is a rectangular hyperbola and has a vertical asymptote with equation $x = -4$ and a horizontal asymptote with equation $y = -1$. These asymptotes intersect at the point $(-4, -1)$. Points $(-3, 0)$ and $(-5, -2)$ are on opposite branches of the curve. Point $(-3, 0)$ is one unit right and one unit up from $(-4, -1)$, while point $(-5, -2)$ is one unit left and one unit down from $(-4, -1)$. This indicates that this hyperbola is congruent to the graph of $y = \frac{1}{x}$ since that graph has

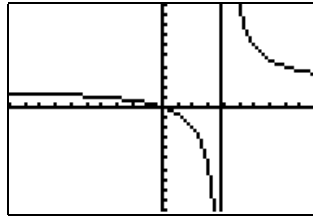
asymptotes which intersect at $(0, 0)$ and the curve passes through $(1, 1)$, which is one unit right and one unit up from $(0, 0)$, and through $(-1, -1)$, which is one unit left and one unit down from $(0, 0)$.

The graph is a translation of the graph of $y = \frac{1}{x}$, four units left and one unit down.

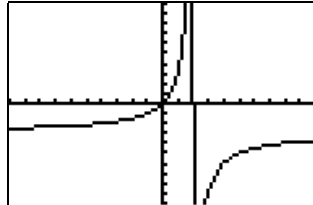
The equation is $f(x) = \frac{1}{x+4} - 1$.

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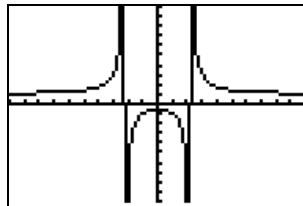
(a) $f(x) = \frac{2x}{x-4}$



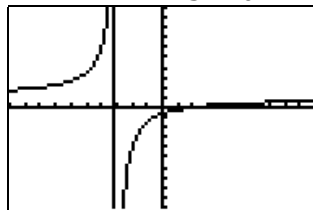
(b) $g(x) = \frac{3x}{2-x}$



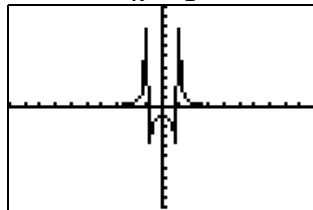
(c) $h(x) = \frac{x^2+3}{x^2-5}$



(d) $h(x) = \frac{x^2-4}{x^2+5x+6}$



(e) $g(x) = \frac{1}{x^2-1}$



(f) $f(x) = \frac{2x}{x^2+4x+3}$

