

concentration for at least 12 hours, even though the concentration is dropping off rapidly after 3 hours. Drug B seems to have a more gentle effect and the concentration reduces more gradually, but it is probably not at an effective level after about 4 hours.

3. Calculate concentrations at hourly intervals, in a spreadsheet.

Concentration of pain-killing drugs A and B for 24 hours after being taken orally:

t	Drug A	Drug B
1	1.67	0.43
2	1.67	0.46
3	1.36	0.39
3.99	1.11	0.32
5	0.93	0.27
6	0.79	0.23
7	0.69	0.20
8	0.61	0.18
9	0.54	0.16
10	0.49	0.15
11	0.45	0.13
12	0.41	0.12
13	0.38	0.11
14	0.35	0.11
15	0.33	0.10
16	0.31	0.09
17	0.29	0.09
18	0.28	0.08
19	0.26	0.08
20	0.25	0.07
21	0.24	0.07
22	0.23	0.07
23	0.22	0.06
24	0.21	0.06

Drug B would need to be taken again after about 4 hours.

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1. (a)

$$\frac{3x}{8} \times \frac{2y}{9} = \frac{3x}{(2)(4)} \times \frac{2y}{(3)(3)}$$

$$= \frac{xy}{12}$$

(b)

$$\frac{6}{a} \times \frac{5a^2}{3b} = \frac{(3)(2)}{a} \times \frac{5(a)(a)}{3b}, a \neq 0, b \neq 0$$

$$= \frac{2}{1} \times \frac{5a}{b}$$

$$= \frac{10a}{b}, a \neq 0, b \neq 0$$

(c)

$$\frac{9x^2y}{4y} \times \frac{12xy}{3x^2y} = \frac{(3)(3x^2y)}{4y} \times \frac{(3x)(4y)}{3x^2y}, x \neq 0, y \neq 0$$

$$= \frac{3}{1} \times \frac{3x}{1}$$

$$= 9x, x \neq 0, y \neq 0$$

(d)

$$\frac{14pq}{5q^2} \times \frac{10q^3}{21pq^2} = \frac{(2)(7pq)}{5q^2} \times \frac{(2)(5q^2)(q)}{(3)(7pq)(q)}, p \neq 0, q \neq 0$$

$$= \frac{2}{1} \times \frac{2}{3}$$

$$= \frac{4}{3}, p \neq 0, q \neq 0$$

(e)

$$\frac{-8x^2}{10y} \times \frac{5y^3}{-2x^2} = \frac{(4)(-2x^2)}{(2)(5y)} \times \frac{(5y)(y^2)}{-2x^2}, x \neq 0, y \neq 0$$

$$= \frac{4y^2}{2}$$

$$= 2y^2, x \neq 0, y \neq 0$$

2. (a)

$$\frac{8a}{9} \div \frac{2b}{3} = \frac{8a}{9} \times \frac{3}{2b}, b \neq 0$$

$$= \frac{(2)(4a)}{(3)(3)} \times \frac{3}{2b}$$

$$= \frac{4a}{3b}, b \neq 0$$

(b)

$$\frac{3m^2}{n} \div \frac{2m}{n} = \frac{3m^2}{n} \times \frac{n}{2m}, m \neq 0, n \neq 0$$

$$= \frac{3m(m)}{n} \times \frac{n}{2m}$$

$$= \frac{3m}{1} \times \frac{1}{2}$$

$$= \frac{3m}{2}, m \neq 0, n \neq 0$$

(c)

$$\frac{-56a^2b^2}{24a^3b} \div \frac{16ab^3}{18b} = \frac{-56a^2b^2}{24a^3b} \times \frac{18b}{16ab^3}, a \neq 0, b \neq 0$$

$$= \frac{(-7)(8a^2b)(b)}{(3)(a)(8a^2b)} \times \frac{(3)(3)(2b)}{(8ab)(2b)(b)}$$

$$= \frac{-7}{a} \times \frac{3}{8ab}$$

$$= \frac{-21}{8a^2b}, a \neq 0, b \neq 0$$

(d)

$$\frac{36pqr^2}{-26p^3qr} \div \frac{-24pq^2r}{39pr}$$

$$= \frac{36pqr^2}{-26p^3qr} \times \frac{39pr}{-24pq^2r}, p \neq 0, q \neq 0, r \neq 0$$

$$= \frac{(3)(12pqr)(r)}{(-2)(13pr)(p^2q)} \times \frac{(3)(13pr)}{(-2)(12pqr)(q)}$$

