

$$13. \quad z_1 = \frac{a}{1+i}, \quad z_2 = \frac{b}{1+2i}$$

$$z_1 + z_2 = 1$$

$$\frac{a}{1+i} + \frac{b}{1+2i} = 1$$

$$\frac{a}{1+i} \times \frac{1-i}{1-i} + \frac{b}{1+2i} \times \frac{1-2i}{1-2i} = 1$$

$$\frac{a(1-i)}{1-i^2} + \frac{b(1-2i)}{1-4i^2} = 1$$

$$\frac{a-ai}{2} + \frac{b-2bi}{5} = 1$$

$$\frac{a}{2} - \frac{a}{2}i + \frac{b}{5} - \frac{2}{5}bi = 1$$

$$\frac{a}{2} + \frac{b}{5} - \frac{a}{2}i - \frac{2}{5}bi = 1$$

Multiply by 10.

$$10\left(\frac{a}{2}\right) + 10\left(\frac{b}{5}\right) - 10\left(\frac{a}{2}i\right) - 10\left(\frac{2b}{5}i\right) = 10(1)$$

$$5a + 2b - 5ai - 4bi = 10$$

$$5a + 2b - (5a + 4b)i = 10 + 0i$$

Equate the real parts on each side of the equation.

$$5a + 2b = 10 \text{ -----①}$$

Equate the imaginary parts.

$$-5a - 4b = 0 \text{ -----②}$$

Add (1) + (2):

$$0a - 2b = 10$$

$$b = -5$$

Substitute $b = -5$ into ①:

$$5a + 2(-5) = 10$$

$$5a = 20$$

$$a = 4$$

14.

$$\frac{z}{z+2} = 2-i$$

$$z = (2-i)(z+2)$$

$$z = 2z + 4 - iz - 2i$$

$$z - 2z + iz = 4 - 2i$$

$$-z + iz = 4 - 2i$$

$$z(-1+i) = 4 - 2i$$

$$z = \frac{4-2i}{-1+i} \times \frac{-1-i}{-1-i}$$

$$= \frac{-4-2i+2i^2}{-1-i^2}$$

$$= \frac{-4-2i+2(-1)}{1-(-1)}$$

$$= \frac{-6-2i}{2}$$

$$= -3-i$$

15. $z\bar{z}$ is always a real number. Therefore, if $\frac{w}{z}$ is a quotient of complex numbers w and z , $\frac{w\bar{z}}{z\bar{z}}$ will have a real number in

the denominator and will be in the form $a + bi$. For example, the quotient $\frac{3-i}{2+i}$ is the same as

$$\frac{3-i}{2+i} \times \frac{2-i}{2-i} = \frac{6-5i+i^2}{4-i^2}$$

$$= \frac{6-5i+(-1)}{4-(-1)}$$

$$= \frac{5-5i}{5} \quad (\text{the denominator is now a real number})$$

$$= 1-i$$

16.

$$z + (1+i)w = 1 - 2i \text{ -----①}$$

$$(1+2i)z - 2iw = 9 + 6i \text{ -----②}$$

Multiply equation ① by $(1+2i)$:

$$(1+2i)z + (1+2i)(1+i)w = (1+2i)(1-2i)$$

$$(1+2i)z + (1+3i+2i^2)w = 1-4i^2$$

$$(1+2i)z + (-1+3i)w = 5 \text{ -----③}$$

$$(1+2i)z - 2iw = 9 + 6i \text{ -----②}$$

Subtract ③ - ②:

$$0z + (-1+3i+2i)w = -4-6i$$

$$(-1+5i)w = -4-6i$$

$$w = \frac{-4-6i}{-1+5i} \times \frac{-1-5i}{-1-5i}$$

$$= \frac{4+26i+30i^2}{1-25i^2}$$

$$= \frac{-26+26i}{26}$$

$$w = -1+i$$

Substitute $w = -1+i$ into equation ①:

$$z + (1+i)(-1+i) = 1-2i$$

$$z + (-1+i^2) = 1-2i$$

$$z - 2 = 1-2i$$

$$z = 1-2i+2$$

$$z = 3-2i$$

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1. (a) $\frac{3}{x}, \frac{5}{y}$;

LCD is xy .

(b) $\frac{-2}{ab}, \frac{1}{b}$;

LCD is ab .

(c) $\frac{4}{x-1}, \frac{2}{x}$;

LCD is $x(x-1)$.

(d) $\frac{b}{b+2}, \frac{3b}{b+1}$;

LCD is $(b+1)(b+2)$.

(e) $\frac{5}{x^2}, \frac{7}{xy}$;

LCD is x^2y .

(f) $\frac{m-1}{2m+3}, \frac{m}{m-3}$;

LCD is $(m-3)(2m+3)$.

(g) $\frac{11x}{yz^2}, \frac{8x}{y^2z}$;

LCD is y^2z^2 .

(h) $\frac{3s}{(s-1)(s+1)}, \frac{s+2}{s-1}$;

LCD is $(s-1)(s+1)$.

(i) $\frac{4}{x^2-4}, \frac{3}{x+2}$ is $\frac{4}{(x-2)(x+2)}, \frac{3}{x+2}$;
LCD is $(x+2)(x-2)$ or x^2-4 .

2. (a)

$$\frac{2}{a} + \frac{7}{b} = \frac{2}{a} \times \frac{b}{b} + \frac{7}{b} \times \frac{a}{a}, a \neq 0, b \neq 0$$

$$= \frac{2b}{ab} + \frac{7a}{ab}$$

$$= \frac{2b+7a}{ab}, a \neq 0, b \neq 0$$

(b)

$$\frac{3}{x} + \frac{2}{x^2} = \frac{3}{x} \times \frac{x}{x} + \frac{2}{x^2}, x \neq 0$$

$$= \frac{3x}{x^2} + \frac{2}{x^2}$$

$$= \frac{3x+2}{x^2}, x \neq 0$$

(c)

$$\frac{4}{xy} + \frac{5}{yz} = \frac{4}{xy} \times \frac{z}{z} + \frac{5}{yz} \times \frac{x}{x}, x \neq 0, y \neq 0, z \neq 0$$

$$= \frac{4z}{xyz} + \frac{5x}{xyz}$$

$$= \frac{4z+5x}{xyz}, x \neq 0, y \neq 0, z \neq 0$$

(d)

$$\frac{7}{x^2y} + \frac{4}{xy^2} = \frac{7}{x^2y} \times \frac{y}{y} + \frac{4}{xy^2} \times \frac{x}{x}, x \neq 0, y \neq 0$$

$$= \frac{7y}{x^2y^2} + \frac{4x}{x^2y^2}$$

$$= \frac{7y+4x}{x^2y^2}, x \neq 0, y \neq 0$$

(e)

$$\frac{6x}{y} + \frac{3}{y^2} = \frac{6x}{y} \times \frac{y}{y} + \frac{3}{y^2}, y \neq 0$$

$$= \frac{6xy}{y^2} + \frac{3}{y^2}$$

$$= \frac{6xy+3}{y^2} \text{ or } \frac{3(2xy+1)}{y^2}, y \neq 0$$

(f)

$$\frac{2x}{x+1} + \frac{5}{x} = \frac{2x}{x+1} \times \frac{x}{x} + \frac{5}{x} \times \frac{(x+1)}{(x+1)}, x \neq 0, -1$$

$$= \frac{2x^2}{x(x+1)} + \frac{5(x+1)}{x(x+1)}$$

$$= \frac{2x^2+5(x+1)}{x(x+1)}$$

$$= \frac{2x^2+5x+5}{x^2+x}, x \neq 0, -1$$

(g)

$$\frac{3}{5y} + \frac{2}{y+2} = \frac{3}{5y} \times \frac{(y+2)}{(y+2)} + \frac{2}{y+2} \times \frac{5y}{5y}, y \neq 0, -2$$

$$= \frac{3(y+2)}{5y(y+2)} + \frac{10y}{5y(y+2)}$$

$$= \frac{3y+6+10y}{5y(y+2)}$$

$$= \frac{13y+6}{5y(y+2)}, y \neq 0, -2$$

(h)

$$\frac{2}{x+3} + \frac{3}{x+1} = \frac{2}{x+3} \times \frac{(x+1)}{(x+1)} + \frac{3}{x+1} \times \frac{(x+3)}{(x+3)}, x \neq -3, -1$$

$$= \frac{2(x+1)}{(x+3)(x+1)} + \frac{3(x+3)}{(x+1)(x+3)}$$

$$= \frac{2x+2+3x+9}{(x+3)(x+1)}$$

$$= \frac{5x+11}{(x+3)(x+1)} \text{ or } \frac{5x+11}{x^2+4x+3}, x \neq -3, -1$$

(i)

$$\frac{x}{x-1} + \frac{2x}{x+3} = \frac{x}{x-1} \times \frac{(x+3)}{(x+3)} + \frac{2x}{x+3} \times \frac{(x-1)}{(x-1)}, x \neq -3, 1$$

$$= \frac{x(x+3)}{(x-1)(x+3)} + \frac{2x(x-1)}{(x+3)(x-1)}$$

$$= \frac{x^2+3x+2x^2-2x}{(x-1)(x+3)}$$

$$= \frac{3x^2+x}{(x-1)(x+3)} \text{ or } \frac{3x^2+x}{x^2+2x-3}, x \neq -3, 1$$

