

19. At the end of the first year, $i = \frac{0.04}{4} = 0.01$ and
 $A = \$4000(1.01)^4 = \$4162.42$.
 At the end of the second year, $i = \frac{0.04 + 0.002}{4} = 0.0105$ and
 $A = \$4162.42(1.0105)^4 = \$4340.01$.
 At the end of the third year, $i = \frac{0.04 + 2(0.002)}{4} = 0.011$ and
 $A = \$4340.01(1.011)^4 = \$4534.14$.

Alternate Solution:

$A = \$4000[(1.01)^4(1.0105)^4(1.011)^4] = \$4534.14$.
 $\therefore$ Bernie's account has a balance of \$4534.14 after three years.

20. For 5 years, $n = 5 \times 12 = 60$ and for 3 years
 $n = 3 \times 12 = 36$.

$$i = \frac{0.048}{12} = 0.004$$

After Sarah's 5th birthday,

$$A = \$500(1.004)^{60} + \$500 = \$1135.32.$$

After Sarah's 10th birthday,

$$A = \$1135.32(1.004)^{60} + \$500 = \$1942.58.$$

After Sarah's 15th birthday,

$$A = \$1942.58(1.004)^{60} + \$500 = \$2968.33.$$

After Sarah's 18th birthday,

$$A = \$2968.33(1.004)^{36} = \$3427.09$$

$\therefore$ the account balance is \$3427.09 on Sarah's 18th birthday.

21. (a) $1 + \frac{0.04}{12} = \frac{301}{300}$, $P_0 = \$100$,

$$P_{n+1} = (P_n) \left(\frac{301}{300} \right) + \$100$$

$$P_1 = \$200.33, P_2 = \$301.00, P_3 = \$402.00, P_4 = \$503.34,$$

$$P_5 = \$605.02, P_6 = \$707.04, P_7 = \$809.40, P_8 = \$912.09,$$

$$P_9 = \$1015.13, P_{10} = \$1118.52, P_{11} = \$1222.25,$$

$$P_{12} = \$1326.32$$

Assuming that he hasn't yet invested \$100 for the first month after the end of the first year, there is

$\$1326.32 - \$100 = \$1226.32$ in Josh's bank account.

(b) $P = \frac{\$1226.32}{\left(\frac{301}{300}\right)^{12}} = \$1178.31$.

22. Matt: $i_1 = \frac{0.08}{2} = 0.04$, $n_1 = 5 \times 2 = 10$,

$$A_1 = \$500(1.04)^{10} = \$740.12$$

Justin: $i_2 = \frac{0.08}{4} = 0.02$, $n_2 = 5 \times 4 = 20$,

$$A_2 = \$500(1.02)^{20} = \$742.97$$

$\therefore$ Justin will have more money after five years (\$2.85 more) because there are more compounding periods associated with his investment.

23.

$$2P = P(1+i)^7$$

$$2 = (1+i)^7$$

$$i = 2^{\frac{1}{7}} - 1$$

$$i = 0.104$$

$$i = 10.4\%$$

$\therefore$ an interest rate of 10.4% would result in a doubled investment in seven years.

24.

$$\$500(1.08)^n = \$600(1.06)^n$$

$$5(1.08)^n = 6(1.06)^n$$

$$\frac{5}{6} = \left(\frac{1.06}{1.08} \right)^n$$

$$n \ln \left(\frac{1.06}{1.08} \right) = \ln \left(\frac{5}{6} \right)$$

$$n \doteq 9.75 \text{ years}$$

$\therefore$ the amounts in both accounts would be equal after approximately 9.75 years.