$$= \frac{3r}{-2p^2q} \times \frac{3}{-2q}$$

$$= \frac{9r}{4p^2q^2}, p \neq 0, q \neq 0, r \neq 0$$

$$\begin{aligned}
 \text{(e)} \quad \frac{-5a^3}{3b^2} \div \frac{-10a^2b}{3b^2} &= \frac{-5a^3}{3b^2} \times \frac{3b^2}{-10a^2b}, a \neq 0, b \neq 0 \\
 &= \frac{(-5a^3)(a)}{(3b^2) \times 2(-5a^2)(b)} \\
 &= \frac{a}{1} \times \frac{1}{2b} \\
 &= \frac{a}{2b}, a \neq 0, b \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{3. (a)} \quad \frac{3x}{x-2} \times \frac{4(x-2)}{6x} &= \frac{(3x)}{(x-2)} \times \frac{(2)(2)(x-2)}{(2)(3x)}, x \neq 0, 2 \\
 &= \frac{1}{1} \times \frac{2}{1} \\
 &= 2, x \neq 0, 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \frac{5y(y+3)}{4y} \times \frac{(y-5)}{(y+3)} &= \frac{(5)(y)(y+3)}{(4)(y)} \times \frac{(y-5)}{(y+3)}, y \neq 0, -3 \\
 &= \frac{5}{4} \times \frac{(y-5)}{1} \\
 &= \frac{5(y-5)}{4} \text{ or } \frac{5y-25}{4}, y \neq 0, -3
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad \frac{(2a-1)}{3a(a+4)} \times \frac{a(a+4)}{(2a-1)} &= \frac{1}{3} \times \frac{1}{1}, a \neq 0, \frac{1}{2}, -4 \\
 &= \frac{1}{3}, a \neq 0, \frac{1}{2}, -4
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad \frac{10x}{x+2} \div \frac{5}{2(x+2)} &= \frac{10x}{(x+2)} \times \frac{2(x+2)}{5}, x \neq -2 \\
 &= \frac{2x}{1} \times \frac{2}{1} \\
 &= 4x, x \neq -2
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad \frac{4b(b+1)}{(b-1)} \div \frac{12(b+1)}{(b+4)} &= \frac{4b(b+1)}{(b-1)} \times \frac{(b+4)}{12(b+1)}, b \neq -4, \pm 1 \\
 &= \frac{b}{(b-1)} \times \frac{(b+4)}{3} \\
 &= \frac{b(b+4)}{3(b-1)}, b \neq -4, \pm 1
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad \frac{(3s+1)}{2s-1} \div \frac{3s(s+1)}{2s-1} &= \frac{(3s+1)}{(2s-1)} \times \frac{(2s-1)}{3s(s+1)}, s \neq -1, 0, \frac{1}{2} \\
 &= \frac{(3s+1)}{1} \times \frac{1}{3s(s+1)} \\
 &= \frac{(3s+1)}{3s(s+1)}, s \neq -1, 0, \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{4. (a)} \quad \frac{12y}{3y-9} \times \frac{4y-12}{6y^2} &= \frac{12y}{3(y-3)} \times \frac{4(y-3)}{6y^2}, y \neq 0, 3 \\
 &= \frac{(2)(6y)}{3(y-3)} \times \frac{4(y-3)}{(6y)y} \\
 &= \frac{2}{3} \times \frac{4}{y} \\
 &= \frac{8}{3y}, y \neq 0, 3
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \frac{6p-12}{5p+5} \times \frac{2p+2}{3p-6} &= \frac{6(p-2)}{5(p+1)} \times \frac{2(p+1)}{3(p-2)}, p \neq -1, 2 \\
 &= \frac{2}{5} \times \frac{2}{1} \\
 &= \frac{4}{5}, p \neq -1, 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad \frac{x^2-y^2}{x^2-16} \times \frac{x-4}{x-y} &= \frac{(x-y)(x+y)}{(x-4)(x+4)} \times \frac{(x-4)}{(x-y)}, x \neq \pm 4, y \\
 &= \frac{x+y}{x+4}, x \neq \pm 4, y
 \end{aligned}$$

$$\begin{aligned}
 \text{5. (a)} \quad \frac{k+1}{k^2-1} \div \frac{k}{k-1} &= \frac{(k+1)}{(k+1)(k-1)} \times \frac{(k-1)}{k}, k \neq 0, \pm 1 \\
 &= \frac{1}{1} \times \frac{1}{k} \\
 &= \frac{1}{k}, k \neq 0, \pm 1
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \frac{x^2-4}{x+3} \div \frac{x-2}{x^2-9} &= \frac{(x-2)(x+2)}{(x+3)} \div \frac{(x-2)}{(x-3)(x+3)}, x \neq \pm 3 \\
 &= \frac{(x-2)(x+2)}{(x+3)} \times \frac{(x-3)(x+3)}{(x-2)}, x \neq \pm 3, 2 \\
 &= \frac{(x+2)}{1} \times \frac{(x-3)}{1} \\
 &= (x+2)(x-3) \\
 &= x^2-x-6, x \neq \pm 3, 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad \frac{1-3q}{2q+1} \div \frac{1-9q^2}{4q^2-1} &= \frac{1-3q}{2q+1} \div \frac{(1-3q)(1+3q)}{(2q-1)(2q+1)}, q \neq \pm \frac{1}{2} \\
 &= \frac{(1-3q)}{(2q+1)} \times \frac{(2q-1)(2q+1)}{(1-3q)(1+3q)}, q \neq \pm \frac{1}{2}, \pm \frac{1}{3} \\
 &= \frac{1}{1} \times \frac{(2q-1)}{(1+3q)} \\
 &= \frac{2q-1}{1+3q}, q \neq \pm \frac{1}{2}, \pm \frac{1}{3}
 \end{aligned}$$

