

21. Organize the information in a chart.

	Distance (km)	Speed (km/h)	Time (h)
Before increase in speed	a	$\frac{a}{x}$	x
After increase in speed	a	$\frac{a}{x-c}$	$x-c$

If the increase in speed is b km/h,

$$\frac{a}{x-c} - \frac{a}{x} = b \quad x \neq 0, c$$

Multiply both sides of the equation by LCD $x(x-c)$.

$$x(x-c)\left(\frac{a}{x-c}\right) - x(x-c)\left(\frac{a}{x}\right) = bx(x-c)$$

$$ax - ax + ac = bx^2 - bcx$$

$$0 = bx^2 - bcx - ac$$

Using the quadratic formula,

$$x = \frac{bc \pm \sqrt{b^2c^2 - 4(b)(-ac)}}{2b}$$

$$= \frac{bc \pm \sqrt{b^2c^2 + 4abc}}{2b}$$

$$x = \frac{bc + \sqrt{b^2c^2 + 4abc}}{2b} \text{ or } x = \frac{bc - \sqrt{b^2c^2 + 4abc}}{2b}$$

Reject the second root, since it will be negative

$$(bc < \sqrt{b^2c^2 + 4abc} \text{ for positive } a, b, c).$$

$$\therefore x = \frac{bc + \sqrt{b^2c^2 + 4ac}}{2b}$$

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1. (a) $(x^2 - 2x - 1) + (3x^2 + x + 2)$

$$= x^2 - 2x - 1 + 3x^2 + x + 2$$

$$= 4x^2 - x + 1$$

(b) $(x^2 - x + 5) + (-x^2 + x - 3)$

$$= x^2 - x + 5 - x^2 + x - 3$$

$$= 2$$

(c) $(-3x^2 - 2x + 2) + (2x^2 - x - 5)$

$$= -3x^2 - 2x + 2 + 2x^2 - x - 5$$

$$= -x^2 - 3x - 3$$

(d) $(x^3 - 4x^2 + 1) - (x^2 - 2x + 4)$

$$= x^3 - 4x^2 + 1 - x^2 + 2x - 4$$

$$= x^3 - 5x^2 + 2x - 3$$

(e) $(2x^4 - 2x^2 - 7) - (3x^4 - x^3 + x^2 - 4)$

$$= 2x^4 - 2x^2 - 7 - 3x^4 + x^3 - x^2 + 4$$

$$= -x^4 + x^3 - 3x^2 - 3$$

(f) $(x^5 + x^4 + x^3 - 4x^2 - x + 5) - (-x^4 - x^3 + 2x + 4)$

$$= x^5 + x^4 + x^3 - 4x^2 - x + 5 + x^4 + x^3 - 2x - 4$$

$$= x^5 + 2x^4 + 2x^3 - 4x^2 - 3x + 1$$

(g) $(-2x^2 + 5x - 3) + (x^2 - 6x + 1) - (-3x^2 - 2x - 4)$

$$= -2x^2 + 5x - 3 + x^2 - 6x + 1 + 3x^2 + 2x + 4$$

$$= 2x^2 + x + 2$$

2. (a) $3x(x-2) - 4x(x+1) = 3x^2 - 6x - 4x^2 - 4x$

$$= -x^2 - 10x$$

(b) $-x(2x-5) - 3x(4-x) = -2x^2 + 5x - 12x + 3x^2$
 $= x^2 - 7x$

(c) $2x^2(x+3) + 4x^2(x-1) = 2x^3 + 6x^2 + 4x^3 - 4x^2$
 $= 6x^3 + 2x^2$

(d) $-2x(x^2 - 5x - 2) - x(2x^2 - x - 4)$
 $= -2x^3 + 10x^2 + 4x - 2x^3 + x^2 + 4x$
 $= -4x^3 + 11x^2 + 8x$

(e) $-4x(x^3 - 3x^2 + 1) + x(3x^2 - 2x - 4)$
 $= -4x^4 + 12x^3 - 4x + 3x^3 - 2x^2 - 4x$
 $= -4x^4 + 15x^3 - 2x^2 - 8x$

(f) $2x^2(3x^3 - 5x + 2) - x^3(2x^3 + 2x - 5)$
 $= 6x^5 - 10x^3 + 4x^2 - 2x^6 - 2x^4 + 5x^3$
 $= -2x^6 + 6x^5 - 2x^4 - 5x^3 + 4x^2$

3. (a) Degree is 2.

(b) Degree is 2.

(c) Degree is 3.

(d) Degree is 3.

(e) Degree is 4.