25. Does $(1 + 2i)^n = 2(1 + i)^n$? No, since powers are involved. Consider $(1 + 0.02)^5 = 2(1 + 0.01)^5$.

$$\text{LHS} = 1.1041, \text{RHS} = 2.1020.$$

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1. (a) $4^3 = 64$

(b) $(-6)^2 = 36$

(c) $-3^2 = -9$

(d) $(-9)^3 = -729$

(e) $15^0 = 1$

(f) $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

(g) $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$

(h) $\left(\frac{-2}{3}\right)^3 = \frac{-8}{27}$

(i) $\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9} = 1\frac{7}{9}$

(j) $\left(\frac{-1}{4}\right)^{-3} = \left(\frac{4}{-1}\right)^3 = -64$

2. (a) $\sqrt{121} = 11$

(b) $\sqrt[3]{8} = 2$

(c) $\sqrt[4]{16} = 2$

(d) $\sqrt[3]{125} = 5$

(e) $\sqrt[5]{1024} = 4$

3. (a) $4^{\frac{1}{2}} = 2$

(b) $(-8)^{\frac{1}{3}} = -2$

(c) $-81^{\frac{1}{4}} = -3$
 (d) $27^{\frac{1}{3}} = 3$
 (e) $125^{\frac{1}{3}} = 5$
 (f) $\left(\frac{1}{16}\right)^{\frac{1}{2}} = \frac{1}{4}$
 (g) $(216)^{-\frac{1}{3}} = 6^{-1} = \frac{1}{6}$
 (h) $(125)^{-\frac{1}{3}} = 5^{-1} = \frac{1}{5}$
 (i) $\left(\frac{4}{9}\right)^{\frac{1}{2}} = \frac{2}{3}$
 (j) $(16)^{-\frac{1}{4}} = 2^{-1} = \frac{1}{2}$

4. (a) $16^{\frac{3}{2}} = 4^3 = 64$
 (b) $8^{\frac{2}{3}} = 2^2 = 4$
 (c) $-16^{\frac{3}{4}} = -2^3 = -8$
 (d) $27^{\frac{4}{3}} = 3^4 = 81$
 (e) $81^{\frac{5}{4}} = (4\sqrt[4]{81})^5 = 3^5 = 243$
 (f) $\left(\frac{1}{8}\right)^{\frac{2}{3}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$
 (g) $(27)^{-\frac{2}{3}} = 3^{-2} = \frac{1}{9}$
 (h) $(125)^{-\frac{4}{3}} = 5^{-4} = \frac{1}{625}$
 (i) $\left(\frac{4}{25}\right)^{\frac{3}{2}} = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$
 (j) $(32)^{-\frac{3}{5}} = 2^{-3} = \frac{1}{8}$

5. (a) $11^{\frac{1}{2}} = 3.32$
 (b) $(-24)^{\frac{1}{3}} = -2.88$
 (c) $-100^{\frac{1}{4}} = -3.16$
 (d) $50^{\frac{1}{3}} = 3.68$
 (e) $165^{\frac{1}{5}} = 2.78$
 (f) $(0.58)^{\frac{1}{2}} = 0.76$
 (g) $(1.05)^{-\frac{1}{3}} = 0.98$
 (h) $(200)^{-\frac{1}{3}} = 0.17$
 (i) $(12.65)^{\frac{1}{6}} = 1.53$
 (j) $(17.6)^{-\frac{1}{4}} = 0.49$

6. (a) $25^{\frac{3}{2}} = 125.00$
 (b) $55^{\frac{2}{3}} = 14.46$
 (c) $1.25^{\frac{3}{4}} = 1.18$
 (d) $12.8^{\frac{2}{5}} = 2.77$
 (e) $167^{\frac{3}{4}} = 46.46$
 (f) $\left(\frac{5}{6}\right)^{\frac{1}{3}} = 0.94$
 (g) $(1034)^{-\frac{2}{3}} = 0.01$
 (h) $(44)^{-\frac{4}{3}} = 0.01$
 (i) $\left(\frac{16}{25}\right)^{\frac{3}{2}} = \frac{64}{125} = 0.51$
 (j) $(85)^{-\frac{2}{5}} = 0.17$

7. (a) $A = \$6000(1.03)^{\frac{4}{12}} = \$6059.41$
 (b) $i = \frac{0.04}{2} = 0.02, A = \$2500(1.02)^{\frac{15}{6}} = \$2626.88$
 (c) $i = \frac{0.044}{4} = 0.011, A = \$12\,300(1.011)^{\frac{22}{3}} = \$13\,327.45$
 (d) $A = \$13\,560(1.065)^{\frac{100}{365}} = \$13\,795.99$
 (e) $i = \frac{0.0365}{2} = 0.018\,25,$
 $A = \$3750(1.018\,25)^{\frac{13}{6}} = \$3899.86$
 (f) $i = \frac{0.052}{52} = 0.001, A = \$6320(1.001)^{\frac{6}{7}} = \$6325.42$