6. When you divide two rational expressions, you take the reciprocal of the second expression and so the numerator of this second expression is now a denominator. This new denominator cannot be zero and so there may be additional

restrictions to state. For example, to divide the expression

$$\frac{x^2 - x}{x^2 - 5x + 6} \div \frac{x^2 - 1}{x - 3}, \text{ the steps are as follows:}$$

Factor numerators and denominators:

$$\frac{x(x-1)}{(x-2)(x-3)} \div \frac{(x-1)(x+1)}{(x-3)}$$

State restrictions: $x \neq 2, 3$

Take the reciprocal of the second expression and change

$\div$ to $\times$:

$$\frac{x(x-1)}{(x-2)(x-3)} \times \frac{(x-3)}{(x-1)(x+1)}$$

Notice that we must now state more restrictions, ie. $x \neq \pm 1$, to ensure the new denominator cannot be zero.

Then we simplify:

$$\begin{aligned} & \frac{x(x-1)}{(x-2)(x-3)} \times \frac{(x-3)}{(x-1)(x+1)} \\ &= \frac{x}{(x-2)(x+1)} \text{ or } \frac{x}{x^2 - x - 2}, x \neq \pm 1, 2, 3 \end{aligned}$$

7. (a)

$$\begin{aligned} & \frac{x^2 + 2x + 1}{x} \times \frac{x^2 - x}{x^2 - 1} \\ &= \frac{(x+1)(x+1)}{x} \times \frac{x(x-1)}{(x-1)(x+1)}, x \neq 0, \pm 1 \\ &= \frac{x+1}{1} \\ &= x+1, x \neq 0, \pm 1 \end{aligned}$$

(b)

$$\begin{aligned} (x+1) \times \frac{1}{3x^2 - 3} &= \frac{(x+1)}{1} \times \frac{1}{3(x-1)(x+1)}, x \neq \pm 1 \\ &= \frac{1}{1} \times \frac{1}{3(x-1)} \\ &= \frac{1}{3(x-1)}, x \neq \pm 1 \end{aligned}$$

(c)

$$\begin{aligned} & \frac{2x^2 - x - 3}{x^2 - 1} \times \frac{x^2 + x - 2}{2x^2 + x - 6} \\ &= \frac{(2x-3)(x+1)}{(x-1)(x+1)} \times \frac{(x+2)(x-1)}{(2x-3)(x+2)}, x \neq \pm 1, \frac{3}{2}, -2 \\ &= \frac{1}{1} \times \frac{1}{1} \\ &= 1, x \neq \pm 1, \frac{3}{2}, -2 \end{aligned}$$

(d)

$$\begin{aligned} & \frac{x^2 + 10x + 25}{x^2 - 8x + 16} \times \frac{x^2 - 16}{x^2 - 25} \\ &= \frac{(x+5)(x+5)}{(x-4)(x-4)} \times \frac{(x-4)(x+4)}{(x-5)(x+5)}, x \neq 4, \pm 5 \\ &= \frac{(x+5)}{(x-4)} \times \frac{(x+4)}{(x-5)} \\ &= \frac{x^2 + 9x + 20}{x^2 - 9x + 20}, x \neq 4, \pm 5 \end{aligned}$$

(e)

$$\begin{aligned} & \frac{x^2 + 5x + 6}{x^2 - 4} \times \frac{x^2 - 6x + 8}{x^2 - x - 12} \\ &= \frac{(x+2)(x+3)}{(x-2)(x+2)} \times \frac{(x-2)(x-4)}{(x-4)(x+3)}, x \neq -3, \pm 2, 4 \\ &= \frac{1}{1} \times \frac{1}{1} \\ &= 1, x \neq -3, \pm 2, 4 \end{aligned}$$

(f)

$$\begin{aligned} & \frac{2x^2 - 5x - 3}{x^2 - 9} \times \frac{2x^2 + 5x + 2}{2x^2 + 5x + 2} \\ &= \frac{(2x+1)(x-3)}{(x-3)(x+3)} \times \frac{(2x-1)(x+3)}{(2x+1)(x+2)}, x \neq -2, -\frac{1}{2}, \pm 3 \\ &= \frac{1}{1} \times \frac{(2x-1)}{(x+2)} \\ &= \frac{2x-1}{x+2}, x \neq -2, -\frac{1}{2}, \pm 3 \end{aligned}$$