(f) Degree is 6.

4. (a) $4[2x - (3 - 5x)] + 3[3x - (4 + x)]$

$$= 4[2x - 3 + 5x] + 3[3x - 4 - x]$$

$$= 4[7x - 3] + 3[2x - 4]$$

$$= 28x - 12 + 6x - 12$$

$$= 34x - 24$$

(b) $5[x - (2x + 3)] - 2[4x - (x + 1)]$

$$= 5[x - 2x - 3] - 2[4x - x - 1]$$

$$= 5[-x - 3] - 2[3x - 1]$$

$$= -5x - 15 - 6x + 2$$

$$= -11x - 13$$

(c) $x[5x - (3x - 8)] + 2x[3x - (4 - x)]$

$$= x[5x - 3x + 8] + 2x[3x - 4 + x]$$

$$= x[2x + 8] + 2x[4x - 4]$$

$$= 2x^2 + 8x + 8x^2 - 8x$$

$$= 10x^2$$

(d) $-3x[x - (5 - 2x)] - x[-2x - (x - 6)]$

$$= -3x[x - 5 + 2x] - x[-2x - x + 6]$$

$$= -3x[3x - 5] - x[-3x + 6]$$

$$= -9x^2 + 15x + 3x^2 - 6x$$

$$= -6x^2 + 9x$$

(e) $2a[a - 2a(a - 2)] + 3a[a + 4a(1 - a)]$

$$= 2a[a - 2a^2 + 4a] + 3a[a + 4a - 4a^2]$$

$$= 2a[5a - 2a^2] + 3a[5a - 4a^2]$$

$$= 10a^2 - 4a^3 + 15a^2 - 12a^3$$

$$= -16a^3 + 25a^2$$

(f) $-a[5a - a(1 - 2a)] - 3a[a - 4a(a + 1)]$

$$= -a[5a - a + 2a^2] - 3a[a - 4a^2 - 4a]$$

$$= -a[4a + 2a^2] - 3a[-3a - 4a^2]$$

$$= -4a^2 - 2a^3 + 9a^2 + 12a^3$$

$$= 10a^3 + 5a^2$$

5. (a) $(x+2)(x^2 - x - 2) = x^3 - x^2 - 2x + 2x^2 - 2x - 4$

$$= x^3 + x^2 - 4x - 4$$

(b) $(1-x)(x^3 - x^2 + 3x) = x^3 - x^2 + 3x - x^4 + x^3 - 3x^2$

$$= -x^4 + 2x^3 - 4x^2 + 3x$$

(c) $(x^2 + 1)(x^2 - 3x + 5) = x^4 - 3x^3 + 5x^2 + x^2 - 3x + 5$

$$= x^4 - 3x^3 + 6x^2 - 3x + 5$$

$$(d) (2 - x^2)(3 + x^2 - x^3) = 6 + 2x^2 - 2x^3 - 3x^2 - x^4 + x^5 \\ = 6 - x^2 - 2x^3 - x^4 + x^5$$

$$(e) (x^2 - 3)(x^3 - 3x^2 + 2x - 1) \\ = x^5 - 3x^4 + 2x^3 - x^2 - 3x^3 + 9x^2 - 6x + 3 \\ = x^5 - 3x^4 - x^3 + 8x^2 - 6x + 3$$

$$(f) (1 - x^3)(2 + 3x - 6x^2 + 2x^3 - x^4) \\ = 2 + 3x - 6x^2 + 2x^3 - x^4 - 2x^3 - 3x^4 + 6x^5 - 2x^6 + x^7 \\ = 2 + 3x - 6x^2 - 4x^4 + 6x^5 - 2x^6 + x^7$$

$$6. (a) (x + 3)(x - 1) - (x - 5)(x - 2) \\ = x^2 + 2x - 3 - (x^2 - 7x + 10) \\ = x^2 + 2x - 3 - x^2 + 7x - 10 \\ = 9x - 13$$

$$(b) 2(x + 1)(x - 4) - 4(x - 2)(x + 2) \\ = 2(x^2 - 3x - 4) - 4(x^2 - 4) \\ = 2x^2 - 6x - 8 - 4x^2 + 16 \\ = -2x^2 - 6x + 8$$

$$(c) 3(2 - x)(x - 4) + (x - 1)^2 \\ = 3(-8 + 6x - x^2) + (x^2 - 2x + 1) \\ = -24 + 18x - 3x^2 + x^2 - 2x + 1 \\ = -2x^2 + 16x - 23$$

