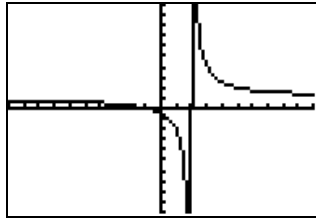
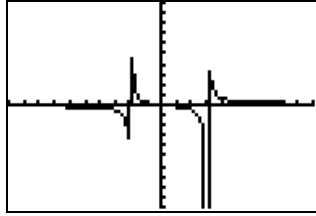


(g) $b(x) = \frac{x+1}{x-2}$



(h) $m(x) = \frac{x}{x^2 - x - 6}$



Practise, Apply, Solve 4.8, page 353

1. In each case, look for values of the variables that would make the denominator zero. Then restrict the values of the variables to exclude these.

(a) $x - 1 = 0$ when $x = 1$

$\therefore$ restriction is $x \neq 1$.

(b) $2y - 1 = 0$ when $y = \frac{1}{2}$

$\therefore$ restriction is $y \neq \frac{1}{2}$.

(c) $6m^2n = 0$ when $m = 0$ or $n = 0$

$\therefore$ restrictions are $m \neq 0, n \neq 0$.

(d) $2x(x - 3) = 0$ when $x = 0$ or $x = 3$

$\therefore$ restrictions are $x \neq 0, x \neq 3$.

(e) $(p - 4)(p + 2) = 0$ when $p = 4$ or $p = -2$

$\therefore$ restrictions are $p \neq -2, p \neq 4$.

(f) $2(a + b) = 0$ when $a = -b$

$\therefore$ restriction is $a \neq -b$.

(g) $8m(m + 3) = 0$ when $m = 0$ or $m = -3$

$\therefore$ restrictions are $m \neq 0, m \neq -3$.

(h) $5(2x - 3) = 0$ when $x = 1.5$

$\therefore$ restriction is $x \neq 1.5$.

(i) $4a(3a + 4) = 0$ when $a = 0$ or $a = -\frac{4}{3}$

$\therefore$ restrictions are $a \neq 0, a \neq -\frac{4}{3}$.

2. (a) $4(x + 3)$

(b) $5(x + 3)$

(c)

$$\frac{4x+12}{5x+15} = \frac{4(x+3)}{5(x+3)}$$

$$= \frac{4}{5}, x \neq -3$$

3. (a)

$$\begin{aligned} -\frac{a+b}{a-b} &= \frac{a+b}{-(a-b)} \\ &= \frac{a+b}{-a+b} \\ &= \frac{a+b}{b-a} \end{aligned}$$

$\therefore$ equivalent.

(b)

$$\begin{aligned} -\frac{a+b}{b-a} &= \frac{a+b}{-(b-a)} \\ &= \frac{a+b}{-b+a} \\ &= \frac{a+b}{a-b} \end{aligned}$$

$\therefore$ not equivalent.

(c) $-\frac{a+b}{-(b-a)} = \frac{a+b}{b-a}$

$\therefore$ equivalent.

(d)

$$\begin{aligned} -\frac{b+a}{a-b} &= \frac{b+a}{-(a-b)} \\ &= \frac{a+b}{-a+b} \\ &= \frac{a+b}{b-a} \end{aligned}$$

$\therefore$ equivalent.

4. (a) $\frac{x+y}{y+x} = 1$ provided $y \neq -x$

(b) $-\frac{(y-x)}{y-x} = -1$ provided $y \neq x$

(c) $\frac{x-y}{-(y-x)} = \frac{x-y}{x-y} = 1$ provided $y \neq x$

(d) $\frac{-(y+x)}{-(x+y)} = 1$ provided $x \neq -y$

(e) $\frac{-(y-x)}{x-y} = \frac{x-y}{x-y} = 1$ provided $x \neq y$

(f) $\frac{x-y}{y-x} = \frac{x-y}{-(x-y)} = -1$ provided $x \neq y$

5. (a) $\frac{x-2}{(x+5)(x-2)}$

Denominator is zero when $x = -5$ or $x = 2$. Restrictions are $x \neq 2, 5$.

Then $\frac{x-2}{(x+5)(x-2)}$ reduces to $\frac{1}{x+5}$.