8. (a)

$$\begin{aligned} & \frac{x^2 - 9}{x^2 - 49} \div \frac{x+3}{x+7} = \frac{(x-3)(x+3)}{(x-7)(x+7)} \div \frac{x+3}{x+7}, x \neq \pm 7 \\ &= \frac{(x-3)(x+3)}{(x-7)(x+7)} \times \frac{x+7}{x+3}, x \neq -3, \pm 7 \\ &= \frac{x-3}{x-7} \times \frac{1}{1} \\ &= \frac{x-3}{x-7}, x \neq -3, \pm 7 \end{aligned}$$

(b)

$$\begin{aligned} & \frac{x^2 - 1}{2x^2 + 3x + 1} \div \frac{x+1}{2x+1} \\ &= \frac{(x-1)(x+1)}{(2x+1)(x+1)} \div \frac{x+1}{2x+1}, x \neq -1, -\frac{1}{2} \\ &= \frac{(x-1)(x+1)}{(2x+1)(x+1)} \times \frac{(2x+1)}{(x+1)}, x \neq -1, -\frac{1}{2} \text{ (c)} \\ &= \frac{x-1}{1} \times \frac{1}{x+1} \\ &= \frac{x-1}{x+1}, x \neq -1, -\frac{1}{2} \end{aligned}$$

(c)

$$\begin{aligned} & \frac{3x+9}{2x^4} \div \frac{x^2 + 5x + 6}{4x^2} \\ &= \frac{3(x+3)}{2x^4} \div \frac{(x+3)(x+2)}{4x^2}, x \neq 0 \\ &= \frac{3(x+3)}{(2x^2)x^2} \times \frac{2(2x^2)}{(x+3)(x+2)}, x \neq -3, -2, 0 \\ &= \frac{3}{x^2} \times \frac{2}{(x+2)} \\ &= \frac{6}{x^2(x+2)}, x \neq -3, -2, 0 \end{aligned}$$

(d)

$$\begin{aligned}
& \frac{x^2 + 4x - 5}{x^2 - 3x} \div \frac{x^2 - 1}{x^2 - 6x + 9} \\
&= \frac{(x+5)(x-1)}{x(x-3)} \div \frac{(x-1)(x+1)}{(x-3)(x-3)}, x \neq 0, 3 \\
&= \frac{(x+5)(x-1)}{x(x-3)} \times \frac{(x-3)(x-3)}{(x-1)(x+1)}, x \neq 0, \pm 1, 3 \\
&= \frac{(x+5)}{x} \times \frac{(x-3)}{(x+1)} \\
&= \frac{(x+5)(x-3)}{x(x+1)} \text{ or } \frac{x^2 + 2x - 15}{x^2 + x}, x \neq 0, \pm 1, 3
\end{aligned}$$

(e)

$$\begin{aligned}
& \frac{x^2 - 2x - 15}{12x + 9} \div \frac{x+3}{6} = \frac{(x-5)(x+3)}{3(4x+3)} \div \frac{x+3}{6}, x \neq -\frac{3}{4} \\
&= \frac{(x-5)(x+3)}{3(4x+3)} \times \frac{6}{(x+3)}, x \neq -\frac{3}{4}, -3 \\
&= \frac{(x-5)}{(4x+3)} \times \frac{2}{1} \\
&= \frac{2(x-5)}{4x+3}, x \neq -\frac{3}{4}, -3
\end{aligned}$$

(f)

$$\begin{aligned}
& \frac{x-2}{x^3} \div \frac{x^2 - 3x + 2}{x^2} \\
&= \frac{x-2}{x^3} \div \frac{(x-2)(x-1)}{x^2}, x \neq 0 \\
&= \frac{x-2}{x(x^2)} \times \frac{x^2}{(x-2)(x-1)}, x \neq 0, 1, 2 \\
&= \frac{1}{x} \times \frac{1}{(x-1)} \\
&= \frac{1}{x(x-1)}, x \neq 0, 1, 2
\end{aligned}$$

9. (a)

$$\begin{aligned}
& \frac{x^2 - 1}{x^2 + 6x + 9} \div \frac{x^2 + 2x - 3}{x^2 + 6x + 9} \\
&= \frac{(x-1)(x+1)}{(x+3)(x+3)} \div \frac{(x+3)(x-1)}{(x+3)(x+3)}, x \neq -3 \\
&= \frac{(x-1)(x+1)}{(x+3)(x+3)} \times \frac{(x+3)(x+3)}{(x+3)(x-1)}, x \neq -3, 1 \\
&= \frac{(x+1)}{(x+3)} \times \frac{1}{1} \\
&= \frac{x+1}{x+3}, x \neq -3, 1
\end{aligned}$$

(b)

$$\begin{aligned}
& \frac{m^2 + 3m + 2}{m^2 - 3m} \times \frac{3-m}{m^2 + 7m + 6} \\
&= \frac{(m+1)(m+2)}{m(m-3)} \times \frac{-1(m-3)}{(m+1)(m+6)}, m \neq -6, -1, 0, 3 \\
&= \frac{m+2}{m} \times \frac{-1}{m+6} \\
&= \frac{-(m+2)}{m(m+6)} \text{ or } \frac{-m-2}{m^2 + 6m}, m \neq -6, -1, 0, 3
\end{aligned}$$