$$(d) 2(3x - 2)^2 - 4(x - 1)(2x + 5) \\ = 2(9x^2 - 12x + 4) - 4(2x^2 + 3x - 5) \\ = 18x^2 - 24x + 8 - 8x^2 - 12x + 20 \\ = 10x^2 - 36x + 28$$

$$(e) (2x^2 - 1)^2 - (4 - x^2)^2 = (4x^4 - 4x^2 + 1) - (16 - 8x^2 + x^4) \\ = 4x^4 - 4x^2 + 1 - 16 + 8x^2 - x^4 \\ = 3x^4 + 4x^2 - 15$$

$$(f) 2x(5x^2 - 2)^2 - 3x(2 - x^2)^2 \\ = 2x(25x^4 - 20x^2 + 4) - 3x(4 - 4x^2 + x^4) \\ = 50x^5 - 40x^3 + 8x - 12x + 12x^3 - 3x^5 \\ = 47x^5 - 28x^3 - 4x$$

$$7. (a) (x - 1)(x + 3)(x + 1) + (2x - 1)(x - 2) \\ = (x - 1)(x^2 + 4x + 3) + (2x^2 - 5x + 2) \\ = x^3 + 4x^2 + 3x - x^2 - 4x - 3 + 2x^2 - 5x + 2 \\ = x^3 + 5x^2 - 6x - 1$$

$$(b) (x - 3)(x + 2)(x + 3) - (3 - x)^2 \\ = (x - 3)(x^2 + 5x + 6) - (9 - 6x + x^2) \\ = x^3 + 5x^2 + 6x - 3x^2 - 15x - 18 - 9 + 6x - x^2 \\ = x^3 + x^2 - 3x - 27$$

$$(c) (2a + 7)^2(a - 2) + 2(a^2 - 1)(4a + 3) \\ = (4a^2 + 28a + 49)(a - 2) + 2(4a^3 + 3a^2 - 4a - 3) \\ = 4a^3 - 8a^2 + 28a^2 - 56a + 49a - 98 + 8a^3 + 6a^2 - 8a - 6 \\ = 12a^3 + 26a^2 - 15a - 104$$

$$(d) (t - 1)(2t - 1)(3t + 4) - (1 - t)^2(t + 4) \\ = (t - 1)(6t^2 + 5t - 4) - (1 - 2t + t^2)(t + 4) \\ = 6t^3 + 5t^2 - 4t - 6t^2 - 5t + 4 - (t + 4 - 2t^2 - 8t + t^3 + 4t^2) \\ = 6t^3 - t^2 - 9t + 4 - (t^3 + 2t^2 - 7t + 4) \\ = 6t^3 - t^2 - 9t + 4 - t^3 - 2t^2 + 7t - 4 \\ = 5t^3 - 3t^2 - 2t$$

$$(e) 3(x - 2)(x + 3)^2 - (3x - 1)(2x + 1)^2 \\ = 3(x - 2)(x^2 + 6x + 9) - (3x - 1)(4x^2 + 4x + 1) \\ = 3(x^3 + 6x^2 + 9x - 2x^2 - 12x - 18) - (12x^3 + 12x^2 + 3x - 4x^2 - 4x - 1) \\ = 3(x^3 + 4x^2 - 3x - 18) - (12x^3 + 8x^2 - x - 1) \\ = 3x^3 + 12x^2 - 9x - 54 - 12x^3 - 8x^2 + x + 1 \\ = -9x^3 + 4x^2 - 8x - 53$$

$$(f) (x - 1)^2(x + 3)^2 - (x + 1)^2(x - 2)^2 \\ = (x^2 - 2x + 1)(x^2 + 6x + 9) - (x^2 + 2x + 1)(x^2 - 4x + 4) \\ = x^4 + 6x^3 + 9x^2 - 2x^3 - 12x - 18x + x^2 + 6x + 9 - (x^4 - 4x^3 + 4x^2 + 2x^3 - 8x^2 + 8x + x^2 - 4x + 4) \\ = x^4 + 4x^3 - 2x^2 - 12x + 9 - (x^4 - 2x^3 - 3x^2 + 4x + 4) \\ = x^4 + 4x^3 - 2x^2 - 12x + 9 - x^4 + 2x^3 + 3x^2 - 4x - 4 \\ = 6x^3 + x^2 - 16x + 5$$

$$8. (a) (x^2 - 3x + 5) - (2x^2 - 5x - 7) \\ = x^2 - 3x + 5 - 2x^2 + 5x + 7 \\ = -x^2 + 2x + 12$$