(b) $\frac{4y(3y-4)}{(3y-4)(3y+4)}$

Denominator is zero when $y = \pm \frac{4}{3}$. Restriction is $y \neq \pm \frac{4}{3}$.

Then $\frac{4y(3y-4)}{(3y-4)(3y+4)}$ reduces to $\frac{4y}{3y+4}$.

$$(c) \frac{(x-1)(3x+5)}{(x-1)(2x+1)}$$

Denominator is zero when $x = 1$ or $x = -\frac{1}{2}$. Restrictions are

$$x \neq -\frac{1}{2}, 1.$$

$$\text{Then } \frac{(x-1)(3x+5)}{(x-1)(2x+1)} \text{ reduces to } \frac{3x+5}{2x+1}.$$

$$6. (a) k^2 - 4 = (k-2)(k+2)$$

$$(b) 3k^2 + 5k - 2 = (3k-1)(k+2)$$

(c)

$$\frac{k^2 - 4}{3k^2 + 5k - 2} = \frac{(k-2)(k+2)}{(3k-1)(k+2)}, k \neq \frac{1}{3} \text{ or } -2$$

$$= \frac{k-2}{3k-1}, \text{ provided } k \neq \frac{1}{3} \text{ or } -2$$

$$7. (a) 6p^2 + 30p + 36 = 6(p^2 + 5p + 6)$$

$$= 6(p+3)(p+2)$$

$$(b) 3p^2 - 3p - 18 = 3(p^2 - p - 6)$$

$$= 3(p-3)(p+2)$$

$$(c) \frac{6p^2 + 30p + 36}{3p^2 - 3p - 18} = \frac{6(p+3)(p+2)}{3(p-3)(p+2)}, p \neq 3 \text{ or } -2$$

$$= \frac{2(p+3)}{(p-3)} \text{ provided } p \neq 3 \text{ or } -2$$

$$8. (a) \frac{2b+8}{5b+20} = \frac{2(b+4)}{5(b+4)} = \frac{2}{5}, b \neq -4$$

$$(b) \frac{3x^2 - 2x}{3x^2 - 6x} = \frac{x(3x-2)}{3x(x-2)}$$

$$= \frac{3x-2}{3(x-2)}, x \neq 0, 2$$

$$(c) \frac{m^2 - 4m}{3m^2 - 12m} = \frac{m(m-4)}{3m(m-4)} = \frac{1}{3}, m \neq 0, 4$$