(c)

$$\begin{aligned}
& (4x^2 - 9y^2) \div \frac{3xy + 2x^2}{2} \\
&= \frac{(2x-3y)(2x+3y)}{1} \div \frac{x(3y+2x)}{2} \\
&= \frac{(2x-3y)(2x+3y)}{1} \times \frac{2}{x(2x+3y)}, x \neq 0, -\frac{3}{2}y \\
&= \frac{(2x-3y)}{1} \times \frac{2}{x} \\
&= \frac{2(2x-3y)}{x} \text{ or } \frac{4x-6y}{x}, x \neq 0, -\frac{3}{2}y
\end{aligned}$$

(d)

$$\begin{aligned}
& \frac{1+4z+4z^2}{z} \times \frac{3z^4}{12z^2-3} \\
&= \frac{(1+2z)(1+2z)}{z} \times \frac{(3)(z^3)(z)}{3(2z-1)(2z+1)}, z \neq 0, \pm \frac{1}{2} \\
&= \frac{1+2z}{1} \times \frac{z^3}{(2z-1)} \\
&= \frac{z^3(1+2z)}{2z-1} \text{ or } \frac{z^3+2z^4}{2z-1}, z \neq 0, \pm \frac{1}{2}
\end{aligned}$$

(e)

$$\begin{aligned}
& \frac{a^2 + 10a + 25}{a^2 + 2a - 15} \times \frac{a^2 - 5a + 6}{a^2 + 3a - 10} \\
&= \frac{(a+5)(a+5)}{(a+5)(a-3)} \times \frac{(a-2)(a-3)}{(a+5)(a-2)}, a \neq -5, 2, 3 \\
&= \frac{1}{1} \times \frac{1}{1} \\
&= 1, a \neq -5, 2, 3
\end{aligned}$$

(f)

$$\begin{aligned}
& \frac{21p-3p^2}{16p^2+4p^3} \div \frac{14-9p+p^2}{12+7p+p^2} \\
&= \frac{3p(7-p)}{4p^2(4+p)} \div \frac{(7-p)(2-p)}{(4+p)(3+p)}, p \neq -4, -3, 0 \\
&= \frac{(3)(p)(7-p)}{4p(p)(4+p)} \times \frac{(4+p)(3+p)}{(7-p)(2-p)}, p \neq -4, -3, 0, 2, 7 \\
&= \frac{3}{4p} \times \frac{(3+p)}{(2-p)} \\
&= \frac{3(3+p)}{4p(2-p)} \text{ or } \frac{9+3p}{8p-4p^2}, p \neq -4, -3, 0, 2, 7
\end{aligned}$$

10. (a)

$$\begin{aligned} & \frac{5x^2+15x}{x^2-2x-15} \times \frac{x^2-7x+10}{10x-20} \\ &= \frac{5x(x+3)}{(x+3)(x-5)} \times \frac{(x-2)(x-5)}{10(x-2)}, x \neq -3, 2, 5 \\ &= \frac{x}{1} \times \frac{1}{2} \\ &= \frac{x}{2}, x \neq -3, 2, 5 \end{aligned}$$

(b)

$$\begin{aligned} & \frac{k^2+k-20}{k^2-16} \div \frac{k^2+3k-10}{k^2+2k-8} \\ &= \frac{(k+5)(k-4)}{(k-4)(k+4)} \div \frac{(k+5)(k-2)}{(k+4)(k-2)}, k \neq 2, \pm 4 \\ &= \frac{(k+5)(k-4)}{(k-4)(k+4)} \times \frac{(k+4)(k-2)}{(k+5)(k-2)}, k \neq -5, 2, \pm 4 \\ &= \frac{1}{1} \times \frac{1}{1} \\ &= 1, k \neq -5, 2, \pm 4 \end{aligned}$$

11.

$$\begin{aligned} f(x) &= \frac{x^2-16}{4x-x^2} = \frac{(x-4)(x+4)}{x(4-x)}, x \neq 0, 4 \\ &= \frac{(x-4)(x+4)}{-x(x-4)} \\ &= \frac{x+4}{-x}, x \neq 0, 4 \end{aligned}$$

(a) $f(1) = \frac{1+4}{-1} = -5$

(b) $f(-1) = \frac{-1+4}{-(-1)} = 3$

(c) $f(5) = \frac{5+4}{-5} = -\frac{9}{5}$ or -1.8

(d) $f(-2) = \frac{-2+4}{-(-2)} = 1$

(e) $f(3) = \frac{3+4}{-3} = -\frac{7}{3}$ or -2.3

(f) $f(-3) = \frac{-3+4}{-(-3)} = \frac{1}{3}$ or $0.\bar{3}$

(g) $f(x)$ is not defined when $x = 0$ or 4 .