3. (a)

$$\frac{3}{4x} - \frac{1}{3x} = \frac{3}{4x} \times \frac{3}{3} - \frac{1}{3x} \times \frac{4}{4}, x \neq 0$$

$$= \frac{9}{12x} - \frac{4}{12x}$$

$$= \frac{5}{12x}, x \neq 0$$

(b)

$$\frac{3}{y^2} - \frac{2}{y} = \frac{3}{y^2} - \frac{2}{y} \times \frac{y}{y}, y \neq 0$$

$$= \frac{3}{y^2} - \frac{2y}{y^2}$$

$$= \frac{3-2y}{y^2}, y \neq 0$$

(c)

$$\frac{2}{3x} - \frac{4}{x} = \frac{2}{3x} - \frac{4}{x} \times \frac{3}{3}, x \neq 0$$

$$= \frac{2}{3x} - \frac{12}{3x}$$

$$= -\frac{10}{3x}, x \neq 0$$

$$\begin{aligned}
 \text{(d)} \quad \frac{10}{x^2y} - \frac{6}{xy^2} &= \frac{10}{x^2y} \times \frac{y}{y} - \frac{6}{xy^2} \times \frac{x}{x}, x \neq 0, y \neq 0 \\
 &= \frac{10y}{x^2y^2} - \frac{6x}{x^2y^2} \\
 &= \frac{10y-6x}{x^2y^2}, x \neq 0, y \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad \frac{9}{4ab^2} - \frac{5}{6a^2b} &= \frac{9}{4ab^2} \times \frac{3a}{3a} - \frac{5}{6a^2b} \times \frac{2b}{2b}, a \neq 0, b \neq 0 \\
 &= \frac{27a}{12a^2b^2} - \frac{10b}{12a^2b^2} \\
 &= \frac{27a-10b}{12a^2b^2}, a \neq 0, b \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad \frac{6}{y+1} - \frac{3}{y-1} &= \frac{6}{y+1} \times \frac{(y-1)}{(y-1)} - \frac{3}{y-1} \times \frac{(y+1)}{(y+1)}, y \neq \pm 1 \\
 &= \frac{6(y-1)}{(y+1)(y-1)} - \frac{3(y+1)}{(y-1)(y+1)} \\
 &= \frac{6y-6-3y-3}{(y+1)(y-1)} \\
 &= \frac{3y-9}{(y+1)(y-1)} \\
 &= \frac{3(y-3)}{(y+1)(y-1)} \text{ or } \frac{3y-9}{y^2-1}, y \neq \pm 1
 \end{aligned}$$

$$\begin{aligned}
 \text{(g)} \quad \frac{3x}{x+5} - \frac{2x}{x-3} &= \frac{3x}{x+5} \times \frac{(x-3)}{(x-3)} - \frac{2x}{x-3} \times \frac{(x+5)}{(x+5)}, x \neq -5, 3 \\
 &= \frac{3x(x-3)}{(x+5)(x-3)} - \frac{2x(x+5)}{(x-3)(x+5)} \\
 &= \frac{3x^2-9x-2x^2-10x}{(x+5)(x-3)} \\
 &= \frac{x^2-19x}{(x+5)(x-3)} \text{ or } \frac{x^2-19x}{x^2+2x-15}, x \neq -5, 3
 \end{aligned}$$

$$\begin{aligned}
 \text{(h)} \quad \frac{t+3}{t-1} - \frac{2t}{t+1} &= \frac{t+3}{t-1} \times \frac{(t+1)}{(t+1)} - \frac{2t}{t+1} \times \frac{(t-1)}{(t-1)}, t \neq \pm 1 \\
 &= \frac{(t+3)(t+1)}{(t-1)(t+1)} - \frac{2t(t-1)}{(t+1)(t-1)} \\
 &= \frac{t^2+4t+3-2t^2+2t}{(t-1)(t+1)} \\
 &= \frac{-t^2+6t+3}{(t-1)(t+1)} \text{ or } \frac{-t^2+6t+3}{t^2-1}, t \neq \pm 1
 \end{aligned}$$

$$\begin{aligned}
 \text{(i)} \quad \frac{4}{x(x+2)} - \frac{3}{x} &= \frac{4}{x(x+2)} - \frac{3}{x} \times \frac{(x+2)}{(x+2)}, x \neq -2, 0 \\
 &= \frac{4}{x(x+2)} - \frac{3(x+2)}{x(x+2)} \\
 &= \frac{4-3x-6}{x(x+2)} \\
 &= \frac{-3x-2}{x(x+2)} \text{ or } \frac{-3x-2}{x^2+2x}, x \neq -2, 0
 \end{aligned}$$

$$\begin{aligned}
 \text{4. (a)} \quad \frac{-2}{x} + \frac{5}{x^2} - \frac{8}{3x} &= \frac{-2}{x} \times \frac{3x}{3x} + \frac{5}{x^2} \times \frac{3}{3} - \frac{8}{3x} \times \frac{x}{x}, x \neq 0 \\
 &= \frac{-6x}{3x^2} + \frac{15}{3x^2} - \frac{8x}{3x^2} \\
 &= \frac{-6x+15-8x}{3x^2} \\
 &= \frac{15-14x}{3x^2}, x \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \frac{6}{pq} - \frac{4}{q} - \frac{8}{p^2q} &= \frac{6}{pq} \times \frac{p}{p} - \frac{4}{q} \times \frac{p^2}{p^2} - \frac{8}{p^2q} \\
 &= \frac{6p}{p^2q} - \frac{4p^2}{p^2q} - \frac{8}{p^2q} \\
 &= \frac{6p-4p^2-8}{p^2q} \text{ or } \frac{-2(2p^2-3p+4)}{p^2q}, p \neq 0, q \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad \frac{2x}{yz} - \frac{3x}{y} - \frac{9x}{y^2z} &= \frac{2x}{yz} \times \frac{y}{y} - \frac{3x}{y} \times \frac{yz}{yz} - \frac{9x}{y^2z} \\
 &= \frac{2xy}{y^2z} - \frac{3xyz}{y^2z} - \frac{9x}{y^2z} \\
 &= \frac{2xy-3xyz-9x}{y^2z}, y \neq 0, z \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad \frac{2a}{(a+1)(a+3)} + \frac{5}{(a+3)} &= \frac{2a}{(a+1)(a+3)} + \frac{5}{(a+3)} \times \frac{(a+1)}{(a+1)} \\
 &= \frac{2a}{(a+1)(a+3)} + \frac{5(a+1)}{(a+3)(a+1)} \\
 &= \frac{2a+5a+5}{(a+1)(a+3)} \\
 &= \frac{7a+5}{(a+1)(a+3)} \text{ or } \frac{7a+5}{a^2+4a+3}, a \neq -3, -1
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad \frac{1}{(x+2)^2} - \frac{3}{(x+2)} &= \frac{1}{(x+2)^2} - \frac{3}{(x+2)} \times \frac{(x+2)}{(x+2)}, x \neq -2 \\
 &= \frac{1}{(x+2)^2} - \frac{3(x+2)}{(x+2)^2} \\
 &= \frac{1-3x-6}{(x+2)^2} \\
 &= \frac{-3x-5}{(x+2)^2}, x \neq -2
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad \frac{1}{2(x+4)} - \frac{3x}{(x+4)^2} + \frac{1}{2}, x \neq -4 \\
 &= \frac{1}{2(x+4)} \times \frac{(x+4)}{(x+4)} - \frac{3x}{(x+4)^2} \times \frac{2}{2} + \frac{1}{2} \times \frac{(x+4)^2}{(x+4)^2} \\
 &= \frac{(x+4)}{2(x+4)^2} - \frac{6x}{2(x+4)^2} + \frac{(x+4)^2}{(x+4)^2} \\
 &= \frac{x+4-6x+x^2+8x+16}{2(x+4)^2} \\
 &= \frac{x^2+3x+20}{2(x+4)^2} \text{ or } \frac{x^2+3x+20}{2x^2+16x+32}, x \neq -4
 \end{aligned}$$

