

13. Let the five consecutive integers be $x, x + 1, x + 2, x + 3, x + 4$, where x is an integer.

(a) The square of the middle integer is $(x + 2)^2 = x^2 + 4x + 4$. The product of the first and fifth integers is $x(x + 4) = x^2 + 4x$. Since $x^2 + 4x + 4$ is 4 more than $x^2 + 4x$, the square of the middle integer is 4 more than the product of the first and fifth integers.

(b) The product of the second and fourth integers is $(x + 1)(x + 3) = x^2 + 4x + 3$. Since $x^2 + 4x + 3$ is 3 more than $x^2 + 4x$, the product of the second and fourth integers is always 3 more than the product of the first and fifth integers.

14. $x + y + z = 1$ and $x^2 + y^2 + z^2 = 3$

$$\begin{aligned}(x + y + z)^2 &= (x + y + z)(x + y + z) \\ &= x^2 + xy + xz + yx + y^2 + yz + zx + zy + z^2 \\ &= x^2 + y^2 + z^2 + 2xy + 2xz + 2yz\end{aligned}$$

$$\text{But } (x + y + z)^2 = 1^2 = 1$$

$$\text{and } x^2 + y^2 + z^2 = 3$$

$$\therefore 1 = 3 + 2xy + 2xz + 2yz$$

$$1 - 3 = 2(xy + xz + yz)$$

$$-2 = 2(xy + xz + yz)$$

$$-1 = xy + xz + yz$$

$$\begin{aligned}15. (x^2 - 4x + 8)(ax^2 + bx + c) \\ &= ax^4 + bx^3 + cx^2 - 4ax^3 - 4bx^2 - 4cx + 8ax^2 + 8bx + 8c \\ &= ax^4 + (b - 4a)x^3 + (c - 4b + 8a)x^2 + (8b - 4c)x + 8c\end{aligned}$$

If this is equal to $5x^4 - 14x^3 + 18x^2 + 40x + 16$ then, comparing coefficients:

$$a = 5 \text{ ----- ①}$$

$$b - 4a = -14 \text{ ----- ②}$$

$$c - 4b + 8a = 18 \text{ ----- ③}$$

$$8b - 4c = 40 \text{ ----- ④}$$

$$8c = 16 \text{ ----- ⑤}$$

$$\text{From ①, } a = 5$$

$$\text{⑤, } c = 2$$

$$\text{④, } 8b - 4(2) = 40$$

$$8b = 48$$

$$b = 6$$

$$\therefore a = 5, b = 6, c = 2$$

$$\text{If } 5x^4 + 18x^2 + 16 = 14x^3 - 40x,$$

$$5x^4 - 14x^3 + 18x^2 + 40x + 16 = 0, \text{ and using the above,}$$

$$(x^2 - 4x + 8)(5x^2 + 6x + 2) = 0$$

$$\text{so } x^2 - 4x + 8 = 0 \text{ or } 5x^2 + 6x + 2 = 0$$

Using the quadratic formula,

$$x^2 - 4x + 8 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 32}}{2}$$

$$= \frac{4 \pm \sqrt{-16}}{2}$$

$$= \frac{4 \pm \sqrt{16i^2}}{2}$$

$$= \frac{4 \pm 4i}{2}$$

$$x = 2 + 2i, 2 - 2i$$

$$5x^2 + 6x + 2 = 0$$

$$x = \frac{-6 \pm \sqrt{36 - 40}}{10}$$

$$= \frac{-6 \pm \sqrt{-4}}{10}$$

$$= \frac{-6 \pm \sqrt{4i^2}}{10}$$

$$= \frac{-6 \pm 2i}{10}$$

$$x = -0.6 + 0.2i, -0.6 - 0.2i$$

$$\therefore x = 2 + 2i, 2 - 2i, -0.6 + 0.2i, \text{ or } -0.6 - 0.2i$$

$$16. x^3 - 1 = (x - 1)(ax^2 + bx + c)$$

$$RS = ax^3 + bx^2 + cx - ax^2 - bx - c$$

$$= ax^3 + (b - a)x^2 + (c - b)x - c$$

If this is the same as $x^3 - 1$, then comparing coefficients:

$$a = 1$$

$$b - a = 0$$

$$c - b = 0$$

$$-c = -1$$

From this, we see that $a = 1, b = a = 1, c = 1$

$$\text{So } x^3 - 1 = (x - 1)(x^2 + x + 1)$$

$$\text{If } x^3 = 1,$$

$$x^3 - 1 = 0$$

$$(x - 1)(x^2 + x + 1) = 0$$

$$x = 1 \text{ or } x^2 + x + 1 = 0$$

Using the quadratic formula,

$$x = \frac{-1 \pm \sqrt{1 - 4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm \sqrt{3i^2}}{2}$$

$$= \frac{-1 \pm \sqrt{3}i}{2}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

The three roots are $x = 1, -0.50 + 0.87i, -0.50 - 0.87i$ (to 2 decimal places).

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1. The vertex form is the form in which the vertex coordinates are clearly visible and so is the maximum or minimum value of the function. $f(x) = a(x - h)^2 + k$ is in vertex form. The vertex is at (h, k) and the maximum or minimum value of this function is k . A quadratic function in standard form, $f(x) = ax^2 + bx + c$ may be put into vertex form by completing the square. The third form for a quadratic function is factored form $f(x) = a(x - p)(x - q)$.

Examples: factored form: $f(x) = 2(x - 1)(x + 4)$

standard form: $f(x) = 3x^2 - 2x + 1$

vertex form: $f(x) = -2(x - 3)^2 + 4$

2. To complete the square for $f(x) = -2x^2 + 6x + 7$

(i) Factor the coefficient of x^2 from the first two terms:

$$f(x) = -2(x^2 - 3x) + 7$$

(ii) Add and subtract the square of half the coefficient of x :

$$f(x) = -2\left(x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right) + 7$$

(iii) Move the fourth term out of the bracket:

$$f(x) = -2\left(x^2 - 3x + \left(\frac{3}{2}\right)^2\right) + 2\left(\frac{3}{2}\right)^2 + 7$$

(iv) Factor the perfect square:

$$f(x) = -2\left(x - \frac{3}{2}\right)^2 + \frac{9}{2} + 7$$

(v) Simplify:

$$f(x) = -2(x - 1.5)^2 + 11.5$$

$$\begin{aligned} 3. (a) f(x) &= x^2 - 6x + 7 \\ &= (x^2 - 6x + 9 - 9) + 7 \\ &= (x^2 - 6x + 9) - 9 + 7 \\ f(x) &= (x - 3)^2 - 2 \end{aligned}$$