12. Simplify the expression for $C(x)$ first.

$$\begin{aligned} C(x) &= \frac{x-2}{100x-x^2} \div \frac{x^2+18x-40}{5x^2+100x} \\ &= \frac{x-2}{x(100-x)} \div \frac{(x+20)(x-2)}{5x(x+20)}, x \neq 0, 100, -20 \\ &= \frac{x-2}{x(100-x)} \times \frac{5x(x+20)}{(x+20)(x-2)}, x \neq 0, 2, 100, -20 \\ &= \frac{1}{(100-x)} \times \frac{5}{1} \\ &= \frac{5}{100-x}, x \neq 0, 2, 100, -20 \end{aligned}$$

(a) $C(90) = \frac{5}{100-90} = 0.5$

It costs $\frac{1}{2}$ billion dollars to keep the atmosphere 90% free of the toxin.

(b) $C(99) = \frac{5}{100-99} = 5$

It costs 5 billion dollars to keep the atmosphere 99% free of the toxin.

(c) $C(99.9) = \frac{5}{100-99.9} = 50$

It costs 50 billion dollars to keep the atmosphere 99.9% free of the toxin.

13.

$$\begin{aligned} & \frac{15x^2+7x-2}{2x^2-x-15} \div \frac{6x^2-11x-10}{4x^2-25} \\ &= \frac{(5x-1)(3x+2)}{(2x+5)(x-3)} \div \frac{(3x+2)(2x-5)}{(2x-5)(2x+5)}, x \neq \pm \frac{5}{2}, 3 \\ &= \frac{(5x-1)(3x+2)}{(2x+5)(x-3)} \times \frac{(2x-5)(2x+5)}{(3x+2)(2x-5)}, x \neq \pm \frac{5}{2}, -\frac{2}{3}, 3 \\ &= \frac{5x-1}{x-3}, x \neq \pm \frac{5}{2}, -\frac{2}{3}, 3 \text{ or } x \neq \pm 2.5, -0.\bar{6}, 3 \end{aligned}$$

If Charlene had attempted to evaluate the original expression for $x = 2.5$, she would have been attempting to divide by zero. This is why she was unable to obtain a value. Note that $x \neq 2.5$ is one of the restrictions on x .

14. (a)

$$\begin{aligned} & \frac{a^2-b^2}{a^2+b^2} \times \frac{a^2+ab}{b^2-a^2} \div a \\ &= \frac{(a-b)(a+b)}{a^2+b^2} \times \frac{a(a+b)}{(b-a)(b+a)} \div \frac{a}{1}, a \neq \pm b \\ &= \frac{(a-b)(a+b)}{a^2+b^2} \times \frac{a(a+b)}{-1(a-b)(a+b)} \times \frac{1}{a}, a \neq 0, \pm b \\ &= \frac{1}{a^2+b^2} \times \frac{a+b}{-1} \times \frac{1}{1} \\ &= -\frac{a+b}{a^2+b^2}, a \neq 0, \pm b \end{aligned}$$

(b)

$$\begin{aligned} & \frac{x^3+4x^2}{x^2-1} \times \frac{x^2-5x+6}{x^2-3x} \div \frac{x^2+2x-8}{x^2-1} \\ &= \frac{x^2(x+4)}{(x-1)(x+1)} \times \frac{(x-3)(x-2)}{x(x-3)} \div \frac{(x+4)(x-2)}{(x-1)(x+1)}, x \neq 0, \pm 1, 3 \\ &= \frac{x(x)(x+4)}{(x-1)(x+1)} \times \frac{(x-3)(x-2)}{x(x-3)} \times \frac{(x-1)(x+1)}{(x+4)(x-2)}, \\ & \quad x \neq 0, \pm 1, 2, 3, -4 \\ &= \frac{x}{1} \times \frac{1}{1} \times \frac{1}{1} \\ &= x, x \neq 0, \pm 1, 2, 3, -4 \end{aligned}$$

15. (a)

$$\begin{aligned} & \frac{x+1}{x-1} \times \frac{x+3}{1-x^2} \div \frac{(x+3)^2}{1-x} \\ &= \frac{(x+1)}{(x-1)} \times \frac{(x+3)}{(1-x)(1+x)} \div \frac{(x+3)(x+3)}{(1-x)}, x \neq \pm 1 \\ &= \frac{(x+1)}{(x-1)} \times \frac{(x+3)}{(1-x)(1+x)} \times \frac{(1-x)}{(x+3)(x+3)}, x \neq \pm 1, -3 \\ &= \frac{1}{(x-1)} \times \frac{1}{1} \times \frac{1}{(x+3)} \\ &= \frac{1}{(x-1)(x+3)} \text{ or } \frac{1}{x^2+2x-3}, x \neq \pm 1, -3 \end{aligned}$$