$$\begin{aligned}
 \text{5. (a)} \quad \frac{x}{3x-6} + \frac{3}{2x-4} &= \frac{x}{3(x-2)} \times \frac{2}{2} + \frac{3}{2(x-2)} \times \frac{3}{3}, x \neq 2 \\
 &= \frac{2x}{6(x-2)} + \frac{9}{6(x-2)} \\
 &= \frac{2x+9}{6(x-2)} \text{ or } \frac{2x+9}{6x-12}, x \neq 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \frac{3x-1}{x^2+3x} + \frac{2}{5x} &= \frac{3x-1}{x(x+3)} + \frac{2}{5x}, x \neq 0, -3 \\
 &= \frac{3x-1}{x(x+3)} \times \frac{5}{5} + \frac{2}{5x} \times \frac{(x+3)}{(x+3)} \\
 &= \frac{5(3x-1)}{5x(x+3)} + \frac{2(x+3)}{5x(x+3)} \\
 &= \frac{15x-5+2x+6}{5x(x+3)} \\
 &= \frac{17x+1}{5x(x+3)} \text{ or } \frac{17x+1}{5x^2+15x}, x \neq 0, -3
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad \frac{6}{y} + \frac{3}{y^2-2y} &= \frac{6}{y} \times \frac{(y-2)}{(y-2)} + \frac{3}{y(y-2)}, y \neq 0, 2 \\
 &= \frac{6(y-2)}{y(y-2)} + \frac{3}{y(y-2)} \\
 &= \frac{6y-12+3}{y(y-2)} \\
 &= \frac{6y-9}{y(y-2)} \text{ or } \frac{6y-9}{y^2-2y}, y \neq 0, 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad \frac{4p}{p^2+3p} - \frac{2}{p+3} &= \frac{4p}{p(p+3)} - \frac{2}{p+3}, p \neq 0, -3 \\
 &= \frac{4}{p+3} - \frac{2}{p+3} \\
 &= \frac{2}{p+3}, p \neq 0, -3
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad \frac{2x+1}{x^2-x-2} + \frac{3}{x-2} \\
 &= \frac{2x+1}{(x-2)(x+1)} + \frac{3}{x-2}, x \neq -1, 2 \\
 &= \frac{2x+1}{(x-2)(x+1)} + \frac{3}{x-2} \times \frac{(x+1)}{(x+1)} \\
 &= \frac{2x+1}{(x-2)(x+1)} + \frac{3(x+1)}{(x-2)(x+1)} \\
 &= \frac{2x+1+3x+3}{(x-2)(x+1)} \\
 &= \frac{5x+4}{(x-2)(x+1)} \text{ or } \frac{5x+4}{x^2-x-2}, x \neq -1, 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad \frac{4a}{a^2-9} - \frac{2}{a+3} &= \frac{4a}{(a-3)(a+3)} - \frac{2}{a+3}, a \neq \pm 3 \\
 &= \frac{4a}{(a-3)(a+3)} - \frac{2}{a+3} \times \frac{a-3}{a-3} \\
 &= \frac{4a}{(a-3)(a+3)} - \frac{2(a-3)}{(a+3)(a-3)} \\
 &= \frac{4a-2a+6}{(a-3)(a+3)} \\
 &= \frac{2a+6}{(a-3)(a+3)} \\
 &= \frac{2(a+3)}{(a-3)(a+3)} \\
 &= \frac{2}{a-3}, a \neq \pm 3
 \end{aligned}$$

$$\begin{aligned}
 \text{(g)} \quad \frac{1}{y+4} - \frac{y-3}{y^2+3y-4} \\
 &= \frac{1}{y+4} - \frac{y-3}{(y+4)(y-1)}, y \neq -4, 1 \\
 &= \frac{1}{y+4} \times \frac{(y-1)}{(y-1)} - \frac{y-3}{(y+4)(y-1)} \\
 &= \frac{y-1}{(y+4)(y-1)} - \frac{y-3}{(y+4)(y-1)} \\
 &= \frac{y-1-y+3}{(y+4)(y-1)} \\
 &= \frac{2}{(y+4)(y-1)} \text{ or } \frac{2}{y^2+3y-4}, y \neq -4, 1
 \end{aligned}$$

$$\begin{aligned}
 & \text{(h)} \quad \frac{3m+1}{2m-3} + \frac{m}{m^2-9} \\
 &= \frac{3m+1}{2m-3} + \frac{m}{(m-3)(m+3)}, \quad m \neq \pm 3, \frac{3}{2} \\
 &= \frac{3m+1}{2m-3} \times \frac{(m^2-9)}{(m^2-9)} + \frac{m}{(m^2-9)} \times \frac{(2m-3)}{(2m-3)} \\
 &= \frac{3m^3 - 27m + m^2 - 9 + 2m^2 - 3m}{(2m-3)(m^2-9)} \\
 &= \frac{3m^3 + 3m^2 - 30m - 9}{(2m-3)(m^2-9)} \text{ or } \frac{3m^3 + 3m^2 - 30m - 9}{2m^3 - 3m^2 - 18m + 27}, \quad m \neq \pm 3, \frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 & \text{6. (a)} \quad \frac{x}{x^2-1} - \frac{1}{x^2-1} = \frac{x}{(x-1)(x+1)} - \frac{1}{(x-1)(x+1)}, \quad x \neq \pm 1 \\
 &= \frac{x-1}{(x-1)(x+1)} \\
 &= \frac{1}{x+1}, \quad x \neq \pm 1
 \end{aligned}$$

$$\begin{aligned}
 & \text{(b)} \quad \frac{x-3}{x^2-7x+10} - \frac{x+2}{x^2-25} \\
 &= \frac{x-3}{(x-2)(x-5)} - \frac{x+2}{(x-5)(x+5)}, \quad x \neq 2, \pm 5 \\
 &= \frac{x-3}{(x-2)(x-5)} \times \frac{(x+5)}{(x+5)} - \frac{x+2}{(x-5)(x+5)} \times \frac{(x-2)}{(x-2)} \\
 &= \frac{(x-3)(x+5)}{(x-2)(x-5)(x+5)} - \frac{(x+2)(x-2)}{(x-5)(x+5)(x-2)} \\
 &= \frac{x^2 + 2x - 15 - (x^2 - 4)}{(x-2)(x-5)(x+5)} \\
 &= \frac{x^2 + 2x - 15 - x^2 + 4}{(x-2)(x-5)(x+5)} \\
 &= \frac{2x-11}{(x-2)(x-5)(x+5)}, \quad x \neq 2, \pm 5
 \end{aligned}$$

$$\begin{aligned}
 & \text{(c)} \quad \frac{3a}{6a^2-a-2} + \frac{2a}{10a^2-a-3} \\
 &= \frac{3a}{(3a-2)(2a+1)} + \frac{2a}{(5a-3)(2a+1)}, \quad a \neq -\frac{1}{2}, \frac{2}{3}, \frac{3}{5} \\
 &= \frac{3a}{(3a-2)(2a+1)} \times \frac{(5a-3)}{(5a-3)} + \frac{2a}{(5a-3)(2a+1)} \times \frac{(3a-2)}{(3a-2)} \\
 &= \frac{3a(5a-3)}{(3a-2)(2a+1)(5a-3)} + \frac{2a(3a-2)}{(3a-2)(2a+1)(5a-3)} \\
 &= \frac{15a^2 - 9a + 6a^2 - 4a}{(3a-2)(2a+1)(5a-3)} \\
 &= \frac{21a^2 - 13a}{(3a-2)(2a+1)(5a-3)} \\
 &= \frac{a(21a-13)}{(3a-2)(2a+1)(5a-3)}, \quad a \neq -\frac{1}{2}, \frac{2}{3}, \frac{3}{5}
 \end{aligned}$$

$$\begin{aligned}
 & \text{(d)} \quad \frac{1}{h} \left(\frac{1}{3x+h} - \frac{1}{3x} \right), \quad h \neq 0, x \neq 0, x \neq -\frac{h}{3} \\
 &= \frac{1}{h} \left(\frac{1}{3x+h} \times \frac{(3x)}{(3x)} - \frac{1}{3} \times \frac{(3x+h)}{(3x+h)} \right) \\
 &= \frac{1}{h} \left(\frac{3x}{3x(3x+h)} - \frac{(3x+h)}{3x(3x+h)} \right) \\
 &= \frac{1}{h} \left(\frac{3x-3x-h}{3x(3x+h)} \right) \\
 &= \frac{1}{h} \left(\frac{-h}{3x(3x+h)} \right) \\
 &= \frac{-1}{3x(3x+h)}, \quad h \neq 0, x \neq 0, x \neq -\frac{h}{3}
 \end{aligned}$$