$$(d) \frac{15x - 5x^2}{x^2 - 3x} = \frac{5x(3-x)}{x(x-3)}$$

$$= \frac{-5x(x-3)}{x(x-3)}$$

$$= -5, x \neq 0, 3$$

$$(e) \frac{x^2 - 4}{x^2 - 5x + 6} = \frac{(x-2)(x+2)}{(x-2)(x-3)}$$

$$= \frac{x+2}{x-3}, x \neq 2, 3$$

$$(f) \frac{p^2 - p - 20}{3p - 15} = \frac{(p-5)(p+4)}{3(p-5)}$$

$$= \frac{p+4}{3}, p \neq 5$$

$$(g) \frac{y^2 - 7y + 12}{y^2 - y - 12} = \frac{(y-3)(y-4)}{(y-4)(y+3)}$$

$$= \frac{y-3}{y+3}, y \neq -3, 4$$

$$(h) \frac{a^2 + a - 20}{a^2 - 11a + 28} = \frac{(a+5)(a-4)}{(a-4)(a-7)}$$

$$= \frac{a+5}{a-7}, a \neq 4, 7$$

$$9. (a) \frac{36x^2y}{-16xy^2} = \frac{(4xy)(9x)}{(4xy)(-4y)}$$

$$= -\frac{9x}{4y}, x \neq 0, y \neq 0$$

$$(b) \frac{6x+24}{4x+16} = \frac{6(x+4)}{4(x+4)} = \frac{3}{2}, x \neq -4$$

$$(c) \frac{9m^2 - 4}{9m^2 - 3m - 2} = \frac{(3m-2)(3m+2)}{(3m-2)(3m+1)}$$

$$= \frac{3m+2}{3m+1}, m \neq -\frac{1}{3}, \frac{2}{3}$$

$$(d) \frac{x^2 - 10xy + 25y^2}{x^2 - 25y^2} = \frac{(x-5y)^2}{(x-5y)(x+5y)}$$

$$= \frac{x-5y}{x+5y}, x \neq \pm 5y$$

$$10. (a) \frac{6m^2 - 2m - 4}{4m^2 - 4} = \frac{2(3m^2 - m - 2)}{4(m^2 - 1)}$$

$$= \frac{2(3m+2)(m-1)}{4(m-1)(m+1)}$$

$$= \frac{3m+2}{2(m+1)}, m \neq \pm 1$$

$$(b) \frac{2a^2 + 5a - 3}{2a^2 - 7a + 3} = \frac{(2a-1)(a+3)}{(2a-1)(a-3)}$$

$$= \frac{a+3}{a-3}, a \neq \frac{1}{2}, 3$$

$$(c) \frac{40 + 6x - 18x^2}{9x^2 - 9x - 10} = \frac{2(20 + 3x - 9x^2)}{(3x+2)(3x-5)}$$

$$= \frac{2(5-3x)(4+3x)}{(3x+2)(3x-5)}$$

$$= \frac{-2(3x-5)(3x+4)}{(3x+2)(3x-5)}$$

$$= \frac{-2(3x+4)}{3x+2}, x \neq -\frac{2}{3}, \frac{5}{3}$$

(d)

$$\begin{aligned}\frac{9x^2 - 1}{9x^3 + 6x^2 + x} &= \frac{(3x-1)(3x+1)}{x(9x^2 + 6x + 1)} \\ &= \frac{(3x-1)(3x+1)}{x(3x+1)^2} \\ &= \frac{3x-1}{x(3x+1)}, x \neq 0, -\frac{1}{3}\end{aligned}$$

11. When referred to rational expressions, restrictions mean restrictions on the values of the variables in the rational expression. Since you cannot divide by zero, the denominator in a rational expression must not be zero. To ensure this, any values of the variables that would make the denominator zero must be excluded. These exclusions are called restrictions. They are written as statements about the variables. For

example, the rational expression $\frac{3xy}{2x(x-1)}$ would be

undefined if x was 0 or 1. Therefore, we write $\frac{3xy}{2x(x-1)}$,

$x \neq 0, 1$, when referring to this expression. It is important to state the restrictions, since they may become less obvious if any reducing is possible. For example,

$$\begin{aligned}\frac{x^2 - 9}{x^2 - 5x + 6} &= \frac{(x-3)(x+3)}{(x-2)(x-3)}, x \neq 2, 3 \\ &= \frac{x+3}{x-2}, x \neq 2, 3\end{aligned}$$

The reduced expression exactly represents the original expression only if both restrictions are in place.

12. (a) $x - y = -(y - x)$ ie. the two expressions are “opposites.”

(b)

$$\begin{aligned}\frac{x^2 + xy - 2y^2}{y^2 - x^2} &= \frac{(x-y)(x+2y)}{(y-x)(y+x)}, x \neq \pm y \\ &= \frac{-(y-x)(x+2y)}{(y-x)(x+y)} \\ &= \frac{-(x+2y)}{x+y}, x \neq \pm y\end{aligned}$$

13. (a)

$$\begin{aligned}\frac{4a^2 - b^2}{b - 2a} &= \frac{(2a-b)(2a+b)}{-(2a-b)} \\ &= -(2a+b), 2a \neq b \left(\text{or } a \neq \frac{1}{2}b \right)\end{aligned}$$

(b)

$$\begin{aligned}\frac{x^2 - 2x + 1}{1 - x} &= \frac{(x-1)^2}{-(x-1)} \\ &= -(x-1) \\ &= 1 - x, x \neq 1\end{aligned}$$

(c)

$$\begin{aligned}\frac{k^2 - 10k + 25}{25 - k^2} &= \frac{(k-5)^2}{(5-k)(5+k)} \\ &= \frac{(k-5)^2}{-(k-5)(5+k)} \\ &= \frac{-(k-5)}{(5+k)} \\ &= \frac{5-k}{5+k}, k \neq \pm 5\end{aligned}$$