If $x = 2$,

$$\frac{1}{x^2+2x-3} = \frac{1}{2^2+2(2)-3} = \frac{1}{5} \text{ or } 0.2$$

(b)

$$\begin{aligned} & \frac{x^2+6x+5}{x^2+7x+12} \times \frac{x^2+2x-8}{x^2-25} \div \frac{x^2-x-2}{x^2-2x-15} \\ &= \frac{(x+5)(x+1)}{(x+3)(x+4)} \times \frac{(x+4)(x-2)}{(x-5)(x+5)} \div \frac{(x-2)(x+1)}{(x-5)(x+3)}, \\ & \quad x \neq -4, -3, \pm 5 \\ &= \frac{(x+5)(x+1)}{(x+3)(x+4)} \times \frac{(x+4)(x-2)}{(x-5)(x+5)} \times \frac{(x-5)(x+3)}{(x-2)(x+1)}, \\ & \quad x \neq -4, -3, -1, 2, \pm 5 \\ &= \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1} \\ &= 1, x \neq -4, -3, -1, 2, \pm 5 \end{aligned}$$

16.

$$\begin{aligned} & \frac{(x-1)f(x)}{3x^2+5x-2} \times \frac{x^2+2x}{2x^2-x-1} \div \frac{x^2g(x)}{6x^2+x-1} = \frac{3}{x} \\ & \frac{(x-1)f(x)}{(3x-1)(x+2)} \times \frac{x(x+2)}{(2x+1)(x-1)} \div \frac{x^2g(x)}{(3x-1)(2x+1)} = \frac{3}{x} \\ & \text{At this point, restrictions on } x \text{ are } x \neq -2, -\frac{1}{2}, \frac{1}{3}, 1, 0. \\ & \frac{(x-1)f(x)}{(3x-1)(x+2)} \times \frac{(x)(x+2)}{(2x+1)(x-1)} \times \frac{(3x-1)(2x+1)}{(x)(x)g(x)} = \frac{3}{x} \\ & \frac{f(x)}{1} \times \frac{1}{1} \times \frac{1}{xg(x)} = \frac{3}{x} \\ & \frac{f(x)}{xg(x)} = \frac{3}{x} \end{aligned}$$

where $x \neq -2, -\frac{1}{2}, 0, \frac{1}{3}, 1$

Multiply both sides by x ,

$$\begin{aligned} \frac{x}{1} \times \frac{f(x)}{xg(x)} &= \frac{3}{x} \times \frac{x}{1} \\ \frac{f(x)}{g(x)} &= \frac{3}{1} \end{aligned}$$

Cross multiply: $f(x) = 3g(x)$

A relationship between $f(x)$ and $g(x)$ is $f(x) = 3g(x)$.

17. (a)

$$\begin{aligned} & \frac{2x^2-3x-2}{6x^2-x-2} \times \frac{3x^2-5x+2}{4x^2-1} \div \frac{x^2-1}{6x^2+7x+2} \\ &= \frac{(2x+1)(x-2)}{(3x-2)(2x+1)} \times \frac{(3x-2)(x-1)}{(2x-1)(2x+1)} \div \frac{(x-1)(x+1)}{(3x+2)(2x+1)}, \\ & \quad x \neq \pm \frac{1}{2}, \pm \frac{2}{3} \\ &= \frac{(2x+1)(x-2)}{(3x-2)(2x+1)} \times \frac{(3x-2)(x-1)}{(2x-1)(2x+1)} \times \frac{(3x+2)(2x+1)}{(x-1)(x+1)}, \\ & \quad x \neq \pm \frac{1}{2}, \pm \frac{2}{3}, \pm 1 \\ &= \frac{(x-2)}{1} \times \frac{1}{(2x-1)} \times \frac{(3x+2)}{(x+1)} \\ &= \frac{(x-2)(3x+2)}{(2x-1)(x+1)} \text{ or } \frac{3x^2-4x-4}{2x^2+x-1}, x \neq \pm \frac{1}{2}, \pm \frac{2}{3}, \pm 1 \end{aligned}$$

(b)

$$\begin{aligned} & \frac{6x^2-7ab-3b^2}{4a^2+19ab-5b^2} \div \frac{9b^2-4a^2}{4a^2-1} \times \frac{8a^2+10ab-3b^2}{4a^2+4a+1} \\ &= \frac{(3a+b)(2a-3b)}{(4a-b)(a+5b)} \div \frac{(3b-2a)(3b+2a)}{(2a-1)(2a+1)} \times \frac{(4a-b)(2a+3b)}{(2a+1)(2a+1)} \\ & \text{Restrictions so far are } a \neq \pm \frac{1}{2}, -5b, \frac{b}{4}. \\ &= \frac{(3a+b)(2a-3b)}{(4a-b)(a+5b)} \times \frac{(2a-1)(2a+1)}{-1(2a-3b)(3b+2a)} \times \frac{(4a-b)(2a+3b)}{(2a+1)(2a+1)} \\ & \text{for } a \neq \pm \frac{1}{2}, -5b, \frac{b}{4}, \pm \frac{3b}{2} \\ &= \frac{(3a+b)}{(a+5b)} \times \frac{(2a-1)}{-1} \times \frac{1}{(2a+1)} \\ &= -\frac{(3a+b)(2a-1)}{(a+5b)(2a+1)} \text{ or } \frac{-6a^2-2ab+3a+b}{2a^2+10ab+a+5b} \\ & a \neq \pm \frac{1}{2}, -5b, \frac{b}{4}, \pm \frac{3b}{2} \text{ or } a \neq \pm 0.5, -5b, 0.25b, \pm 1.5b \end{aligned}$$