8. (a) $P = \frac{\$3000}{(1.05)^{\frac{300}{365}}} = \$2882.08$
 (b) $i = \frac{0.06}{2} = 0.03, P = \frac{\$10\,500}{(1.03)^{\frac{4}{6}}} = \$10\,295.11$
 (c) $i = \frac{0.06}{4} = 0.015, P = \frac{\$7400}{(1.015)^{\frac{37}{3}}} = \$6158.63$
 (d) $i = \frac{0.065}{2} = 0.0325, P = \frac{\$100\,000}{(1.0325)^{\frac{29}{6}}} = \$85\,677.09$
 (e) $P = \frac{\$150}{(1.082)^{\frac{8}{12}}} = \$142.32$
 (f) $i = \frac{0.11}{2} = 0.055, P = \frac{\$3400}{(1.055)^{\frac{15}{6}}} = \$2974.05$

9. (a) $V_0 = \$25\,000$
 (b) $1 - 0.85 = 0.15 = 15\%$
 (c) $V_3 = \$25\,000(0.85)^3 = \$15\,353.13$
 (d) $V_{4\frac{4}{12}} = \$25\,000(0.85)^{\frac{13}{3}} = \$12\,362$

10. (a) $V_t = \$500(1.06)^t$
 (b) $V_5 = \$500(1.06)^5 = \$669.11$
 (c) $V_{\frac{300}{365}} = \$500(1.06)^{\frac{300}{365}} = \$524.53$

11. (a) $M_t = 700\left(\frac{1}{2}\right)^{\frac{60 \times 3}{20}}$, t in 20 minutes

(b) $M_{3h} = 700\left(\frac{1}{2}\right)^{\frac{60 \times 3}{20}} \doteq 1.367 \text{ g}$

(c) $M_{5 \text{ min}} = 700\left(\frac{1}{2}\right)^{\frac{5}{20}} \doteq 588.627 \text{ g}$

12. $r = 1 - \left[\frac{\$85000}{\$125000}\right]^{\frac{1}{3}} = 12.1\%$

$\therefore$ the depreciation rate for the truck is 12.1%.

13. (a) $M_t = 40\left(\frac{1}{2}\right)^{\frac{t}{5}}$, t in hours

(b) $M_{15h} = 40\left(\frac{1}{2}\right)^3 \doteq 5 \text{ mg}$

(c) $M_{2h} = 40\left(\frac{1}{2}\right)^{\frac{2}{5}} \doteq 30.3 \text{ mg}$

14. (a) $(25x^2)^{\frac{1}{2}} = \sqrt{25x^2} = 5x$

(b) $(-27x^3)^{\frac{1}{3}} = (-27)^{\frac{1}{3}}(x^3)^{\frac{1}{3}} = -3x$

(c) $(16x^4)^{\frac{1}{4}} = (16)^{\frac{1}{4}}(x^4)^{\frac{1}{4}} = 2x$

(d) $(-16x^4)^{\frac{1}{2}}$, complex root, $\therefore$ no solution

(e) $(243x^5)^{\frac{1}{5}} = (243)^{\frac{1}{5}}(x^5)^{\frac{1}{5}} = 3x$

(f) $[(x+2)^4]^{\frac{1}{4}} = x+2$

(g) $(-64x^8)^{\frac{1}{4}}$, complex root, $\therefore$ no solution

(h) $[(2x+5)^7]^{\frac{1}{7}} = 2x+5$

15.

(	$\pm$	1	2	5	)	$\wedge$	(
$\pm$	5	$\div$	3	)	=		

16. (a) $(25x^4)^{\frac{3}{2}} = (5^2)^{\frac{3}{2}}(x^4)^{\frac{3}{2}} = 5^3 x^6 = 125x^6$

(b) $(27a^3b^3)^{\frac{2}{3}} = (3^3)^{\frac{2}{3}}(a^3)^{\frac{2}{3}}(b^3)^{\frac{2}{3}} = 3^2 a^2 b^2 = 9a^2 b^2$

(c) $\left(\frac{8x^3}{125}\right)^{\frac{2}{3}} = (8)^{\frac{2}{3}}(x^3)^{\frac{2}{3}}\left(\frac{1}{125}\right)^{\frac{2}{3}} = \frac{4x^2}{25}$

(d) $\left(\frac{27}{64y^3}\right)^{\frac{2}{3}} = (3^3)^{\frac{2}{3}}\left(\frac{1}{4^3}\right)^{\frac{2}{3}}\left(\frac{1}{y^3}\right)^{\frac{2}{3}} = \frac{9}{16y^2}$