(b)

$$\begin{aligned} f(x) &= x^2 - 5x + 1 \\ &= \left(x^2 - 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2\right) + 1 \\ &= \left(x^2 - 5x + \left(\frac{5}{2}\right)^2\right) - \left(\frac{5}{2}\right)^2 + 1 \\ &= \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 1 \\ &= (x - 2.5)^2 - 5.25 \end{aligned}$$

(c)

$$\begin{aligned} f(x) &= 2x^2 + 7x - 3 \\ &= 2\left(x^2 + \frac{7}{2}x\right) - 3 \\ &= 2\left(x^2 + \frac{7}{2}x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right) - 3 \\ &= 2\left(x^2 + \frac{7}{2}x + \left(\frac{7}{4}\right)^2\right) - 2\left(\frac{7}{4}\right)^2 - 3 \\ &= 2\left(x + \frac{7}{4}\right)^2 - \frac{49}{8} - 3 \end{aligned}$$

$$f(x) = 2(x + 1.75)^2 - 9.125$$

(d)

$$\begin{aligned} f(x) &= -4x^2 - 3x + 2 \\ &= -4\left(x^2 + \frac{3}{4}x\right) + 2 \\ &= -4\left(x^2 + \frac{3}{4}x + \left(\frac{3}{8}\right)^2 - \left(\frac{3}{8}\right)^2\right) + 2 \\ &= -4\left(x^2 + \frac{3}{4}x + \left(\frac{3}{8}\right)^2\right) + 4\left(\frac{9}{64}\right) + 2 \\ &= -4(x + 0.375)^2 + 2.5625 \end{aligned}$$

(e)

$$\begin{aligned} f(x) &= -1.6x^2 - 2.4x - 1 \\ &= -1.6(x^2 + 1.5x) - 1 \\ &= -1.6(x^2 + 1.5x + (0.75)^2 - (0.75)^2) - 1 \\ &= -1.6(x^2 + 1.5x + (0.75)^2) + (1.6)(0.75)^2 - 1 \\ &= -1.6(x + 0.75)^2 - 0.1 \end{aligned}$$

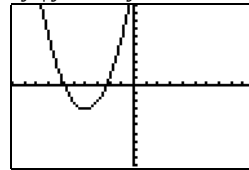
(f)

$$\begin{aligned} f(x) &= \frac{2}{3}x^2 - \frac{4}{9}x + 2 \\ &= \frac{2}{3}\left(x^2 - \frac{2}{3}x\right) + 2 \\ &= \frac{2}{3}\left(x^2 - \frac{2}{3}x + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right) + 2 \\ &= \frac{2}{3}\left(x^2 - \frac{2}{3}x + \left(\frac{1}{3}\right)^2\right) - \frac{2}{3}\left(\frac{1}{9}\right) + 2 \\ &= \frac{2}{3}\left(x - \frac{1}{3}\right)^2 + \frac{52}{27} \end{aligned}$$

$$\begin{aligned} 4. (a) f(x) &= x^2 + 8x + 13 \\ &= x^2 + 8x + 16 - 16 + 13 \\ &= x^2 + 8x + 16 - 3 \\ &= (x + 4)^2 - 3 \end{aligned}$$

Vertex is at $(-4, -3)$. Domain is $\mathbf{R}$. Range is

$$\{y \mid y \in \mathbf{R}, y \geq -3\}$$



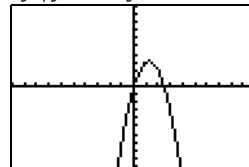
(b)

$$\begin{aligned} f(x) &= -2x^2 + 5x \\ &= -2\left(x^2 - \frac{5}{2}x\right) \\ &= -2\left(x^2 - \frac{5}{2}x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right) \\ &= -2\left(x^2 - \frac{5}{2}x + \left(\frac{5}{4}\right)^2\right) + 2\left(\frac{5}{4}\right)^2 \end{aligned}$$

$$f(x) = -2(x - 1.25)^2 + 3.125$$

Vertex is at $(1.25, 3.125)$. Domain is $\mathbf{R}$. Range is

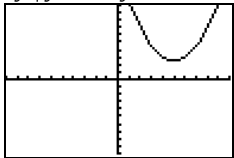
$$\{y \mid y \in \mathbf{R}, y \leq 3.125\}$$



(c)

$$\begin{aligned}
 f(x) &= \frac{1}{2}x^2 - 5x + 15 \\
 &= \frac{1}{2}(x^2 - 10x) + 15 \\
 &= \frac{1}{2}(x^2 - 10x + 25 - 25) + 15 \\
 &= \frac{1}{2}(x^2 - 10x + 25) - \frac{1}{2}(25) + 15 \\
 f(x) &= \frac{1}{2}(x - 5)^2 + 2.5
 \end{aligned}$$

Vertex is at (5, 2.5). Domain is $\mathbf{R}$. Range is $\{y \mid y \in \mathbf{R}, y \geq 2.5\}$



$$\begin{aligned}
 5. \quad h(t) &= 0.8 + 29.4t - 4.9t^2 \\
 &= -4.9(t^2 - 6t) + 0.8 \\
 &= -4.9(t^2 - 6t + 9 - 9) + 0.8 \\
 &= -4.9(t^2 - 6t + 9) + 4.9(9) + 0.8 \\
 h(t) &= -4.9(t - 3)^2 + 44.9
 \end{aligned}$$

Maximum height reached by the ball is 44.9 metres. The ball reaches maximum height when $t = 3$, ie. 3 seconds after it is struck. At the instant the ball was hit, $t = 0$.

$$h(0) = 0.8 + 29.4(0) - 4.9(0)^2$$

The ball was 0.8 metres above the ground when it was hit.

6. (a) The y-intercept and direction of opening are clear from a quadratic equation in standard form.

(b) The x-intercepts – roots of the function – as well as the direction of opening are clear from the factored form of a quadratic function.

(c) The coordinates of the vertex and the direction of opening are clear from a quadratic function written in vertex form.

Examples:

(a) $f(x) = -3x^2 + 2x + 7$ is in standard form. Since $a < 0$ the graph will open down. The y-intercept is (0, 7).

(b) $f(x) = 3(x - 2)(x + 1)$ is in factored form. Since $a > 0$ this graph opens up. It has x-intercepts (2, 0) and (-1, 0), so the roots of the function are 2 and -1.

(c) $f(x) = -2(x + 4)^2 - 3$ is in vertex form. Since $a < 0$ the graph opens down. The vertex is at (-4, -3). The function has a maximum value of -3. This value occurs when $x = -4$.