14.

$$\begin{aligned}\frac{(2t^3 + t^2 - t)a}{48t^2 - 24t} &= \frac{t(2t^2 + t - 1)a}{24t(2t - 1)} \\ &= \frac{t(2t-1)(t+1)a}{24t(2t-1)} \\ &= \frac{(t+1)a}{24}, t \neq 0, \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\frac{(3t^2 - 2t)a}{3t^2 + 34t - 24} &= \frac{t(3t-2)a}{(3t-2)(t+12)} \\ &= \frac{ta}{t+12}, t \neq \frac{2}{3}, -12\end{aligned}$$

(a) We may use either expression to calculate the dose for a one-year-old child.

$$\text{Using } \frac{(t+1)a}{24}, \text{ dose} = \frac{(1+1)250}{24} \doteq 20.8 \text{ mg}$$

$$\text{Using } \frac{ta}{t+12}, \text{ dose} = \frac{1(250)}{13} \doteq 19.2 \text{ mg}$$

Dose for a one-year-old child is about 20 mg.

(b) $t = \frac{2}{3}$ and so the second formula may not be used.

$$\text{Using } \frac{(t+1)a}{24}, \text{ dose} = \frac{\frac{2}{3}+1}{24}(250) \doteq 17.4 \text{ mg}$$

Dose for an eight-month-old child is about 17 mg.

15. For a cylindrical can with height h and base radius r ,
surface area $= 2\pi r^2 + 2\pi rh$
volume $= \pi r^2 h$

$$\begin{aligned}\text{Ratio } \frac{\text{surface area}}{\text{volume}} &= \frac{2\pi r^2 + 2\pi rh}{\pi r^2 h} \\ &= \frac{2\pi r(r+h)}{\pi r^2 h} \\ &= \frac{2(r+h)}{rh}, r \neq 0, h \neq 0\end{aligned}$$

Notice that in this expression, r and h have equal weighting. If r and h are initially the same size then a change in the height will have the same effect as a similar change in the radius. If r is initially less than h , then increasing or decreasing the radius has a larger effect than increasing or decreasing the height. If h is initially less than r , then change in the height has a greater effect than change in radius. You can investigate this further using a spreadsheet.

r	h	$\frac{2(r+h)}{rh}$	$\frac{2(r+2h)}{2rh}$	$\frac{2(2r+h)}{2rh}$	D-C	E-C
2	5	1.40	1.20	0.90	-0.20	-0.50
4	6	0.83	0.67	0.58	-0.17	-0.25
6	7	0.62	0.48	0.45	-0.14	-0.17
2	20	1.10	1.05	0.60	-0.05	-0.50
3	3	1.33	1.00	1.00	-0.33	-0.33
3	2	1.67	1.17	1.33	-0.50	-0.33
5	4	0.90	0.65	0.70	-0.25	-0.20
8	6	0.58	0.42	0.46	-0.17	-0.13
5	5	0.80	0.60	0.60	-0.20	-0.20
12	3	0.83	0.50	0.75	-0.33	-0.08

r	h	$\frac{2(r+h)}{rh}$	$\frac{2(r+0.5h)}{0.5rh}$	$\frac{2(0.5r+h)}{0.5rh}$	D-C	E-C
2	5	1.40	1.80	2.40	0.40	1.00
4	6	0.83	1.17	1.33	0.33	0.50
6	7	0.62	0.90	0.95	0.29	0.33
2	20	1.10	1.20	2.10	0.10	1.00
3	3	1.33	2.00	2.00	0.67	0.67
3	2	1.67	2.67	2.33	1.00	0.67
5	4	0.90	1.40	1.30	0.50	0.40
8	6	0.58	0.92	0.83	0.33	0.25
5	5	0.80	1.20	1.20	0.40	0.40
12	3	0.83	1.50	1.00	0.67	0.17

16. Factor the numerator and denominator of the expression.

$$\frac{a^2 - 5b^2}{2a^2 - 9ab - 5b^2} = \frac{(a - \sqrt{5}b)(a + \sqrt{5}b)}{(2a + b)(a - 5b)}$$

Look for values of the variables that would make the denominator equal to zero. Exclude these by stating restrictions.