$$\begin{aligned}
 & \text{(e)} \quad \frac{k+1}{2k^2-7k+6} - \frac{k-3}{2k^2-k-3} \\
 &= \frac{k+1}{(2k-3)(k-2)} - \frac{k-3}{(2k-3)(k+1)}, \quad k \neq -1, \frac{3}{2}, 2 \\
 &= \frac{k+1}{(2k-3)(k-2)} \times \frac{(k+1)}{(k+1)} - \frac{k-3}{(2k-3)(k+1)} \times \frac{(k-2)}{(k-2)} \\
 &= \frac{(k+1)(k+1)}{(2k-3)(k-2)(k+1)} - \frac{(k-3)(k-2)}{(2k-3)(k+1)(k-2)} \\
 &= \frac{k^2 + 2k + 1 - (k^2 - 5k + 6)}{(2k-3)(k-2)(k+1)} \\
 &= \frac{k^2 + 2k + 1 - k^2 + 5k - 6}{(2k-3)(k-2)(k+1)} \\
 &= \frac{7k-5}{(2k-3)(k-2)(k+1)}, \quad k \neq -1, \frac{3}{2}, 2
 \end{aligned}$$

$$\begin{aligned}
 & \text{(f)} \quad \frac{1}{a-3} + \frac{1}{a^2-9} - \frac{1}{a^2-2a-3} \\
 &= \frac{1}{a-3} + \frac{1}{(a+3)(a-3)} - \frac{1}{(a-3)(a+1)}, \quad a \neq -1, \pm 3 \\
 &= \frac{1}{a-3} \times \frac{(a+3)(a+1)}{(a+3)(a+1)} + \frac{1}{(a+3)(a-3)} \times \frac{(a+1)}{(a+1)} \\
 &\quad - \frac{1}{(a-3)(a+1)} \times \frac{(a+3)}{(a+3)} \\
 &= \frac{(a+3)(a+1)}{(a-3)(a+3)(a+1)} + \frac{(a+1)}{(a+3)(a-3)(a+1)} \\
 &\quad - \frac{(a+3)}{(a-3)(a+1)(a+3)} \\
 &= \frac{a^2 + 4a + 3 + a + 1 - a - 3}{(a-3)(a+3)(a+1)} \\
 &= \frac{a^2 + 4a + 1}{(a-3)(a+3)(a+1)}, \quad a \neq -1, \pm 3
 \end{aligned}$$

7. Finding the LCD instead of just multiplying the denominators when adding or subtracting rational expressions reduces the number and degree of terms in both the numerator and denominator of the sum or difference.

For example, consider the difference:

$$\frac{x}{x^2-4} - \frac{3x}{3x^2+5x-2} = \frac{x}{(x-2)(x+2)} - \frac{3x}{(3x-1)(x+2)},$$

$$x \neq \frac{1}{3}, \pm 2$$

If we simply multiply the denominators to find a common denominator, we need to multiply each term by a product of binomials:

$$\frac{x}{(x-2)(x+2)} \times \frac{(3x-1)(x+2)}{(3x-1)(x+2)} - \frac{3x}{(3x-1)(x+2)} \times \frac{(x-2)(x+2)}{(x-2)(x+2)}$$

A better way is to find the LCD, and use this for a common denominator. Since the factor $(x+2)$ is common to both denominators, this only needs to appear once in the LCD.

The LCD is $(x-2)(x+2)(3x-1)$. The difference is

$$\frac{x}{(x-2)(x+2)} \times \frac{(3x-1)}{(3x-1)} - \frac{3x}{(3x-1)(x+2)} \times \frac{(x-2)}{(x-2)}.$$

These expressions are much easier to deal with.

8. (a)

$$\frac{5x-7y}{12x} + \frac{2x-9y}{8y}, x \neq 0, y \neq 0$$

$$= \frac{5x-7y}{12x} \times \frac{(2y)}{(2y)} + \frac{2x-9y}{8y} \times \frac{(3x)}{(3x)}$$

$$= \frac{2y(5x-7y)}{24xy} + \frac{3x(2x-9y)}{24xy}$$

$$= \frac{10xy-14y^2+6x^2-27xy}{24xy}$$

$$= \frac{6x^2-17xy-14y^2}{24xy} \text{ or } \frac{(3x+2y)(2x-7y)}{24xy}, x \neq 0, y \neq 0$$

(b)

$$\frac{1}{2x+8} - \frac{3x}{(x+4)^2} + \frac{1}{2}$$

$$= \frac{1}{2(x+4)} - \frac{3x}{(x+4)^2} + \frac{1}{2}, x \neq -4$$

$$= \frac{1}{2(x+4)} \times \frac{(x+4)}{(x+4)} - \frac{3x}{(x+4)^2} \times \frac{2}{2} + \frac{1}{2} \times \frac{(x+4)^2}{(x+4)^2}$$

$$= \frac{(x+4)}{2(x+4)^2} - \frac{6x}{2(x+4)^2} + \frac{(x+4)^2}{2(x+4)^2}$$

$$= \frac{x+4-6x+x^2+8x+16}{2(x+4)^2}$$

$$= \frac{x^2+3x+20}{2(x+4)^2}, x \neq -4$$

9. (a)

$$\frac{x-2}{x^2-7x+10} + \frac{x+2}{x^2-4x-5}$$

$$= \frac{x-2}{(x-2)(x-5)} + \frac{x+2}{(x-5)(x+1)}, x \neq -1, 2, 5$$

$$= \frac{x-2}{(x-2)(x-5)} \times \frac{(x+1)}{(x+1)} + \frac{x+2}{(x-5)(x+1)} \times \frac{(x-2)}{(x-2)}$$

$$= \frac{(x-2)(x+1)}{(x-2)(x-5)(x+1)} + \frac{(x+2)(x-2)}{(x-5)(x+1)(x-2)}$$

$$= \frac{x^2-x-2+x^2-4}{(x-2)(x-5)(x+1)}$$

$$= \frac{2x^2-x-6}{(x-2)(x-5)(x+1)}$$

$$= \frac{2x^2-x-6}{(x-2)(x-5)(x+1)}$$

$$= \frac{(2x+3)(x-2)}{(x-2)(x-5)(x+1)}$$

$$= \frac{2x+3}{(x-5)(x+1)} \text{ or } \frac{2x+3}{x^2-4x-5}, x \neq -1, 2, 5$$

(b)

$$\frac{x-3}{x^2+x-12} - \frac{x-2}{x^2+3x-4}$$

$$= \frac{x-3}{(x+4)(x-3)} - \frac{x-2}{(x+4)(x-1)}, x \neq -4, 1, 3$$

$$= \frac{1}{(x+4)} \times \frac{(x-1)}{(x-1)} - \frac{x-2}{(x+4)(x-1)}$$

$$= \frac{x-1}{(x+4)(x-1)} - \frac{x-2}{(x+4)(x-1)}$$

$$= \frac{x-1-(x-2)}{(x+4)(x-1)}$$

$$= \frac{x-1-x+2}{(x+4)(x-1)}$$

$$= \frac{1}{(x+4)(x-1)} \text{ or } \frac{1}{x^2+3x-4}, x \neq -4, 1, 3$$

(c)

$$\frac{3k}{6k^2+13k-5} + \frac{2k+1}{6k^2+7k-3}$$

$$= \frac{3k}{(3k-1)(2k+5)} + \frac{2k+1}{(3k-1)(2k+3)}, k \neq -\frac{5}{2}, -\frac{3}{2}, \frac{1}{3}$$

$$= \frac{3k}{(3k-1)(2k+5)} \times \frac{(2k+3)}{(2k+3)} + \frac{2k+1}{(3k-1)(2k+3)} \times \frac{(2k+5)}{(2k+5)}$$

$$= \frac{3k(2k+3)}{(3k-1)(2k+5)(2k+3)} + \frac{(2k+1)(2k+5)}{(3k-1)(2k+3)(2k+5)}$$

$$= \frac{6k^2+9k+4k^2+12k+5}{(3k-1)(2k+5)(2k+3)}$$

$$= \frac{10k^2+21k+5}{(3k-1)(2k+5)(2k+3)}, k \neq -\frac{5}{2}, -\frac{3}{2}, \frac{1}{3}$$