7. If the graph of $f(x) = ax^2 + bx$ is translated vertically c units, the new graph represents the function

$f(x) = ax^2 + bx + c$. It is easy to put $f(x) = ax^2 + bx$ into factored form $f(x) = x(ax + b)$. In this form, the x-intercepts 0 and $-\frac{b}{a}$ are easily obtained. The x-coordinate of the vertex is

halfway between these intercepts, because of the symmetry of the quadratic function. The maximum or minimum value of the function may then be determined by using this x-coordinate in the function equation. Since the graph of $f(x) = ax^2 + bx + c$ is simply a vertical translation of

$f(x) = ax^2 + bx$, the x-coordinate of the vertex will be the same for both graphs. The maximum or minimum value of $f(x) = ax^2 + bx + c$ is then found by substituting this x-value in the function equation as before.

8. The value of a in the function equation, $f(x) = ax^2 + bx + c$, $f(x) = a(x - p)(x - q)$ or $f(x) = a(x - h)^2 + k$, indicates whether the function has a maximum or a minimum value. If $a > 0$, the graph opens up and the function has a minimum value. If $a < 0$, the graph opens down and the function has a maximum value.

(a) $f(x) = -2(x - 1)^2 + 6$

i. Since $a < 0$, $f(x)$ has a maximum value.

ii. x-coordinate of the vertex is 1. This is the x-value that produces the maximum value.

iii. Maximum value is the y-coordinate of the vertex. This is 6.

(b) $f(x) = (x - 3)(x + 7)$

i. Since $a > 0$, $f(x)$ has a minimum value.

ii. x-coordinate of the vertex is halfway between the intercepts -7 and 3. This is -2.

iii. Minimum value is $f(-2) = (-2 - 3)(-2 + 7) = -25$

(c) $h(t) = -5t^2 + 20t$

i. Since $a < 0$, $h(t)$ has a maximum value.

ii. $t = \frac{-b}{2a}$ is the t-coordinate of the vertex.

$$t = \frac{-20}{-10} = 2$$

$t = 2$ is the value of t that produces the maximum value.

iii. Maximum value for $h(t)$ is $h(2) = -5(2^2) + 20(2) = 20$

(d) $g(x) = -0.1(3.2 - x)(2.1 + x)$

i. Since $a < 0$, $g(x)$ has a maximum value.

ii. x-intercepts are -2.1 and 3.2, so x-coordinate of vertex is $\frac{-2.1 + 3.2}{2} = 0.55$. This gives the maximum value.

iii. Maximum value for $g(x)$ is

$$g(0.55) = -0.1(3.2 - 0.55)(2.1 + 0.55) = -0.70225$$

(e) $f(x) = 3.2(x - 1.1)^2 - 4.5$

i. Since $a > 0$, $f(x)$ has a minimum value.

ii. Vertex is (1.1, -4.5). Value of x that produces minimum value is 1.1.

iii. Minimum value is -4.5.

(f) $C(x) = 0.04x^2 + 18x + 198$

i. Since $a > 0$, $C(x)$ has a minimum value.

ii. x-coordinate of vertex is

$$\frac{-b}{2a} = \frac{-18}{0.08} = -225$$

This is the value of x that produces the minimum C .

iii. Minimum C is $C(-225)$.

$$C(-225) = 0.04(-225)^2 + 18(-225) + 198 = -1827$$

9. (a) Revenue = number sold $\times$ price

$$R(x) = x(-5x + 22) = -5x^2 + 22x$$

(b) Profit = Revenue - Cost

$$P(x) = -5x^2 + 22x - (2x + 15)$$

$$= -5x^2 + 22x - 2x - 15$$

$$= -5x^2 + 20x - 15$$

$$\begin{aligned}
 \text{(c)} \quad P(x) &= -5(x^2 - 4x) - 15 \\
 &= -5(x^2 - 4x + 2^2 - 2^2) - 15 \\
 &= -5(x^2 - 4x + 2^2) + 5(2)^2 - 15 \\
 &= -5(x - 2)^2 + 5
 \end{aligned}$$

The value of x that will maximize profits is 2, which represents 2000 items.

(d) The break-even quantities are the zeros of $P(x)$.

$$\begin{aligned}
 \text{If } -5(x - 2)^2 + 5 &= 0 \\
 -5(x - 2)^2 &= -5 \\
 (x - 2)^2 &= 1
 \end{aligned}$$

Taking square roots,

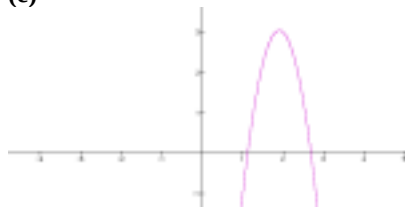
$$x - 2 = \pm 1$$

$$x = 2 \pm 1$$

$$x = 1 \text{ or } 3$$

Break-even quantities are approximately 1000 and 3000 items.

(e)



10. Let the number of empty seats be x . Then the number of passengers is $22 - x$ and the fare charged is $\$(60 + 5x)$. Revenue from fares collected is $\$(60 + 5x)(22 - x)$. Cost to run bus is $\$225 + \$30(22 - x)$.

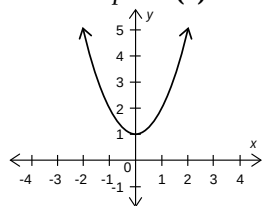
$$\begin{aligned}
 \text{Profit, } P(x) &= R(x) - C(x) \\
 &= (60 + 5x)(22 - x) - [225 + 30(22 - x)] \\
 &= 1320 + 50x - 5x^2 - [225 + 660 - 30x] \\
 &= 1320 + 50x - 5x^2 - [885 - 30x] \\
 &= 1320 + 50x - 5x^2 - 885 + 30x \\
 P(x) &= 435 + 80x - 5x^2 \text{ where } x \text{ is a whole number.}
 \end{aligned}$$

Complete the square to find the vertex.

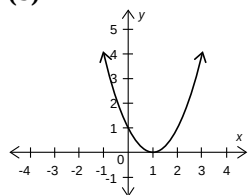
$$\begin{aligned}
 P(x) &= -5(x^2 - 16x) + 435 \\
 &= -5(x^2 - 16x + 64 - 64) + 435 \\
 &= -5(x^2 - 16x + 64) + 5(64) + 435 \\
 &= -5(x - 8)^2 + 755
 \end{aligned}$$

Profit will be at a maximum value when $x = 8$. (A profit of $\$755$ on the trip.) The bus should run with 8 empty seats to maximize the profit.

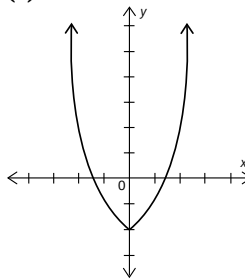
11. Examples: (a)



(b)



(c)



The zeros of a function are the x -values that make the function equal to zero. On the graph, these are the x -intercepts.