$$(b) x(2x - 3)^2 - 2x(x - 4) \\ = x(4x^2 - 12x + 9) - 2x^2 + 8x \\ = 4x^3 - 12x^2 + 9x - 2x^2 + 8x \\ = 4x^3 - 14x^2 + 17x$$

$$(c) (1 - x + x^2 - x^3 - x^4)(1 + x) \\ = 1 - x + x^2 - x^3 - x^4 + x - x^2 + x^3 - x^4 - x^5 \\ = 1 - 2x^4 - x^5$$

$$9. (x + 1)^2 - (x + 2)(x - 3) = (x^2 + 2x + 1) - (x^2 - x - 6) \\ = x^2 + 2x + 1 - x^2 + x + 6 \\ = 3x + 7$$

(where Jake was incorrect)

Use brackets around the second product of binomials to avoid Jake's error.

$$10. (a) (2a - 3)(a^3 - 2a^2 + a - 1) \\ = 2a^4 - 4a^3 + 2a^2 - 2a - 3a^3 + 6a^2 - 3a + 3 \\ = 2a^4 - 7a^3 + 8a^2 - 5a + 3$$

$$(b) (y^2 + 3y - 5)(2y^2 - y + 6) \\ = 2y^4 - y^3 + 6y^2 + 6y^3 - 3y^2 + 18y - 10y^2 + 5y - 30 \\ = 2y^4 + 5y^3 - 7y^2 + 23y - 30$$

$$(c) (x^2 - x + 1)^2 = (x^2 - x + 1)(x^2 - x + 1) \\ = x^4 - x^3 + x^2 - x^3 + x^2 - x + x^2 - x + 1 \\ = x^4 - 2x^3 + 3x^2 - 2x + 1$$

$$(d) (x - 1)^2(x + 3)^2 = (x^2 - 2x + 1)(x^2 + 6x + 9) \\ = x^4 + 6x^3 + 9x^2 - 2x^3 - 12x^2 - 18x + x^2 + 6x + 9 \\ = x^4 + 4x^3 - 2x^2 - 12x + 9$$

11.

Product of binomials – Product of a monomial and a trinomial

$$= (x + 1)(x - 3) - 3x(x^2 - 4x + 2) \\ = x^2 - 2x - 3 - 3x^3 + 12x^2 - 6x \\ = -3x^3 + 13x^2 - 8x - 3$$

The resulting polynomial has degree 3.

$$12. (a) (a^2 + 2a - 1)^2 + (a^2 - 2a + 1)^2 \\ = (a^2 + 2a - 1)(a^2 + 2a - 1) + (a^2 - 2a + 1)(a^2 - 2a + 1) \\ = a^4 + 2a^3 - a^2 + 2a^3 + 4a^2 - 2a - a^2 - 2a + 1 + a^4 - 2a^3 + a^2 - 2a^3 - 4a^2 + 2a + 1 \\ = 2a^4 + 8a^2 - 8a + 2$$

$$(b) (a^2 - ab + b^2)^2 - (a^2 - 2ab + b^2)^2$$

This is a difference of squares.

$$= [(a^2 - ab + b^2) + (a^2 - 2ab + b^2)][(a^2 - ab + b^2) - (a^2 - 2ab + b^2)] \\ = (a^2 - ab + b^2 + a^2 - 2ab + b^2)(a^2 - ab + b^2 - a^2 + 2ab - b^2) \\ = (2a^2 - 3ab + 2b^2)(ab) \\ = 2a^3b - 3a^2b^2 + 2ab^3$$

13. Let the five consecutive integers be $x, x + 1, x + 2, x + 3, x + 4$, where x is an integer.

(a) The square of the middle integer is $(x + 2)^2 = x^2 + 4x + 4$. The product of the first and fifth integers is $x(x + 4) = x^2 + 4x$. Since $x^2 + 4x + 4$ is 4 more than $x^2 + 4x$, the square of the middle integer is 4 more than the product of the first and fifth integers.

(b) The product of the second and fourth integers is $(x + 1)(x + 3) = x^2 + 4x + 3$. Since $x^2 + 4x + 3$ is 3 more than $x^2 + 4x$, the product of the second and fourth integers is always 3 more than the product of the first and fifth integers.