17. (a) $(16x^2)^{\frac{-3}{2}} = (4x)^{-3} = \frac{1}{64x^3}$

(b) $(49c^6)^{\frac{-3}{2}} = (7c^3)^{-3} = \frac{1}{343c^9}$

(c) $\left(\frac{-8x^3}{216}\right)^{\frac{-1}{3}} = \left(\frac{-216}{8x^3}\right)^{\frac{1}{3}} = \frac{-6}{2x} = \frac{-3}{x}$

(d) $\left(\frac{16}{81y^8}\right)^{\frac{-3}{4}} = \left(\frac{2}{3y^2}\right)^{-3} = \left(\frac{3y^2}{2}\right)^3 = \frac{27y^6}{8}$

18. $i = \frac{0.05}{2} = 0.025$, $P = \frac{\$921.35}{(1.025)^{\frac{50}{6}}} = \$750.00$

$\therefore$ Sami invested \$750.00 in the account two months ago.

19. (a) $V_{500} = 1.5(0.99)^{500} \doteq 0.01 > 0.001$
 $\therefore$ since the voltage will be 0.01 V, the watch will still run after 500 days.

(b) $V = 1.5(0.99)^{\frac{3}{24}} = 1.498$
 $\therefore$ the battery has a voltage of 1.498 V after 3 h.

20. $L = \left(150^{\frac{2}{3}} + 200^{\frac{2}{3}}\right)^{\frac{3}{2}} = 493.28 \text{ cm}$

$\therefore$ the longest piece of plywood that a carpenter can carry around the corner of the two hallways is 493.28 cm.

21. (a) **Method #1:** $(32)^{\frac{-3}{5}} = (5\sqrt[5]{32})^{-3} = 2^{-3} = \frac{1}{8}$

Method #2: $(32)^{\frac{-3}{5}} = (2^5)^{\frac{-3}{5}} = 2^{-3} = \frac{1}{8}$

22. (a) $P = (\$7146.03)(1.005)^{-66}(1.02)^{-18} = \$3599.99$
 $\therefore$ Leon originally invested \$3599.99.

(b) $n = \frac{10 \times 12}{6} = 20$

$\$7146.03 = \$3599.99\left(1 + \frac{i}{2}\right)^{20}$

$1.98501^{\frac{1}{20}} = 1 + \frac{i}{2}$

$i = 2\left(1.98501^{\frac{1}{20}} - 1\right)$

$i = 0.0698$

$i = 6.98\%$

$\therefore$ the interest rate would have to be 6.98% in order to generate the same amount of money.

23. (a) $t = (1.66 \times 10^{-4})(6370 + 450)^{1.5} = 93.49 \text{ min}$
 OR 93 min 29.7 s.
 $\therefore$ a satellite situated 450 km above the Earth takes 93 min and 29.7 s to orbit the Earth once.

(b)

$$24(60) = (1.66 \times 10^{-4})(6370 + h)^{1.5}$$

$$\frac{1440}{1.66 \times 10^{-4}} = (6370 + h)^{1.5}$$

$$8\,674\,698.795 = (6370 + h)^{1.5}$$

$$(8\,674\,698.795)^{\frac{1}{1.5}} = 6370 + h$$

$$h = (8\,674\,698.795)^{\frac{1}{1.5}} - 6370$$

$$h = 35\,848.51 \text{ km}$$

$\therefore$ the television satellite must be 35 848.51 km above the Earth's surface.

Practise, Apply, Solve 1.10, page 85

1. (a) $(x^4)(x^3) = x^{4+3} = x^7$

(b) $(c^4)^5 = c^{(4)(5)} = c^{20}$

(c) $x^6 \div x^3 = x^{6-3} = x^3$

(d) $(ab)^3 = a^3b^3$

(e) $(d^4)(d^2)(d^{-3}) = d^{4+2-3} = d^3$

(f) $b^{-3} \div b^2 = b^{-3-2} = b^{-5} = \frac{1}{b^5}$

(g) $(2x^3)^2 = 2^2x^{3(2)} = 4x^6$

(h) $\left(\frac{2x}{-3y^2}\right)^3 = \frac{(2x)^3}{(-3y^2)^3} = \frac{2^3x^3}{(-3)^3y^{2 \times 3}} = \frac{8x^3}{-27y^6}$