- If a graph does not cross the x -axis, it is the graph of a function with no zeros. Graph (a) is such a function.
- If a graph just touches the x -axis in one point, it is a graph of a function with exactly one zero. Graph (b) is a function with one zero.
- If a graph crosses the x -axis in two places, like graph (c), it represents a function with two zeros.

12. The discriminant, for a function $f(x) = ax^2 + bx + c$ is the value of $b^2 - 4ac$. This is the part of the quadratic function which lies inside the square root.

- If $b^2 - 4ac < 0$, the function has no zeros.
- If $b^2 - 4ac = 0$, the function has exactly one zero.
- If $b^2 - 4ac > 0$, the function has two zeros.

$$\begin{aligned}
 \text{(a)} \quad f(x) &= 2x^2 + 3x + 5 \text{ has no zeros, since} \\
 b^2 - 4ac &= 3^2 - 4(2)(5) \\
 &= 9 - 40 \\
 &= -31
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad f(x) &= 2x^2 + 4x + 2 \text{ has exactly one zero, since} \\
 b^2 - 4ac &= 4^2 - 4(2)(2) \\
 &= 16 - 16 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad f(x) &= 2x^2 + 5x + 3 \text{ has two zeros, since} \\
 b^2 - 4ac &= 5^2 - 4(2)(3) \\
 &= 25 - 24 \\
 &= 1
 \end{aligned}$$

13. A quadratic function $f(x) = a(x - h)^2 + k$ has

- two zeros if a and k have opposite signs
- one zero if $k = 0$
- no zeros if a and k have the same sign.

(a) $f(x) = 2(x - 3)^2 + 4$ has no zeros, since $a = 2$ and $k = 4$ have the same sign.

(b) $f(x) = 2(x - 3)^2$ has one zero, since $k = 0$.

(c) $f(x) = 2(x - 3)^2 - 4$ has two zeros, since $a = 2$ and $k = -4$ have opposite signs.

14. (a) $f(x) = 3x^2 + 4x + 1$

$$\text{i. } b^2 - 4ac = 4^2 - 4(3)(1) = 4$$

Since $b^2 - 4ac > 0$, $f(x)$ has two zeros.

ii. Graph of $y = f(x)$ intersects the x -axis at two points.

$$\text{(b)} \quad g(x) = -2(x + 3.6)^2 + 4.1$$

$$\text{i. } a = -2, k = 4.1$$

Since a, k have opposite signs $g(x)$ has two zeros.

ii. Graph of $g(x)$ intersects the x -axis at two points.

(c) $h(x) = -5(x + 1.3)^2$

i. Since $k = 0$, $h(x)$ has one zero.

ii. Graph of $h(x)$ touches the x -axis at one point.

(d) $m(x) = 4(x + 1)^2 + 0.5$

i. $a = 4$, $k = 0.5$

Since a and k have the same sign, $m(x)$ has no zeros.

ii. Graph of $m(x)$ does not meet the x -axis.

(e) $p(x) = -x^2 - 6x - 9$

i. $b^2 - 4ac = (-6)^2 - 4(-1)(-9)$

$$= 36 - 36$$

$$= 0$$

$p(x)$ has one zero.

ii. Graph of $p(x)$ touches the x -axis at one point.

(f) $p(x) = -3.2(x + 1.1)(x - 2.4)$

i. This function clearly has two zeros, $x = -1.1$ and $x = 2.4$.

ii. The graph of $y = p(x)$ intersects the x -axis at two points.

15. (a) $f(x) = kx^2 - 8x + 2$

$$b^2 - 4ac = (-8)^2 - 4(k)(2)$$

$$= 64 - 8k$$

i. for two intercepts, $b^2 - 4ac > 0$

$$64 - 8k > 0$$

$$64 > 8k$$

$$8 > k$$

Graph has two intercepts if $k < 8$.

ii. for one intercept, $b^2 - 4ac = 0$

$$64 - 8k = 0$$

$$64 = 8k$$

$$8 = k$$

Graph has one intercept if $k = 8$.

iii. Graph has no intercepts if $k > 8$.

(b) $f(x) = 2x^2 + kx + 50$

$$b^2 - 4ac = k^2 - 4(2)(50)$$

$$= k^2 - 400$$

i. for two intercepts,

$$k^2 - 400 > 0$$

$$k^2 > 400$$

$$k < -20 \text{ or } k > 20$$

ii. for one intercept,

$$k^2 - 400 = 0$$

$$k^2 = 400$$

$$k = \pm 20$$

iii. for no intercepts,

$$-20 < k < 20$$

(c) $f(x) = 9x^2 + kx + 4$

$$b^2 - 4ac = k^2 - 4(9)(4)$$

$$= k^2 - 144$$

i. for two intercepts

$$k^2 - 144 > 0$$

$$k^2 > 144$$

$$k < -12 \text{ or } k > 12$$

ii. for one intercept

$$k^2 - 144 = 0$$

$$k = \pm 12$$

iii. for no intercepts,

$$-12 < k < 12$$

(d) $f(x) = x^2 + (k + 1)x + 1$

$$b^2 - 4ac = (k + 1)^2 - 4(1)(1)$$

$$= k^2 + 2k + 1 - 4$$

$$= k^2 + 2k - 3$$

$$= (k + 3)(k - 1)$$

i. for two intercepts,

$$(k + 3)(k - 1) > 0$$

$$k + 3 > 0 \text{ and } k - 1 > 0$$

$$k > -3 \text{ and } k > 1, \text{ so } k > 1$$

$$\text{OR } k + 3 < 0 \text{ and } k - 1 < 0$$

$$k < -3 \text{ and } k < 1, \text{ so } k < -3$$

Graph has two intercepts if $k < -3$ or if $k > 1$.

ii. for one intercept,

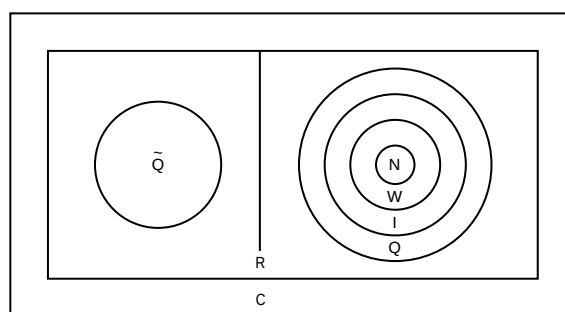
$$(k + 3)(k - 1) = 0$$

$$k = -3 \text{ or } k = 1$$

iii. for no intercepts

$$-3 < k < 1$$

16.



17. Unlike real numbers, complex numbers are made up of two parts: a real part and an imaginary part. The real part can be any real number, and the imaginary part is any real multiple of the imaginary number, i , which is the number such that $i^2 = -1$. A complex number takes the form $a + bi$, where a, b are real. Complex numbers are needed to provide solutions for quadratic equations that do not have any real roots. They may also be used to represent quantities that have two components.