14. $x + y + z = 1$ and $x^2 + y^2 + z^2 = 3$

$$\begin{aligned}(x + y + z)^2 &= (x + y + z)(x + y + z) \\ &= x^2 + xy + xz + yx + y^2 + yz + zx + zy + z^2 \\ &= x^2 + y^2 + z^2 + 2xy + 2xz + 2yz\end{aligned}$$

$$\text{But } (x + y + z)^2 = 1^2 = 1$$

$$\text{and } x^2 + y^2 + z^2 = 3$$

$$\therefore 1 = 3 + 2xy + 2xz + 2yz$$

$$1 - 3 = 2(xy + xz + yz)$$

$$-2 = 2(xy + xz + yz)$$

$$-1 = xy + xz + yz$$

$$\begin{aligned}15. (x^2 - 4x + 8)(ax^2 + bx + c) \\ &= ax^4 + bx^3 + cx^2 - 4ax^3 - 4bx^2 - 4cx + 8ax^2 + 8bx + 8c \\ &= ax^4 + (b - 4a)x^3 + (c - 4b + 8a)x^2 + (8b - 4c)x + 8c\end{aligned}$$

If this is equal to $5x^4 - 14x^3 + 18x^2 + 40x + 16$ then, comparing coefficients:

$$a = 5 \text{ ----- ①}$$

$$b - 4a = -14 \text{ ----- ②}$$

$$c - 4b + 8a = 18 \text{ ----- ③}$$

$$8b - 4c = 40 \text{ ----- ④}$$

$$8c = 16 \text{ ----- ⑤}$$

$$\text{From ①, } a = 5$$

$$\text{⑤, } c = 2$$

$$\text{④, } 8b - 4(2) = 40$$

$$8b = 48$$

$$b = 6$$

$$\therefore a = 5, b = 6, c = 2$$

$$\text{If } 5x^4 + 18x^2 + 16 = 14x^3 - 40x,$$

$$5x^4 - 14x^3 + 18x^2 + 40x + 16 = 0, \text{ and using the above,}$$

$$(x^2 - 4x + 8)(5x^2 + 6x + 2) = 0$$

$$\text{so } x^2 - 4x + 8 = 0 \text{ or } 5x^2 + 6x + 2 = 0$$

Using the quadratic formula,

$$x^2 - 4x + 8 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 32}}{2}$$

$$= \frac{4 \pm \sqrt{-16}}{2}$$

$$= \frac{4 \pm \sqrt{16i^2}}{2}$$

$$= \frac{4 \pm 4i}{2}$$

$$x = 2 + 2i, 2 - 2i$$

$$5x^2 + 6x + 2 = 0$$

$$x = \frac{-6 \pm \sqrt{36 - 40}}{10}$$

$$= \frac{-6 \pm \sqrt{-4}}{10}$$

$$= \frac{-6 \pm \sqrt{4i^2}}{10}$$

$$= \frac{-6 \pm 2i}{10}$$

$$x = -0.6 + 0.2i, -0.6 - 0.2i$$

$$\therefore x = 2 + 2i, 2 - 2i, -0.6 + 0.2i, \text{ or } -0.6 - 0.2i$$

$$16. x^3 - 1 = (x - 1)(ax^2 + bx + c)$$

$$RS = ax^3 + bx^2 + cx - ax^2 - bx - c$$

$$= ax^3 + (b - a)x^2 + (c - b)x - c$$

If this is the same as $x^3 - 1$, then comparing coefficients:

$$a = 1$$

$$b - a = 0$$

$$c - b = 0$$

$$-c = -1$$

From this, we see that $a = 1, b = a = 1, c = 1$

$$\text{So } x^3 - 1 = (x - 1)(x^2 + x + 1)$$

$$\text{If } x^3 = 1,$$

$$x^3 - 1 = 0$$

$$(x - 1)(x^2 + x + 1) = 0$$

$$x = 1 \text{ or } x^2 + x + 1 = 0$$

Using the quadratic formula,

$$x = \frac{-1 \pm \sqrt{1 - 4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm \sqrt{3i^2}}{2}$$

$$= \frac{-1 \pm \sqrt{3}i}{2}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

The three roots are $x = 1, -0.50 + 0.87i, -0.50 - 0.87i$ (to 2 decimal places).

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1. The vertex form is the form in which the vertex coordinates are clearly visible and so is the maximum or minimum value of the function. $f(x) = a(x - h)^2 + k$ is in vertex form. The vertex is at (h, k) and the maximum or minimum value of this function is k . A quadratic function in standard form, $f(x) = ax^2 + bx + c$ may be put into vertex form by completing the square. The third form for a quadratic function is factored form $f(x) = a(x - p)(x - q)$.

Examples: factored form: $f(x) = 2(x - 1)(x + 4)$

standard form: $f(x) = 3x^2 - 2x + 1$

vertex form: $f(x) = -2(x - 3)^2 + 4$