(d)

$$\begin{aligned}
& \frac{4-5q}{24q^2+2q-12} - \frac{5-4q}{12q^2-15q-18} \\
&= \frac{4-5q}{2(12q^2+q-6)} - \frac{5-4q}{3(4q^2-5q-6)} \\
&= \frac{4-5q}{2(4q+3)(3q-2)} - \frac{5-4q}{3(4q+3)(q-2)}, q \neq -\frac{3}{4}, \frac{2}{3}, 2 \\
&= \frac{4-5q}{2(4q+3)(3q-2)} \times \frac{3(q-2)}{3(q-2)} - \frac{5-4q}{3(4q+3)(q-2)} \times \frac{2(3q-2)}{2(3q-2)} \\
&= \frac{3(-5q^2+14q-8) - 2(-12q^2+23q-10)}{6(4q+3)(3q-2)(q-2)} \\
&= \frac{-15q^2+42q-24+24q^2-46q+20}{6(4q+3)(3q-2)(q-2)} \\
&= \frac{9q^2-4q-4}{6(4q+3)(3q-2)(q-2)}, q \neq -\frac{3}{4}, \frac{2}{3}, 2
\end{aligned}$$

(e)

$$\begin{aligned}
& \left(\frac{3x-2}{2x^2-5x-3} - \frac{x+2}{x^2-9} \right) \div \frac{2x-3}{2x^2+7x+3} \\
&= \left(\frac{3x-2}{(2x+1)(x-3)} - \frac{x+2}{(x-3)(x+3)} \right) \div \frac{2x-3}{(2x+1)(x+3)}, \\
& \quad x \neq -\frac{1}{2}, \pm 3 \\
&= \left(\frac{3x-2}{(2x+1)(x-3)} \times \frac{(x+3)}{(x+3)} - \frac{x+2}{(x-3)(x+3)} \times \frac{(2x+1)}{(2x+1)} \right) \\
& \quad \times \frac{(2x+1)(x+3)}{(2x-3)}, x \neq -\frac{1}{2}, \pm 3, \frac{3}{2} \\
&= \left(\frac{(3x-2)(x+3)}{(2x+1)(x-3)(x+3)} - \frac{(x+2)(2x+1)}{(x-3)(x+3)(2x+1)} \right) \\
& \quad \times \frac{(2x+1)(x+3)}{(2x-3)} \\
&= \left(\frac{3x^2+7x-6-(2x^2+5x+2)}{(2x+1)(x-3)(x+3)} \right) \times \frac{(2x+1)(x+3)}{(2x-3)} \\
&= \left(\frac{3x^2+7x-6-2x^2-5x-2}{(2x+1)(x-3)(x+3)} \right) \times \frac{(2x+1)(x+3)}{(2x-3)} \\
&= \frac{x^2+2x-8}{(2x+1)(x-3)(x+3)} \times \frac{(2x+1)(x+3)}{(2x-3)} \\
&= \frac{(x+4)(x-2)}{(2x+1)(x-3)(x+3)} \times \frac{(2x+1)(x+3)}{(2x-3)} \\
&= \frac{(x+4)(x-2)}{(x-3)(2x-3)} \text{ or } \frac{x^2+2x-8}{2x^2-9x+9}
\end{aligned}$$

Alternate solution:

$$\begin{aligned}
& \left(\frac{3x-2}{(2x+1)(x-3)} - \frac{x+2}{(x-3)(x+3)} \right) \div \frac{2x-3}{(2x+1)(x+3)}, \\
& \quad x \neq -\frac{1}{2}, \pm 3 \\
&= \left(\frac{3x-2}{(2x+1)(x-3)} - \frac{x+2}{(x-3)(x+3)} \right) \times \frac{(2x+1)(x+3)}{2x-3}, \\
& \quad x \neq -\frac{1}{2}, \frac{3}{2}, \pm 3 \\
&= \frac{3x-2}{(2x+1)(x-3)} \times \frac{(2x+1)(x+3)}{2x-3} - \frac{x+2}{(x-3)(x+3)} \\
& \quad \times \frac{(2x+1)(x+3)}{2x-3} \\
&= \frac{(3x-2)(x+3)}{(x-3)(2x-3)} - \frac{(x+2)(2x+1)}{(x-3)(2x-3)} \\
&= \frac{3x^2+7x-6-(2x^2+5x+2)}{(x-3)(2x-3)} \\
&= \frac{3x^2+7x-6-2x^2-5x-2}{(x-3)(2x-3)} \\
&= \frac{x^2+2x-8}{(x-3)(2x-3)} \\
&= \frac{(x+4)(x-2)}{(x-3)(2x-3)}
\end{aligned}$$

(f)

$$\begin{aligned}
& \frac{a-2}{6a^2-7a-5} \div \frac{2a}{3a^2-5a} - \frac{3a+2}{2a^2+11a+5} \\
&= \frac{a-2}{(3a-5)(2a+1)} \div \frac{2a}{a(3a-5)} - \frac{3a+2}{(2a+1)(a+5)}, \\
& \quad a \neq -5, -\frac{1}{2}, 0, \frac{5}{3} \\
&= \frac{a-2}{(3a-5)(2a+1)} \times \frac{a(3a-5)}{2a} - \frac{3a+2}{(2a+1)(a+5)} \\
&= \frac{a-2}{2(2a+1)} - \frac{3a+2}{(2a+1)(a+5)} \\
&= \frac{a-2}{2(2a+1)} \times \frac{(a+5)}{(a+5)} - \frac{3a+2}{(2a+1)(a+5)} \times \frac{2}{2} \\
&= \frac{(a-2)(a+5)}{2(2a+1)(a+5)} - \frac{2(3a+2)}{2(2a+1)(a+5)} \\
&= \frac{a^2+3a-10-6a-4}{2(2a+1)(a+5)} \\
&= \frac{a^2-3a-14}{2(2a+1)(a+5)} \text{ or } \frac{a^2-3a-14}{4a^2+22a+10}
\end{aligned}$$

10. (a)

$$\begin{aligned}
 f(x) &= 3x + 1 + \frac{1}{x+1}, \quad x \neq -1 \\
 &= \frac{(3x+1)}{1} \times \frac{(x+1)}{(x+1)} + \frac{1}{x+1} \\
 &= \frac{(3x+1)(x+1)}{x+1} + \frac{1}{x+1} \\
 &= \frac{3x^2 + 4x + 1 + 1}{x+1}
 \end{aligned}$$

$$f(x) = \frac{3x^2 + 4x + 2}{x+1}, \quad x \neq -1$$

Zeros occur where $f(x) = 0$.

$$0 = \frac{3x^2 + 4x + 2}{x+1}$$

$$0 = 3x^2 + 4x + 2$$

Use the quadratic formula.

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{-4 \pm \sqrt{-8i^2}}{6}$$

$$= \frac{-4 \pm 2\sqrt{2}i}{6}$$

$$= -\frac{2}{3} \pm \frac{\sqrt{2}}{3}i$$

Zeros are $x = -\frac{2}{3} + \frac{\sqrt{2}}{3}i$ or $x = -\frac{2}{3} - \frac{\sqrt{2}}{3}i$.

(b)

$$\begin{aligned}
 f(x) &= 6x - 17 + \frac{28}{x+2} \\
 &= \frac{(6x-17)}{1} \times \frac{(x+2)}{(x+2)} + \frac{28}{x+2} \\
 &= \frac{(6x-17)(x+2)}{(x+2)} + \frac{28}{x+2} \\
 &= \frac{6x^2 - 5x - 34 + 28}{x+2} \\
 &= \frac{6x^2 - 5x - 6}{x+2} \\
 &= \frac{(3x+2)(2x-3)}{x+2}
 \end{aligned}$$

Zeros occur where $f(x) = 0$.

$$\frac{(3x+2)(2x-3)}{x+2} = 0$$

$$(3x+2)(2x-3) = 0$$

Zeros are $x = -\frac{2}{3}$ or $x = \frac{3}{2}$.

$$11. (a) \frac{x-2}{x} + \frac{4}{5x} = -\frac{1}{5}, \quad x \neq 0$$

Method 1:

$$\frac{(x-2)}{x} \times \frac{5}{5} + \frac{4}{5x} = -\frac{1}{5}$$

$$\frac{5(x-2)}{5x} + \frac{4}{5x} = -\frac{1}{5}$$

$$\frac{5x-10+4}{5x} = -\frac{1}{5}$$

$$\frac{5x-6}{5x} = -\frac{1}{5}$$

Cross multiply:

$$5(5x-6) = -5x$$

$$5x-6 = -1$$

$$5x = -1+6$$

$$5x = 5$$

$$x = 1$$

Method 2:

LCD is $5x$.