18. If a quadratic equation has no real roots, it has two complex conjugate roots in the form $a + bi$, $a - bi$, where a and b are real. This will occur if the discriminant $b^2 - 4ac$ is negative.

19. (a) 0.85 is rational (**Q**), real (**R**) and complex (**C**).

(b) -7 is an integer (**I**), rational (**Q**), real (**R**) and complex (**C**).

(c) $\sqrt{-3}$ is complex (**C**).

(d) $\frac{5}{9}$ is rational (**Q**), real (**R**) and complex (**C**).

(e) $2 + 5i$ is complex (**C**).

(f) $\sqrt{5}$ is irrational (**Q**), real (**R**) and complex (**C**).

(g) $2.\overline{45}$ is rational (**Q**), real (**R**) and complex (**C**).

(h) $-\frac{3}{7}$ is rational (**Q**), real (**R**) and complex (**C**).

(i) 0 is whole (**W**), an integer (**I**), rational (**Q**), real (**R**) and complex (**C**).

(j) $\sqrt{9}$ is natural (**N**), whole (**W**), an integer (**I**), rational (**Q**), real (**R**) and complex (**C**).

20. (a) $2 + i$

(b) $1 - 3i$

(c) $-4 + 5i$

(d) $-5 - 6i$

(e) $13 + 13i$

(f) $3 - \sqrt{3}i$

21. (a)

$$x^2 + 81 = 0$$

$$x^2 = -81$$

$$x = \pm\sqrt{-81}$$

$$= \pm\sqrt{81i^2}$$

$$x = \pm 9i$$

(b)

$$-2x^2 = 72$$

$$x^2 = -36$$

$$x = \pm\sqrt{-36}$$

$$= \pm\sqrt{36i^2}$$

$$x = \pm 6i$$

(c)

$$x^2 - 2x + 3 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-8}}{2}$$

$$= \frac{2 \pm \sqrt{8i^2}}{2}$$

$$= \frac{2 \pm 2\sqrt{2}i}{2}$$

$$= 1 \pm \sqrt{2}i$$

$$x = 1 + \sqrt{2}i \text{ or } 1 - \sqrt{2}i$$

$$\approx 1 + 1.41i \text{ or } 1 - 1.41i \text{ (to 2 decimal places)}$$

(d)

$$x^2 - 4x + 20 = 0$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(20)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{-64}}{2}$$

$$= \frac{4 \pm \sqrt{64i^2}}{2}$$

$$= \frac{4 \pm 8i}{2}$$

$$= 2 \pm 4i$$

$$x = 2 + 4i \text{ or } 2 - 4i$$

(e)

$$2x^2 = 3x - 2$$

$$2x^2 - 3x + 2 = 0$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(2)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{-7}}{4}$$

$$= \frac{3 \pm \sqrt{7i^2}}{4}$$

$$x = \frac{3 + \sqrt{7i^2}}{4}, \frac{3 - \sqrt{7i^2}}{4}$$

(f)

$$3x(x + 4) = -14$$

$$3x^2 + 12x + 14 = 0$$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(3)(14)}}{2(3)}$$

$$= \frac{-12 \pm \sqrt{-24}}{6}$$

$$= \frac{-12 \pm \sqrt{-24i^2}}{6}$$

$$= \frac{-12 \pm 2\sqrt{6}i}{6}$$

$$= \frac{-6 \pm \sqrt{6}i}{3}$$

$$x = \frac{-6 + \sqrt{6}i}{3}, \frac{-6 - \sqrt{6}i}{3}$$

22. To add two complex numbers, you add the real parts and then add the imaginary parts separately. For example,

$$(2 - 3i) + (-1 + 6i) = (2 + (-1)) + (-3 + 6)i$$

$$= 1 + 3i$$

If you add two complex conjugates, the answer is a real number. The imaginary part is zero. For example,

$$(-3 + 2i) + (-3 - 2i) = (-3 + (-3)) + (2 + (-2))i$$

$$= -6 + 0i$$

$$= -6$$

23. Use the distributive property when multiplying complex numbers. For example,

$$(1 - 2i)(3 + i) = 3 + i - 6i - 2i^2$$

$$= 3 - 5i - 2(-1)$$

$$= 3 - 5i + 2$$

$$= 5 - 5i$$

When you multiply complex conjugates, the result is a real number. The imaginary part is zero. For example,

$$(-2 + 3i)(-2 - 3i) = 4 - 9i^2 \text{ (results in a difference of squares)}$$

$$= 4 - 9(-1)$$

$$= 13 \text{ or } 13 + 0i$$

24. (a) $(3i)^2 = 9i^2$

$$= 9(-1)$$

$$= -9$$

$$\begin{aligned}
 \text{(b)} \quad i^5 &= i^4(i) \\
 &= (i^2)^2(i) \\
 &= (-1)^2(i) \\
 &= 1i \\
 &= i
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad (2-i)(5+2i) &= 10 - i - 2i^2 \\
 &= 10 - i - 2(-1) \\
 &= 10 - i + 2 \\
 &= 12 - i
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad (-3-4i)(2-3i) &= -6 + i + 12i^2 \\
 &= -6 + i + 12(-1) \\
 &= -6 + i - 12 \\
 &= -18 + i
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad (-1+7i)(-1-7i) &= (-1)^2 - 49i^2 \\
 &= 1 - 49(-1) \\
 &= 1 + 49 \\
 &= 50
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad -i(3-5i) &= -3i + 5i^2 \\
 &= -3i + 5(-1) \\
 &= -3i - 5 \\
 &= -5 - 3i
 \end{aligned}$$

$$\begin{aligned}
 \text{(g)} \quad (-1+4i)^2 &= 1 - 8i + 16i^2 \\
 &= 1 - 8i + 16(-1) \\
 &= 1 - 8i - 16 \\
 &= -15 - 8i
 \end{aligned}$$

$$\begin{aligned}
 \text{(h)} \quad \frac{1}{(2+i)(2-i)} &= \frac{1}{4-i^2} \\
 &= \frac{1}{4-(-1)} \\
 &= \frac{1}{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{(i)} \quad (7-2i)^2 - (2-3i)^2 + 2i \\
 &= 49 - 28i + 4i^2 - (4 - 12i + 9i^2) + 2i \\
 &= 49 - 28i + 4(-1) + 4 + 12i - 9(-1) + 2i \\
 &= 49 - 28i - 4 + 4 + 12i + 9 + 2i \\
 &= 58 - 14i
 \end{aligned}$$

$$\begin{aligned}
 \text{(j)} \quad (5+3i) + (8-2i) - (6-3i) \\
 &= (5+8-6) + (3-2+3)i \\
 &= 7 + 4i
 \end{aligned}$$