If $2a + b = 0$

$$2a = -b$$

$$a = -\frac{1}{2}b$$

If $a - 5b = 0$

$$a = 5b$$

$\therefore$ restrictions on variables are $a \neq -\frac{1}{2}b, 5b$.

The expression does not reduce.

Craig is incorrect. He must have incorrectly factored the numerator.

17.

$$\begin{aligned} \text{LS} &= \frac{-4x^3 - 7x^2 - 3x}{12x + 13x^2 - 4x^3} \\ &= \frac{-x(4x^2 + 7x + 3)}{-x(4x^2 - 13x - 12)} \\ &= \frac{-x(4x + 3)(x + 1)}{-x(4x + 3)(x - 4)} \\ &= \frac{x + 1}{x - 4}, x \neq -\frac{3}{4}, 0, 4 \end{aligned}$$

$$\begin{aligned} \text{RS} &= \frac{2x^3 + x^2 - x}{2x^3 - 9x^2 + 4x} \\ &= \frac{x(2x^2 + x - 1)}{x(2x^2 - 9x + 4)} \\ &= \frac{x(2x - 1)(x + 1)}{x(2x - 1)(x - 4)} \\ &= \frac{x + 1}{x - 4}, x \neq 0, \frac{1}{2}, 4 \end{aligned}$$

The identity is true provided all restrictions apply. That is, provided $x \neq -\frac{3}{4}, 0, \frac{1}{2}, 4$.

18.

$$\begin{aligned} \text{Drug A concentration} &= \frac{5t^4 - 10t^2}{t^5 - 4t} \\ &= \frac{5t^2(t^2 - 2)}{t(t^4 - 4)} \\ &= \frac{5t^2(t^2 - 2)}{t(t^2 - 2)(t^2 + 2)} \\ C_A &= \frac{5t}{t^2 + 2}, t \neq \pm\sqrt{2}, 0 \end{aligned}$$

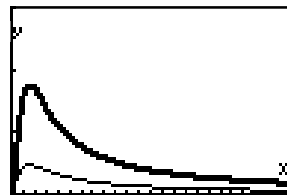
$$\begin{aligned} \text{Drug B concentration} &= \frac{3t^2 - 12t}{2t^3 - 8t^2 + 5t - 20} \\ &= \frac{3t(t - 4)}{2t^2(t - 4) + 5(t - 4)} \\ &= \frac{3t(t - 4)}{(t - 4)(2t^2 + 5)} \\ C_B &= \frac{3t}{2t^2 + 5}, t \neq 4 \end{aligned}$$

The answer to the question “which drug needs to be taken more often?” can be obtained in several ways.

1. Examine the two reduced expressions for C_A and C_B .

For any value of t , the denominator for C_B is always larger than that for C_A , and the numerator for C_A is always larger than that for C_B . This means that the concentration C_A is always larger than C_B . Drug B would need to be taken more often.

2. Graph the two relationships on a graphing calculator, showing the effect of one dose over 24 hours.



Plot1	Plot2	Plot3
$Y_1 = 5X/(X^2 + 2)$		
$Y_2 = 3X/(2X^2 + 5)$		
$Y_3 =$		
$Y_4 =$		
$Y_5 =$		
$Y_6 =$		
$Y_7 =$		

WINDOW
Xmin=0
Xmax=24
Xscl=1
Ymin=0
Ymax=3
Yscl=1
Xres=1

The graph shows that the concentration for drug A is always greater than that for drug B. Drug A maintains an effective

concentration for at least 12 hours, even though the concentration is dropping off rapidly after 3 hours. Drug B seems to have a more gentle effect and the concentration reduces more gradually, but it is probably not at an effective level after about 4 hours.