Multiply both sides of the equation by $5x$.

$$5x\left(\frac{x-2}{x}\right) + 5x\left(\frac{4}{5x}\right) = 5x\left(-\frac{1}{5}\right)$$

$$5(x-2) + 4 = -x$$

$$5x-10+4 = -x$$

$$5x-6 = -x$$

$$6x = 6$$

$$x = 1$$

$$(b) \frac{2x+3}{x-1} - \frac{3}{x} = 2, \quad x \neq 0, 1$$

Method 1:

$$\frac{2x+3}{x-1} \times \frac{x}{x} - \frac{3}{x} \times \frac{(x-1)}{(x-1)} = 2$$

$$\frac{x(2x+3)}{x(x-1)} - \frac{3(x-1)}{x(x-1)} = 2$$

$$\frac{2x^2 + 3x - 3x + 3}{x(x-1)} = 2$$

$$\frac{2x^2 + 3}{x(x-1)} = 2$$

$$2x^2 + 3 = 2x(x-1)$$

$$2x^2 + 3 = 2x^2 - 2x$$

$$3 = -2x$$

$$-\frac{3}{2} = x \text{ or } x = -1.5$$

Method 2:

LCD is $x(x-1)$.

Multiply both sides of the equation by $x(x-1)$.

$$x(x-1)\left(\frac{2x+3}{x-1}\right) - x(x-1)\left(\frac{3}{x}\right) = 2x(x-1)$$

$$x(2x+3) - 3(x-1) = 2x^2 - 2x$$

$$2x^2 + 3x - 3x + 3 = 2x^2 - 2x$$

$$3 = -2x$$

$$-\frac{3}{2} = x$$

Method 2 is used for the remaining parts of question 11.

$$(c) \frac{1}{x} = \frac{2}{x+1} + \frac{1}{1-x}, x \neq 0, \pm 1$$

LCD is $x(1+x)(1-x)$.

$$x(1+x)(1-x)\left(\frac{1}{x}\right) = x(1+x)(1-x)\left(\frac{2}{1+x}\right) + x(1+x)(1-x)\left(\frac{1}{1-x}\right)$$

$$1-x^2 = 2x(1-x) + x(1+x)$$

$$1-x^2 = 2x - 2x^2 + x + x^2$$

$$1-x^2 = 3x - x^2$$

$$1 = 3x$$

$$\frac{1}{3} = x$$

(d)

$$\frac{3x}{x^2-1} = \frac{x}{x+1} - 4$$

$$\frac{3x}{(x-1)(x+1)} = \frac{x}{x+1} - 4, x \neq \pm 1$$

LCD is $(x-1)(x+1)$.

Multiply both sides of the equation by $(x-1)(x+1)$.

$$(x-1)(x+1)\left(\frac{3x}{(x-1)(x+1)}\right) = (x-1)(x+1)\left(\frac{x}{x+1}\right) - 4(x-1)(x+1)$$

$$3x = x(x-1) - 4(x^2-1)$$

$$3x = x^2 - x - 4x^2 + 4$$

$$3x^2 + 4x - 4 = 0$$

$$(3x-2)(x+2) = 0$$

$$x = \frac{2}{3} \text{ or } -2$$

(e)

$$\frac{15}{x^2-1} = \frac{4}{x-1} - \frac{3}{x+1}$$

$$\frac{15}{(x-1)(x+1)} = \frac{4}{x-1} - \frac{3}{x+1}, x \neq \pm 1$$

LCD is $(x-1)(x+1)$.

Multiply both sides of the equation by $(x-1)(x+1)$.

$$(x-1)(x+1)\left(\frac{15}{(x-1)(x+1)}\right) = (x-1)(x+1)\left(\frac{4}{x-1}\right) - (x-1)(x+1)\left(\frac{3}{x+1}\right)$$

$$15 = 4(x+1) - 3(x-1)$$

$$15 = 4x + 4 - 3x + 3$$

$$15 = x + 7$$

$$8 = x$$

(f)

$$\frac{x}{x-2} + 2 = \frac{5x}{x+2} + \frac{3x+1}{x^2-4}$$

$$\frac{x}{x-2} + 2 = \frac{5x}{x+2} + \frac{3x+1}{(x-2)(x+2)}, x \neq \pm 2$$

Multiply both sides of the equation by LCD $(x-2)(x+2)$.

$$(x-2)(x+2)\left(\frac{x}{x-2}\right) + 2(x-2)(x+2)$$

$$= (x-2)(x+2)\left(\frac{5x}{x+2}\right) + (x-2)(x+2)\left(\frac{3x+1}{(x-2)(x+2)}\right)$$

$$x(x+2) + 2(x^2-4) = 5x(x-2) + 3x+1$$

$$x^2 + 2x + 2x^2 - 8 = 5x^2 - 10x + 3x + 1$$

$$0 = 2x^2 - 9x + 9$$

$$0 = (2x-3)(x-3)$$

$$x = \frac{3}{2} \text{ or } x = 3$$

12. Let x km/h represent Sarah's speed.

	Distance (km)	Speed (km/h)	Time (h)
Sarah	42	x	$\frac{42}{x}$
Arshia	42	$x + 0.4$	$\frac{42}{x + 0.4}$

Arshia stopped for $\frac{1}{2}$ hour but finished only $\frac{1}{4}$ hour after

Sarah. It took her $\frac{1}{4}$ hour less time to cover the 42 km course than it took Sarah.

$$\therefore \text{Sarah's time} - \text{Arshia's time} = \frac{1}{4}$$

$$\frac{42}{x} - \frac{42}{x+0.4} = \frac{1}{4}, x \neq 0, -0.4$$

$$\frac{42}{x} \times \frac{(x+0.4)}{(x+0.4)} - \frac{42}{x+0.4} \times \frac{x}{x} = \frac{1}{4}$$

$$\frac{42(x+0.4) - 42x}{x(x+0.4)} = \frac{1}{4}$$

$$\frac{42x + 16.8 - 42x}{x(x+0.4)} = \frac{1}{4}$$

$$\frac{16.8}{x(x+0.4)} = \frac{1}{4}$$

LCD is $x(x+0.4)$.

Multiply both sides of the equation by $x(x+0.4)$.

$$x(x+0.4) \times \frac{16.8}{x(x+0.4)} = \frac{1}{4} \times x(x+0.4)$$

$$16.8 = \frac{1}{4}x(x+0.4)$$

$$4 \times 16.8 = \frac{1}{4}x(x+0.4) \times 4$$

$$67.2 = x(x+0.4)$$

$$67.2 = x^2 + 0.4x$$

$$0 = x^2 + 0.4x - 67.2$$

Use the quadratic formula.

$$x = \frac{-0.4 \pm \sqrt{0.4^2 - 4(1)(-67.2)}}{2(1)}$$

$$x = \frac{-0.4 \pm 16.4}{2}$$

Since x is positive, we take only the positive root. i.e. $x = 8$

Sarah drove at an average speed of 8 km/h and Arshia drove at an average speed of 8.4 km/h. It took Sarah $\frac{42}{8} = 5$ hours and 15 min to finish the course. It took Arshia $\frac{42}{8.4} = 5$ hours to finish the course.

13. Let the speed of the current be x km/h.

	Distance (km)	Speed (km/h)	Time (h)
Down river	39	$8 + x$	$\frac{39}{8 + x}$
Up river	39	$8 - x$	$\frac{39}{8 - x}$

If the total time taken is 16 h,

$$\frac{39}{8 + x} + \frac{39}{8 - x} = 16, x \neq \pm 8$$

Multiply through by the LCD $(8 + x)(8 - x)$.

$$(8 + x)(8 - x)\left(\frac{39}{8 + x}\right) + (8 + x)(8 - x)\left(\frac{39}{8 - x}\right) = 16(8 + x)(8 - x)$$

$$39(8 - x) + 39(8 + x) = 16(64 - x^2)$$

$$312 - 39x + 312 + 39x = 1024 - 16x^2$$

$$624 = 1024 - 16x^2$$

$$16x^2 = 400$$

$$x^2 = 25$$

$$x = \pm 5$$

The speed of the current is 5 km/h.