$$\begin{aligned}
 \text{25. (a)} \quad z &= 3 - i \\
 z + \bar{z} &= 3 - i + 3 + i \\
 &= 6 \\
 z - \bar{z} &= (3 - i) - (3 + i) \\
 &= 3 - i - 3 - i \\
 &= -2i \\
 z\bar{z} &= (3 - i)(3 + i) \\
 &= 9 - i^2 \\
 &= 9 - (-1) \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad z &= 3 + 4i \\
 z + \bar{z} &= 3 + 4i + 3 - 4i \\
 &= 6 \\
 z - \bar{z} &= (3 + 4i) - (3 - 4i) \\
 &= 3 + 4i - 3 + 4i \\
 &= 8i
 \end{aligned}$$

$$\begin{aligned}
 z\bar{z} &= (3 + 4i)(3 - 4i) \\
 &= 9 - 16i^2 \\
 &= 9 - 16(-1) \\
 &= 25
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad z &= -5 + 2i \\
 z + \bar{z} &= -5 + 2i + (-5 - 2i) \\
 &= -10 \\
 z - \bar{z} &= (-5 + 2i) - (-5 - 2i) \\
 &= -5 + 2i + 5 + 2i \\
 &= 4i \\
 z\bar{z} &= (-5 + 2i)(-5 - 2i) \\
 &= 25 - 4i^2 \\
 &= 25 - 4(-1) \\
 &= 29
 \end{aligned}$$

$$\begin{aligned}
 \text{26.} \quad f(x) &= -2x^2 - x + 2 \\
 f(3-2i) &= -2(3-2i)^2 - (3-2i) + 2 \\
 &= -2(9 - 12i + 4i^2) - 3 + 2i + 2 \\
 &= -2(9 - 12i + 4(-1)) - 1 + 2i \\
 &= -2(5 - 12i) - 1 + 2i \\
 &= -10 + 24i - 1 + 2i \\
 &= -11 + 26i
 \end{aligned}$$

27. Roots come in conjugate pairs.

(a) If $3i$ is one root, the other root is $-3i$. Equation is:

$$\begin{aligned}
 (x-3i)(x+3i) &= 0 \\
 x^2 - 9i^2 &= 0 \\
 x^2 - 9(-1) &= 0 \\
 x^2 + 9 &= 0
 \end{aligned}$$

(b) If $1-2i$ is one root, the other root is $1+2i$. Equation is:

$$\begin{aligned}
 (x-(1-2i))(x-(1+2i)) &= 0 \\
 (x-1+2i)(x-1-2i) &= 0 \\
 ((x-1)+2i)((x-1)-2i) &= 0 \\
 (x-1)^2 - 4i^2 &= 0 \\
 x^2 - 2x + 1 - 4(-1) &= 0 \\
 x^2 - 2x + 5 &= 0
 \end{aligned}$$

(c) If $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$ is one root, the other is $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$.

$$\begin{aligned} & \left(x - \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) \right) \left(x - \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) \right) \\ &= \left(\left(x + \frac{1}{2} \right) + \frac{\sqrt{3}}{2}i \right) \left(\left(x + \frac{1}{2} \right) - \frac{\sqrt{3}}{2}i \right) \\ &= \left(x + \frac{1}{2} \right)^2 - \left(\frac{\sqrt{3}}{2}i \right)^2 \\ &= x^2 + x + \frac{1}{4} - \frac{3}{4}i^2 \\ &= x^2 + x + \frac{1}{4} - \frac{3}{4}(-1) \\ &= x^2 + x + 1 \end{aligned}$$

28. For the function $f(x) = \frac{1}{x}$, as x increases, the value of the function decreases, and as x decreases, the value of the function increases. This is why it is said to display inverse variation. The graph of $f(x) = \frac{1}{x}$ has a horizontal asymptote which is the x -axis, and a vertical asymptote which is the y -axis. It has two congruent parts, one in the first quadrant and one in the third. It is called a rectangular hyperbola. Because you cannot divide by zero, x cannot take this value and 0 must be excluded from the domain of $f(x)$. The domain is all real numbers other than zero. The range is all real numbers other than zero.

29. At each zero of $f(x)$ the graph of $y = \frac{1}{f(x)}$ has a vertical asymptote. The graph of $y = \frac{1}{f(x)}$ always passes through the points where $f(x) = 1$ or $f(x) = -1$.

When $f(x)$ is close to 1, $\frac{1}{f(x)}$ is also close to 1. When $f(x)$ is close to -1 , $\frac{1}{f(x)}$ is also close to -1 .

Both graphs occur in the same quadrants. When $f(x)$ is positive, $\frac{1}{f(x)}$ is positive. When $f(x)$ is negative, $\frac{1}{f(x)}$ is negative.

As $f(x)$ gets larger, $\frac{1}{f(x)}$ gets smaller. As $f(x)$ gets smaller, $\frac{1}{f(x)}$ gets larger.

30. (a) As x increases, y decreases, so this may be inverse variation. Is $y = \frac{k}{x}$? If so, then $xy = k$. For each line of the chart, $xy = 60$. $\therefore y = \frac{60}{x}$. Yes, this is inverse variation, with equation $y = \frac{60}{x}$ or $xy = 60$.

(b) As x decreases, y increases, so this may be inverse variation. $xy = 2880$ for each row of the chart. Yes, this is

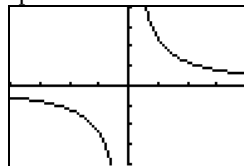
inverse variation, with equation $y = \frac{2880}{x}$ or $xy = 2880$.

(c) As x increases so does y . This is not inverse variation.

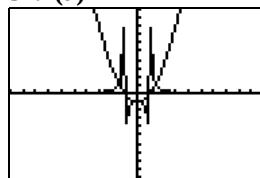
31. Let t represent time, and a represent acceleration. If t and a vary inversely, $at = k$ describes the relationship between t and a . When $a = 12$, $t = 5$. $\therefore at = 12 \times 5 = 60$

An equation to model the relation is $t = \frac{60}{a}$. If $a = 15$,

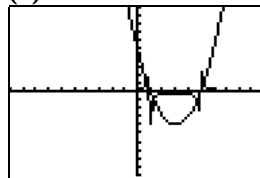
$t = \frac{60}{15} = 4$. It takes the car 4 seconds to reach the same speed when the acceleration is 15 m/s^2 .