(c)

$$\begin{aligned}
& \frac{m^2 - mn}{6m^2 + 11mn + 3n^2} \div \frac{m^2 - n^2}{2m^2 - mn - 6n^2} \\
&= \frac{4m^2 - 7mn - 2n^2}{3m^2 + 7mn + 2n^2} \\
&= \frac{m(m-n)}{(3m+n)(2m+3n)} \div \frac{(m-n)(m+n)}{(2m+3n)(m-2n)} \div \frac{(4m+n)(m-2n)}{(3m+n)(m+2n)} \\
&\quad \text{for } m \neq -\frac{n}{3}, -\frac{3n}{2}, \pm 2n \\
&= \frac{m(m-n)}{(3m+n)(2m+3n)} \times \frac{(2m+3n)(m-2n)}{(m-n)(m+n)} \times \frac{(3m+n)(m+2n)}{(4m+n)(m-2n)} \\
&\quad \text{for } m \neq -\frac{n}{4}, -\frac{n}{3}, -\frac{3n}{2}, \pm n, \pm 2n \\
&= \frac{m}{1} \times \frac{1}{(m+n)} \times \frac{(m+2n)}{(4m+n)} \\
&= \frac{m(m+2n)}{(m+n)(4m+n)} \quad \text{or} \quad \frac{m^2 + 2mn}{4m^2 + 5mn + n^2}, \\
&\quad m \neq -\frac{n}{4}, -\frac{n}{3}, -\frac{3n}{2}, \pm n, \pm 2n \text{ or} \\
&\quad m \neq -0.25n, -0.3n, -1.5n, \pm n, \pm 2n
\end{aligned}$$

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1. (a) $1 + i$

(b) $-1 - 2i$

(c) $\sqrt{3} + i$

(d) $-2 - \sqrt{2}i$

(e) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$

(f) $x - yi$

2. (a)

$$\begin{aligned}
\frac{1}{1-i} \times \frac{1+i}{1+i} &= \frac{1+i}{1-i^2} \\
&= \frac{1+i}{1-(-1)} \\
&= \frac{1}{2} - \frac{1}{2}i
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{1}{-1+2i} \times \frac{-1-2i}{-1-2i} &= \frac{-1-2i}{1-4i^2} \\
&= \frac{-1-2i}{1-4(-1)} \\
&= -\frac{1}{5} - \frac{2}{5}i
\end{aligned}$$

(c)

$$\begin{aligned}
\frac{1}{\sqrt{3}-i} \times \frac{\sqrt{3}+i}{\sqrt{3}+i} &= \frac{\sqrt{3}+i}{3-i^2} \\
&= \frac{\sqrt{3}+i}{3-(-1)} \\
&= \frac{\sqrt{3}}{4} + \frac{1}{4}i
\end{aligned}$$

(d)

$$\begin{aligned}
\frac{1}{-2+\sqrt{2}i} \times \frac{-2-\sqrt{2}i}{-2-\sqrt{2}i} &= \frac{-2-\sqrt{2}i}{4-2i^2} \\
&= \frac{-2-\sqrt{2}i}{4-2(-1)} \\
&= -\frac{2}{6} - \frac{\sqrt{2}}{6}i \\
&= -\frac{1}{3} - \frac{\sqrt{2}}{6}i
\end{aligned}$$

(e)

$$\begin{aligned}
\frac{1}{-\frac{1}{2} - \frac{\sqrt{3}}{2}i} \times \frac{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}{-\frac{1}{2} + \frac{\sqrt{3}}{2}i} &= \frac{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}{\frac{1}{4} - \frac{3}{4}i^2} \\
&= \frac{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}{\frac{1}{4} - \frac{3}{4}(-1)} \\
&= -\frac{1}{2} + \frac{\sqrt{3}}{2}i
\end{aligned}$$

(f)

$$\begin{aligned}
\frac{1}{x+yi} \times \frac{x-yi}{x-yi} &= \frac{x-yi}{x^2-y^2i^2} \\
&= \frac{x-yi}{x^2-y^2(-1)} \\
&= \frac{x}{x^2+y^2} - \frac{y}{x^2+y^2}i
\end{aligned}$$

x, y not both zero.

3. (a)

$$\begin{aligned}
\frac{30}{3+i} \times \frac{3-i}{3-i} &= \frac{90-30i}{9-i^2} \\
&= \frac{90-30i}{9-(-1)} \\
&= 9-3i
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{2}{1+i} \times \frac{1-i}{1-i} &= \frac{2-2i}{1-i^2} \\
&= \frac{2-2i}{1-(-1)} \\
&= 1+i
\end{aligned}$$