14. Let John's rowing speed be x km/h.

	Distance (km)	Speed (km/h)	Time (h)
Up river	16	$x - 2$	$\frac{16}{x - 2}$
Down river	16	$x + 2$	$\frac{16}{x + 2}$

Total time taken is 6 h.

$$\therefore \frac{16}{x - 2} + \frac{16}{x + 2} = 6$$

Multiply both sides by LCD $(x - 2)(x + 2)$.

$$(x - 2)(x + 2)\left(\frac{16}{x - 2}\right) + (x - 2)(x + 2)\left(\frac{16}{x + 2}\right) = 6(x - 2)(x + 2)$$

$$16(x + 2) + 16(x - 2) = 6(x^2 - 4)$$

$$16x + 32 + 16x - 32 = 6x^2 - 24$$

$$32x = 6x^2 - 24$$

$$0 = 6x^2 - 32x - 24$$

$$0 = 3x^2 - 16x - 12$$

$$0 = (3x + 2)(x - 6)$$

$$x = -\frac{2}{3} \text{ or } x = 6$$

Reject $-\frac{2}{3}$ since $x \geq 0$. John's rowing speed is 6 km/h.

15. Let x represent the number of T-shirts Jaime bought. If she paid \$450 for x shirts, each shirt cost $\frac{\$450}{x}$. Since Jaime kept 2 shirts for herself, she sold $(x - 2)$ shirts. If she sold $(x - 2)$ shirts for \$560, each shirt cost $\frac{\$560}{x - 2}$. If she made a profit of \$10 on each shirt, this second cost must be \$10 more than the first cost, so

$$\frac{560}{x - 2} - \frac{450}{x} = 10, x \neq 0, 2.$$

Multiply both sides of the equation by the LCD, which is $x(x - 2)$.

$$x(x - 2)\left(\frac{560}{x - 2}\right) - x(x - 2)\left(\frac{450}{x}\right) = 10x(x - 2)$$

$$560x - 450(x - 2) = 10x^2 - 20x$$

$$560x - 450x + 900 = 10x^2 - 20x$$

$$110x + 900 = 10x^2 - 20x$$

$$0 = 10x^2 - 130x - 900$$

$$0 = 10(x - 18)(x + 5)$$

$$x = 18 \text{ or } -5$$

Reject the negative root since we can assume $x > 2$. Jaime bought a block of 18 shirts.

16. Let x represent the number of days Stuart expected to take to paint the house.

	Total Earnings (\$)	Number of days to complete job	Daily rate of pay
Expected	900	x	$\frac{900}{x}$
Actual	900	$x + 4$	$\frac{900}{x + 4}$

Since Stuart earned \$18.75 less per day than he expected,

$$\frac{900}{x} - \frac{900}{x + 4} = \frac{75}{4}, x \neq -4, 0$$

Multiply both sides of the equation by the LCD $4x(x + 4)$.

$$4x(x + 4)\left(\frac{900}{x}\right) - 4x(x + 4)\left(\frac{900}{x + 4}\right) = \frac{75}{4} \times 4x(x + 4)$$

$$3600x + 14400 - 3600x = 75x(x + 4)$$

$$14400 = 75x^2 + 300x$$

$$0 = 75x^2 + 300x - 14400$$

$$0 = 75(x^2 + 4x - 192)$$

$$0 = 75(x - 12)(x + 16)$$

$$x = 12 \text{ or } -16$$

We reject the negative root since $x > 0$.

Stuart expected to paint the house in 12 days.

17. Let x represent the number of students that had lunch. If the bill of \$239.25 is divided equally among the students, each student pays $\frac{\$239.25}{x}$. If 4 students do not pay, then $(x - 4)$ students pay. If the bill of \$239.25 is divided equally among $(x - 4)$ students, each student pays $\frac{\$239.25}{x - 4}$. If this

second amount is \$1 more than the first amount,

$$\frac{239.25}{x-4} - \frac{239.25}{x} = 1$$

Multiply both sides of the equation by the LCD, $x(x-4)$.

$$x(x-4)\left(\frac{239.25}{x-4}\right) - x(x-4)\left(\frac{239.25}{x}\right) = x(x-4)$$

$$239.25x - 239.25x + 957 = x^2 - 4x$$

$$0 = x^2 - 4x - 957$$

$$0 = (x-33)(x+29)$$

$$x = 33 \text{ or } -29$$

Since $a > 0$, then reject -29 as an answer. Therefore, 33 students had lunch.

18. To subtract $\frac{x-2}{x^2-4x-32} - \frac{x-1}{x^2-2x-48}$

Factor the denominators and state any restrictions.

$$\frac{x-2}{(x-8)(x+4)} - \frac{x-1}{(x-8)(x+6)} \quad x \neq -6, -4, 8$$

- Put each expression over the lowest common denominator.

$$\begin{aligned} & \frac{x-2}{(x-8)(x+4)} \times \frac{(x+6)}{(x+6)} - \frac{x-1}{(x-8)(x+6)} \times \frac{(x+4)}{(x+4)} \\ &= \frac{(x-2)(x+6)}{(x-8)(x+4)(x+6)} - \frac{(x-1)(x+4)}{(x-8)(x+6)(x+4)} \end{aligned}$$

- Expand and subtract the numerators.

$$\begin{aligned} \frac{x^2+4x-12-(x^2+3x-4)}{(x-8)(x+4)(x+6)} &= \frac{x^2+4x-12-x^2-3x+4}{(x-8)(x+4)(x+6)} \\ &= \frac{x-8}{(x-8)(x+4)(x+6)} \end{aligned}$$

- Reduce the expression to lowest terms.

$$= \frac{1}{(x+4)(x+6)} \text{ or } \frac{1}{x^2+10x+24}$$

19. $\frac{2x-1}{(x+1)(3x+2)} = \frac{A}{x+1} + \frac{B}{3x+2} \quad x \neq -1, -\frac{2}{3}$

Find A and B by combining the expressions on the right side and then comparing numerators on each side.

$$\begin{aligned} \text{R.S.} &= \frac{A}{(x+1)} \times \frac{(3x+2)}{(3x+2)} + \frac{B}{(3x+2)} \times \frac{(x+1)}{(x+1)} \\ &= \frac{A(3x+2)}{(x+1)(3x+2)} + \frac{B(x+1)}{(x+1)(3x+2)} \\ &= \frac{A(3x+2)+B(x+1)}{(x+1)(3x+2)} \end{aligned}$$

$$\text{So } \frac{2x-1}{(x+1)(3x+2)} = \frac{A(3x+2)+B(x+1)}{(x+1)(3x+2)}$$

Since the denominators are the same, the numerators must also be the same.

$$2x-1 = A(3x+2) + B(x+1)$$

$$2x-1 = 3Ax + 2A + Bx + B$$

$$2x-1 = (3A+B)x + 2A+B$$

$$\text{Comparing coefficients of } x: 2 = 3A + B \text{ -----①}$$

$$\text{Comparing constant terms: } -1 = 2A + B \text{ -----②}$$

$$\text{Subtract ① - ②: } 3 = A$$

$$\begin{aligned} \text{Substitute } A = 3 \text{ in ①: } 2 &= 3(3) + B \\ -7 &= B \end{aligned}$$

$$\therefore A = 3 \text{ and } B = -7$$

$$\frac{2x-1}{(x+1)(3x+2)} = \frac{3}{x+1} - \frac{7}{3x+2}$$

20.

$$\frac{(x-1)}{(x+1)(x-2)^2} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}, \quad x \neq -1, 2$$

$$\text{R.S.} = \frac{A}{(x+1)} \times \frac{(x-2)^2}{(x-2)^2} + \frac{B}{x-2} \times \frac{(x+1)(x-2)}{(x+1)(x-2)}$$

$$+ \frac{C}{(x-2)^2} \times \frac{(x+1)}{(x+1)}$$

$$= \frac{A(x-2)^2}{(x+1)(x-2)^2} + \frac{B(x+1)(x-2)}{(x+1)(x-2)^2}$$

$$+ \frac{C(x+1)}{(x+1)(x-2)^2}$$

$$= \frac{A(x^2-4x+4) + B(x^2-x-2) + C(x+1)}{(x+1)(x-2)^2}$$

$$= \frac{(A+B)x^2 + (-4A-B+C)x + 4A-2B+C}{(x+1)(x-2)^2}$$

$$\text{If } \frac{(x-1)}{(x+1)(x-2)^2}$$

$$= \frac{(A+B)x^2 + (-4A-B+C)x + 4A-2B+C}{(x+1)(x-2)^2},$$

the numerators must be equal, since the denominators are equal.