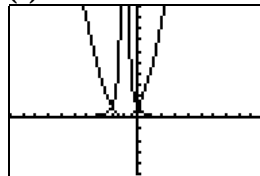
32. (a)



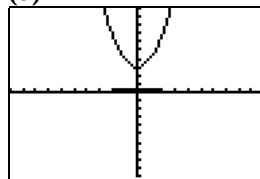
(b)



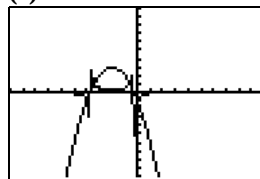
(c)



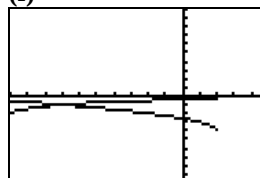
(d)



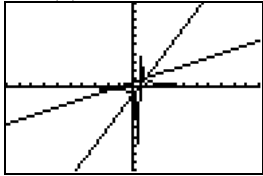
(e)



(f)



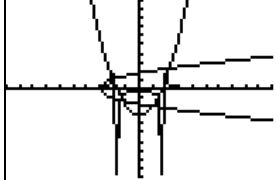
33. (a)



For $f(x)$ and $f^{-1}(x)$ the domain and range are both the set of real numbers.

For $\frac{1}{f(x)}$ the domain is all real numbers other than $\frac{1}{2}$. The range is all real numbers except zero.

(b)



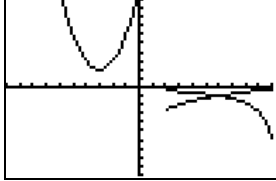
For $f(x)$, $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \in \mathbf{R}, y \geq -3\}$

For $\frac{1}{f(x)}$, $D = \{x \mid x \in \mathbf{R}, x \neq \pm 1.7\}$,

$R = \{y \mid y \in \mathbf{R}, y \leq -\frac{1}{3} \text{ or } y > 0\}$.

For $f^{-1}(x)$, $D = \{x \mid x \in \mathbf{R}, x \geq -3\}$, $R = \{y \mid y \in \mathbf{R}\}$.

(c)



For $f(x)$, $D = \{x \mid x \in \mathbf{R}, x \geq 2\}$, $R = \{y \mid y \in \mathbf{R}, y \geq -3\}$.

For $\frac{1}{f(x)}$, $D = \{x \mid x \in \mathbf{R}, x \geq 2, x \neq 11\}$,

$R = \{y \mid y \in \mathbf{R}, y \neq 0\}$.

For $f^{-1}(x)$, $D = \{x \mid x \in \mathbf{R}, x \geq -3\}$, $R = \{y \mid y \in \mathbf{R}, y \geq 2\}$

34. A rational expression is a sum, difference, product, or quotient of terms, each of which could be written as a quotient of polynomials. You can simplify a rational expression by dividing the numerator and denominator by a factor common to both. Therefore, it is necessary to factor the numerator and denominator in order to identify any common factors. It is also necessary to exclude any variable values that would make the denominator zero, and this is easily done when the denominator has been factored.

35. A rational expression is not defined if its denominator is zero. When simplifying a rational expression you must state and exclude any values of the variables that would make the denominator zero.

36. (a)

$$\begin{aligned}\frac{3x+9}{-5x-15} &= \frac{3(x+3)}{-5(x+3)}, x \neq -3 \\ &= -\frac{3}{5}, x \neq -3\end{aligned}$$

(b)

$$\begin{aligned}\frac{48p^3q^2}{-18pq^3} &= \frac{(6pq^2)(8p^2)}{(6pq^2)(-3q)}, p \neq 0, q \neq 0 \\ &= -\frac{8p^2}{3q}, p \neq 0, q \neq 0\end{aligned}$$

(c)

$$\begin{aligned}\frac{16x^2-9}{4x^2+x-3} &= \frac{(4x-3)(4x+3)}{(4x-3)(x+1)}, x \neq -1, \frac{3}{4} \\ &= \frac{4x+3}{x+1}, x \neq -1, \frac{3}{4}\end{aligned}$$

(d)

$$\begin{aligned}\frac{a^2-2a-8}{a^2-a-12} &= \frac{(a-4)(a+2)}{(a-4)(a+3)}, a \neq -3, 4 \\ &= \frac{(a+2)}{(a+3)}\end{aligned}$$

(e)

$$\begin{aligned}\frac{10x^2-11x-6}{10x^2+19x+6} &= \frac{(5x+2)(2x-3)}{(5x+2)(2x+3)}, x \neq -\frac{2}{5}, -\frac{3}{2} \\ &= \frac{(2x-3)}{(2x+3)}, x \neq -\frac{2}{5}, -\frac{3}{2}\end{aligned}$$

(f)

$$\begin{aligned}\frac{x^2-6x+9}{9-x^2} &= \frac{(x-3)(x-3)}{(3-x)(3+x)}, x \neq \pm 3 \\ &= \frac{-1(3-x)(x-3)}{(3-x)(3+x)} \\ &= \frac{-1(x-3)}{(x+3)} \text{ or } \frac{3-x}{3+x}, x \neq \pm 3\end{aligned}$$

(g)

$$\begin{aligned}\frac{2x^2-5xy+2y^2}{y^2-3xy+2x^2} &= \frac{(2x-y)(x-2y)}{(y-2x)(y-x)}, x \neq -\frac{1}{2}y, y \\ &= \frac{-1(y-2x)(x-2y)}{(y-2x)(y-x)} \\ &= \frac{-1(x-2y)}{y-x} \text{ or } \frac{2y-x}{y-x}, x \neq \frac{1}{2}y, y\end{aligned}$$

37.

$$\begin{aligned}C(t) &= \frac{5t^5+10t^3}{t^6+4t^4+4t^2} \\ &= \frac{5t^3(t^2+2)}{t^2(t^2+2)(t^2+2)}, t \neq 0, \pm\sqrt{2} \\ &= \frac{5t}{t^2+2}, t \neq 0, \pm\sqrt{2}\end{aligned}$$

$$(a) \quad C\left(\frac{1}{2}\right) = \frac{5\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)^2+2} \doteq 1.1$$

$$(b) \quad C(1) = \frac{5(1)}{1+2} \doteq 1.7$$

$$(c) \quad C(2) = \frac{5(2)}{4+2} \doteq 1.7$$

(d) If the concentration = 1,

$$\frac{5t}{t^2 + 2} = 1$$

$$5t = t^2 + 2$$

$$0 = t^2 - 5t + 2$$

$$t = \frac{5 \pm \sqrt{25 - 4(1)(2)}}{2(1)}$$

$$t = \frac{5 \pm 4.12}{2}$$

$$t \doteq 0.44 \text{ or } 4.56$$

It takes approximately 5 hours for the concentration to drop below 1.

38. To multiply rational expressions,

- factor the numerators and denominators. This enables the next step:
- state any restrictions. Factors cannot be divided out unless these are in place.
- divide out any common factors
- multiply the numerators and multiply the denominators, and write the result as a single rational expression.