3. Calculate concentrations at hourly intervals, in a spreadsheet.

Concentration of pain-killing drugs A and B for 24 hours after being taken orally:

t	Drug A	Drug B
1	1.67	0.43
2	1.67	0.46
3	1.36	0.39
3.99	1.11	0.32
5	0.93	0.27
6	0.79	0.23
7	0.69	0.20
8	0.61	0.18
9	0.54	0.16
10	0.49	0.15
11	0.45	0.13
12	0.41	0.12
13	0.38	0.11
14	0.35	0.11
15	0.33	0.10
16	0.31	0.09
17	0.29	0.09
18	0.28	0.08
19	0.26	0.08
20	0.25	0.07
21	0.24	0.07
22	0.23	0.07
23	0.22	0.06
24	0.21	0.06

Drug B would need to be taken again after about 4 hours.

Practise, Apply, Solve 4.9, page 359

1. (a)

$$\frac{3x}{8} \times \frac{2y}{9} = \frac{3x}{(2)(4)} \times \frac{2y}{(3)(3)}$$

$$= \frac{xy}{12}$$

(b)

$$\frac{6}{a} \times \frac{5a^2}{3b} = \frac{(3)(2)}{a} \times \frac{5(a)(a)}{3b}, a \neq 0, b \neq 0$$

$$= \frac{2}{1} \times \frac{5a}{b}$$

$$= \frac{10a}{b}, a \neq 0, b \neq 0$$

(c)

$$\frac{9x^2y}{4y} \times \frac{12xy}{3x^2y} = \frac{(3)(3x^2y)}{4y} \times \frac{(3x)(4y)}{3x^2y}, x \neq 0, y \neq 0$$

$$= \frac{3}{1} \times \frac{3x}{1}$$

$$= 9x, x \neq 0, y \neq 0$$

(d)

$$\frac{14pq}{5q^2} \times \frac{10q^3}{21pq^2} = \frac{(2)(7pq)}{5q^2} \times \frac{(2)(5q^2)(q)}{(3)(7pq)(q)}, p \neq 0, q \neq 0$$

$$= \frac{2}{1} \times \frac{2}{3}$$

$$= \frac{4}{3}, p \neq 0, q \neq 0$$

(e)

$$\frac{-8x^2}{10y} \times \frac{5y^3}{-2x^2} = \frac{(4)(-2x^2)}{(2)(5y)} \times \frac{(5y)(y^2)}{-2x^2}, x \neq 0, y \neq 0$$

$$= \frac{4y^2}{2}$$

$$= 2y^2, x \neq 0, y \neq 0$$

2. (a)

$$\frac{8a}{9} \div \frac{2b}{3} = \frac{8a}{9} \times \frac{3}{2b}, b \neq 0$$

$$= \frac{(2)(4a)}{(3)(3)} \times \frac{3}{2b}$$

$$= \frac{4a}{3b}, b \neq 0$$

(b)

$$\frac{3m^2}{n} \div \frac{2m}{n} = \frac{3m^2}{n} \times \frac{n}{2m}, m \neq 0, n \neq 0$$

$$= \frac{3m(m)}{n} \times \frac{n}{2m}$$

$$= \frac{3m}{1} \times \frac{1}{2}$$

$$= \frac{3m}{2}, m \neq 0, n \neq 0$$

(c)

$$\frac{-56a^2b^2}{24a^3b} \div \frac{16ab^3}{18b} = \frac{-56a^2b^2}{24a^3b} \times \frac{18b}{16ab^3}, a \neq 0, b \neq 0$$

$$= \frac{(-7)(8a^2b)(b)}{(3)(a)(8a^2b)} \times \frac{(3)(3)(2b)}{(8ab)(2b)(b)}$$

$$= \frac{-7}{a} \times \frac{3}{8ab}$$

$$= \frac{-21}{8a^2b}, a \neq 0, b \neq 0$$

(d)

$$\frac{36pqr^2}{-26p^3qr} \div \frac{-24pq^2r}{39pr}$$

$$= \frac{36pqr^2}{-26p^3qr} \times \frac{39pr}{-24pq^2r}, p \neq 0, q \neq 0, r \neq 0$$

$$= \frac{(3)(12pqr)(r)}{(-2)(13pr)(p^2q)} \times \frac{(3)(13pr)}{(-2)(12pqr)(q)}$$

$$= \frac{3r}{-2p^2q} \times \frac{3}{-2q}$$

$$= \frac{9r}{4p^2q^2}, p \neq 0, q \neq 0, r \neq 0$$