$$\therefore x-1 = (A+B)x^2 + (-4A-B+C)x + 4A-2B+C$$

$$\text{Compare coefficients of } x^2: 0 = A+B \text{ -----①}$$

$$\text{Compare coefficients of } x: 1 = -4A-B+C \text{ -----②}$$

$$\text{Compare constant terms: } -1 = 4A-2B+C \text{ -----③}$$

$$\text{Solve the system of equations: From ①, } A = -B$$

$$\text{Substitute } A = -B \text{ into equations ② and ③:}$$

$$\text{②: } 1 = -4(-B) - B + C$$

$$1 = 3B + C \text{ -----④}$$

$$\text{③: } -1 = 4(-B) - 2B + C$$

$$-1 = -6B + C \text{ -----⑤}$$

$$\text{Subtract ④ - ⑤: } 2 = 9B$$

$$\frac{2}{9} = B, \quad A = -\frac{2}{9}$$

$$\text{Substitute in ③:}$$

$$-1 = 4\left(-\frac{2}{9}\right) - 2\left(\frac{2}{9}\right) + C$$

$$-1 = -\frac{8}{9} - \frac{4}{9} + C$$

$$-1 = -\frac{12}{9} + C$$

$$-1 = -\frac{4}{3} + C$$

$$\frac{1}{3} = C$$

$$\frac{(x-1)}{(x+1)(x-2)^2} = -\frac{2}{9(x+1)} + \frac{2}{9(x-2)} + \frac{1}{3(x-2)^2}$$

21. Organize the information in a chart.

	Distance (km)	Speed (km/h)	Time (h)
Before increase in speed	a	$\frac{a}{x}$	x
After increase in speed	a	$\frac{a}{x-c}$	$x-c$

If the increase in speed is b km/h,

$$\frac{a}{x-c} - \frac{a}{x} = b \quad x \neq 0, c$$

Multiply both sides of the equation by LCD $x(x-c)$.

$$x(x-c)\left(\frac{a}{x-c}\right) - x(x-c)\left(\frac{a}{x}\right) = bx(x-c)$$

$$ax - ax + ac = bx^2 - bcx$$

$$0 = bx^2 - bcx - ac$$

Using the quadratic formula,

$$x = \frac{bc \pm \sqrt{b^2c^2 - 4(b)(-ac)}}{2b}$$

$$= \frac{bc \pm \sqrt{b^2c^2 + 4abc}}{2b}$$

$$x = \frac{bc + \sqrt{b^2c^2 + 4abc}}{2b} \text{ or } x = \frac{bc - \sqrt{b^2c^2 + 4abc}}{2b}$$

Reject the second root, since it will be negative

$$(bc < \sqrt{b^2c^2 + 4abc} \text{ for positive } a, b, c).$$

$$\therefore x = \frac{bc + \sqrt{b^2c^2 + 4ac}}{2b}$$

Practise, Apply, Solve 4.12, page 374

1. (a) $(x^2 - 2x - 1) + (3x^2 + x + 2)$

$$= x^2 - 2x - 1 + 3x^2 + x + 2$$

$$= 4x^2 - x + 1$$

(b) $(x^2 - x + 5) + (-x^2 + x - 3)$

$$= x^2 - x + 5 - x^2 + x - 3$$

$$= 2$$

(c) $(-3x^2 - 2x + 2) + (2x^2 - x - 5)$

$$= -3x^2 - 2x + 2 + 2x^2 - x - 5$$

$$= -x^2 - 3x - 3$$

(d) $(x^3 - 4x^2 + 1) - (x^2 - 2x + 4)$

$$= x^3 - 4x^2 + 1 - x^2 + 2x - 4$$

$$= x^3 - 5x^2 + 2x - 3$$

(e) $(2x^4 - 2x^2 - 7) - (3x^4 - x^3 + x^2 - 4)$

$$= 2x^4 - 2x^2 - 7 - 3x^4 + x^3 - x^2 + 4$$

$$= -x^4 + x^3 - 3x^2 - 3$$

(f) $(x^5 + x^4 + x^3 - 4x^2 - x + 5) - (-x^4 - x^3 + 2x + 4)$

$$= x^5 + x^4 + x^3 - 4x^2 - x + 5 + x^4 + x^3 - 2x - 4$$

$$= x^5 + 2x^4 + 2x^3 - 4x^2 - 3x + 1$$

(g) $(-2x^2 + 5x - 3) + (x^2 - 6x + 1) - (-3x^2 - 2x - 4)$

$$= -2x^2 + 5x - 3 + x^2 - 6x + 1 + 3x^2 + 2x + 4$$

$$= 2x^2 + x + 2$$

2. (a) $3x(x-2) - 4x(x+1) = 3x^2 - 6x - 4x^2 - 4x$

$$= -x^2 - 10x$$

(b) $-x(2x-5) - 3x(4-x) = -2x^2 + 5x - 12x + 3x^2$
 $= x^2 - 7x$

(c) $2x^2(x+3) + 4x^2(x-1) = 2x^3 + 6x^2 + 4x^3 - 4x^2$
 $= 6x^3 + 2x^2$

(d) $-2x(x^2 - 5x - 2) - x(2x^2 - x - 4)$
 $= -2x^3 + 10x^2 + 4x - 2x^3 + x^2 + 4x$
 $= -4x^3 + 11x^2 + 8x$

(e) $-4x(x^3 - 3x^2 + 1) + x(3x^2 - 2x - 4)$
 $= -4x^4 + 12x^3 - 4x + 3x^3 - 2x^2 - 4x$
 $= -4x^4 + 15x^3 - 2x^2 - 8x$

(f) $2x^2(3x^3 - 5x + 2) - x^3(2x^3 + 2x - 5)$
 $= 6x^5 - 10x^3 + 4x^2 - 2x^6 - 2x^4 + 5x^3$
 $= -2x^6 + 6x^5 - 2x^4 - 5x^3 + 4x^2$

3. (a) Degree is 2.

(b) Degree is 2.

(c) Degree is 3.

(d) Degree is 3.

(e) Degree is 4.

(f) Degree is 6.

4. (a) $4[2x - (3 - 5x)] + 3[3x - (4 + x)]$

$$= 4[2x - 3 + 5x] + 3[3x - 4 - x]$$

$$= 4[7x - 3] + 3[2x - 4]$$

$$= 28x - 12 + 6x - 12$$

$$= 34x - 24$$

(b) $5[x - (2x + 3)] - 2[4x - (x + 1)]$

$$= 5[x - 2x - 3] - 2[4x - x - 1]$$

$$= 5[-x - 3] - 2[3x - 1]$$

$$= -5x - 15 - 6x + 2$$

$$= -11x - 13$$

(c) $x[5x - (3x - 8)] + 2x[3x - (4 - x)]$

$$= x[5x - 3x + 8] + 2x[3x - 4 + x]$$

$$= x[2x + 8] + 2x[4x - 4]$$

$$= 2x^2 + 8x + 8x^2 - 8x$$

$$= 10x^2$$

(d) $-3x[x - (5 - 2x)] - x[-2x - (x - 6)]$

$$= -3x[x - 5 + 2x] - x[-2x - x + 6]$$

$$= -3x[3x - 5] - x[-3x + 6]$$

$$= -9x^2 + 15x + 3x^2 - 6x$$

$$= -6x^2 + 9x$$

(e) $2a[a - 2a(a - 2)] + 3a[a + 4a(1 - a)]$

$$= 2a[a - 2a^2 + 4a] + 3a[a + 4a - 4a^2]$$

$$= 2a[5a - 2a^2] + 3a[5a - 4a^2]$$

$$= 10a^2 - 4a^3 + 15a^2 - 12a^3$$

$$= -16a^3 + 25a^2$$

(f) $-a[5a - a(1 - 2a)] - 3a[a - 4a(a + 1)]$

$$= -a[5a - a + 2a^2] - 3a[a - 4a^2 - 4a]$$

$$= -a[4a + 2a^2] - 3a[-3a - 4a^2]$$

$$= -4a^2 - 2a^3 + 9a^2 + 12a^3$$

$$= 10a^3 + 5a^2$$

5. (a) $(x+2)(x^2 - x - 2) = x^3 - x^2 - 2x + 2x^2 - 2x - 4$

$$= x^3 + x^2 - 4x - 4$$

(b) $(1-x)(x^3 - x^2 + 3x) = x^3 - x^2 + 3x - x^4 + x^3 - 3x^2$

$$= -x^4 + 2x^3 - 4x^2 + 3x$$

(c) $(x^2 + 1)(x^2 - 3x + 5) = x^4 - 3x^3 + 5x^2 + x^2 - 3x + 5$

$$= x^4 - 3x^3 + 6x^2 - 3x + 5$$