2. (a) $2^4 \times 2^5 = 2^{4+5} = 2^9$

(b) $2^n \times 2^m = 2^{n+m}$

(c) $(2^4)^x = 2^{4x}$

(d) $4^3 = (2^2)^3 = 2^6$

(e) $8^n = (2^3)^n = 2^{3n}$

(f) $2^6 \div 2^{-2} = 2^{6+2} = 2^8$

(g) $4^4 \div 2^m = (2^2)^4 \div 2^m = 2^{8-m}$

(h) $16^{3x-1} = (2^4)^{3x-1} = 2^{12x-4}$

3. (a) $3^5 = 243$

(b) $7^0 = 1$

(c) $8^{-2} = \frac{1}{64}$

(d) $(-3)^4 = (-1)^43^4 = 81$

(e) $-16^{\frac{-3}{4}} = (-1)(\sqrt[4]{16})^{-3} = \frac{-1}{8}$

(f) $16^{\frac{1}{4}} = \sqrt[4]{16} = 2$

(g) $32^{\frac{2}{5}} = (\sqrt[5]{32})^2 = 2^2 = 4$

(h) $4^{\frac{3}{2}} = (\sqrt{4})^3 = 2^3 = 8$

(i) $125^{\frac{-1}{3}} = (\sqrt[3]{125})^{-1} = \frac{1}{5}$

(j) $100^{\frac{-1}{2}} = \frac{1}{\sqrt{100}} = \frac{1}{10}$

(k) $-27^{\frac{1}{3}} = -\sqrt[3]{27} = -3$

(l) $256^{\frac{3}{4}} = (16^2)^{\frac{3}{4}} = 16^{\frac{3}{2}} = (\sqrt{16})^3 = 4^3 = 64$

(m) $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$

(n) $128^{\frac{-3}{7}} = (\sqrt[7]{128})^{-3} = 2^{-3} = \frac{1}{8}$

(o) $64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = 4^2 = 16$

4. (a) $(x^{-3})^2 = x^{-6} = \frac{1}{x^6}$

(b) $\sqrt[3]{x} = x^{\frac{1}{3}}$

(c) $\sqrt[4]{c^5} = c^{\frac{5}{4}}$

(d) $\sqrt[3]{a^{-2}} = a^{\frac{-2}{3}} = \frac{1}{a^{\frac{2}{3}}}$

(e) $(\sqrt{c^{-3}})^2 = \left(c^{\frac{-3}{2}}\right)^2 = c^{-3} = \frac{1}{c^3}$

5. (a) $\frac{(xy)^4}{xy} = (xy)^3 = x^3y^3$

(b) $\frac{3(ab)^4}{(-a^2)^2} = \frac{3(a^2)^2b^4}{(-1)^2(a^2)^2} = 3b^4$

(c) $(a^3b)^2\left(\frac{-a}{b}\right)^3 = \frac{a^3b^2(-1)^3a^3}{b^3} = \frac{-a^6}{b}$

(d) $(-d^3)^4\left(\frac{c}{d}\right)^6 = \frac{(-1)^4d^{12}c^6}{d^6} = c^6d^6$

(e) $\left(\frac{-1}{b}\right)^2(a^3b)^2 = \left(\frac{1}{b^2}\right)a^6b^2 = a^6$

(f) $\frac{(a^4b^2)^3}{(a^2b^2)^2} = \frac{a^{12}b^6}{a^4b^4} = a^8b^2$

(g) $\left(\frac{x}{-y}\right)^3(-xy)^4 = \frac{(-1)^3x^3}{y^3}(-1)^4x^4y^4 = -x^7y$

(h) $\left(\frac{x^3}{y^4}\right)^4\left(\frac{y^2}{x^4}\right)^3 = \left(\frac{x^{12}}{y^{16}}\right)\left(\frac{y^6}{x^{12}}\right) = \frac{1}{y^{10}}$

6. (a) $49^{\frac{1}{2}} + 16^{\frac{1}{4}} = \sqrt{49} + \sqrt[4]{16} = 7 + 2 = 9$

(b) $27^{\frac{2}{3}} - 81^{\frac{3}{4}} = (\sqrt[3]{27})^2 - (\sqrt[4]{81})^3 = 3^2 - 3^3 = 9 - 27 = -18$

(c) $16^{\frac{3}{4}} + 16^{\frac{3}{4}} - 81^{\frac{-1}{4}} = 2(2^4)^{\frac{3}{4}} - (3^4)^{\frac{-1}{4}} = 2(8) - 3^{-1} = 15\frac{2}{3}$

(d) $128^{\frac{-5}{7}} - 16^{-0.75} = (2^7)^{\frac{-5}{7}} - (2^4)^{\frac{-3}{4}} = 2^{-5} - 2^{-3} = \frac{1}{32} - \frac{1}{8} = \frac{-3}{32}$