If you are dividing rational expressions then the third step is to take the reciprocal of the second expression and change $\div$ to $\times$. Then you must state any additional restrictions on the variables. After this, follow steps 3 and 4 as above.

39. There may be extra restrictions when you divide rational expressions because neither the numerator nor the denominator, of the expression you are dividing by, can be zero.

40. (a)

$$\frac{6x}{8y} \times \frac{2y^2}{3x} = \frac{2y}{4}, x \neq 0, y \neq 0$$

$$= \frac{y}{2}, x \neq 0, y \neq 0$$

(b)

$$\frac{10m^2}{3n} \times \frac{6mn}{20m^2} = \frac{m}{1}, m \neq 0, n \neq 0$$

$$= m, m \neq 0, n \neq 0$$

(c)

$$\frac{2ab}{5bc} \div \frac{6ac}{10b}, b \neq 0, c \neq 0$$

$$= \frac{2ab}{5bc} \times \frac{10b}{6ac}, a \neq 0, b \neq 0, c \neq 0$$

$$= \frac{2b}{3c^2}, a \neq 0, b \neq 0, c \neq 0$$

(d)

$$\frac{5p}{8pq} \div \frac{3p}{12q}, p \neq 0, q \neq 0$$

$$= \frac{5p}{8pq} \times \frac{12q}{3p}, p \neq 0, q \neq 0$$

$$= \frac{5}{2p}, p \neq 0, q \neq 0$$

41. (a)

$$\frac{3}{x^2 - 9} \div \frac{3x - 6}{x - 3} = \frac{3}{(x - 3)(x + 3)} \div \frac{3(x - 2)}{x - 3}, x \neq \pm 3$$

$$= \frac{3}{(x - 3)(x + 3)} \times \frac{(x - 3)}{3(x - 2)}, x \neq 2, \pm 3$$

$$= \frac{1}{(x + 3)(x - 2)} \text{ or } \frac{1}{x^2 + x - 6}, x \neq 2, \pm 3$$

(b)

$$\frac{1}{10a - 2a^2} \div \frac{a}{4a - 20} = \frac{1}{2a(5 - a)} \div \frac{a}{4(a - 5)}, a \neq 0, 5$$

$$= \frac{1}{2a(5 - a)} \times \frac{4(a - 5)}{a}$$

$$= \frac{1}{2a(5 - a)} \times \frac{-4(5 - a)}{a}$$

$$= \frac{-2}{a^2}, a \neq 0, 5$$

(c)

$$\frac{3}{y^2 - 2y - 15} \times \frac{y^2 - 9}{6y - 18}$$

$$= \frac{3}{(y - 5)(y + 3)} \times \frac{(y - 3)(y + 3)}{6(y - 3)}, y \neq \pm 3, 5$$

$$= \frac{1}{2(y - 5)} \text{ or } \frac{1}{2y - 10}, y \neq \pm 3, 5$$

(d)

$$\frac{x^2 + 4x - 21}{x^2 - 6x - 16} \times \frac{x^2 - 8x + 15}{x^2 + 9x + 14}$$

$$= \frac{(x + 7)(x - 3)}{(x - 8)(x + 2)} \times \frac{(x - 3)(x - 5)}{(x + 2)(x + 7)}, x \neq -2, -7, 8$$

$$= \frac{(x - 3)^2(x - 5)}{(x - 8)(x + 2)^2}, x \neq -2, -7, 8$$

42.

$$f(x) = \frac{x^2 - 9}{3x - x^2}$$

$$= \frac{(x - 3)(x + 3)}{x(3 - x)}, x \neq 0, 3$$

$$= \frac{-(3 - x)(x + 3)}{x(3 - x)}$$

$$= \frac{-(x + 3)}{x}, x \neq 0, 3$$

$$g(x) = \frac{x^2 - 5x + 6}{2x^2 - 4x}$$

$$= \frac{(x - 2)(x - 3)}{2x(x - 2)}, x \neq 0, 2$$

$$= \frac{(x - 3)}{2x}, x \neq 0, 2$$

(a) $f(1) = \frac{-(1 + 3)}{1} = -4$

(b) $f(-1) = \frac{-(-1 + 3)}{-1} = 2$

$$(c) \quad g(5) = \frac{(5-3)}{2(5)} = \frac{1}{5}$$

$$(d) \quad g(-2) = \frac{(-2-3)}{2(-2)} = \frac{5}{4}$$

$$(e) \quad f(3) \text{ does not exist, } x \neq 3.$$

$$(f) \quad f(-3) = \frac{-(-3+3)}{-3} = 0$$

$$\begin{aligned} (g) \quad \frac{f(x)}{g(x)} &= \frac{(x-3)(x+3)}{x(3-x)} \div \frac{(x-2)(x-3)}{2x(x-2)}, \quad x \neq 0, 2, 3 \\ &= \frac{(x-3)(x+3)}{x(3-x)} \times \frac{2x(x-2)}{(x-2)(x-3)}, \quad x \neq 0, 2, 3 \\ &= \frac{2(x+3)}{3-x}, \quad x \neq 0, 2, 3 \end{aligned}$$

43. (a)

$$\begin{aligned} \frac{x^2}{2xy} \times \frac{x}{2y^2} \div \frac{(3x)^2}{xy^2}, \quad x \neq 0, y \neq 0 \\ = \frac{x^2}{2xy} \times \frac{x}{2y^2} \times \frac{xy^2}{9x^2}, \quad x \neq 0, y \neq 0 \\ = \frac{x}{36y}, \quad x \neq 0, y \neq 0 \end{aligned}$$

$$\begin{aligned} (b) \quad \frac{x^2-5x+6}{x^2-1} \times \frac{x^2-4x-5}{x^2-4} \div \frac{x-5}{x^2+3x+2} \\ = \frac{(x-2)(x-3)}{(x-1)(x+1)} \times \frac{(x-5)(x+1)}{(x-2)(x+2)} \div \frac{x-5}{(x+1)(x+2)}, \\ x \neq \pm 1, \pm 2 \\ = \frac{(x-2)(x-3)}{(x-1)(x+1)} \times \frac{(x-5)(x+1)}{(x-2)(x+2)} \times \frac{(x+1)(x+2)}{(x-5)}, \\ x \neq \pm 1, \pm 2, 5 \end{aligned}$$

$$= \frac{(x-3)(x+1)}{(x-1)} \text{ or } \frac{x^2-2x-3}{x-1}, \quad x \neq \pm 1, \pm 2, 5$$

$$\begin{aligned} (c) \quad \frac{1-x^2}{1+y} \times \frac{1-y^2}{x+x^2} \div \frac{y^3-y}{x^2} \\ = \frac{(1-x)(1+x)}{1+y} \times \frac{(1-y)(1+y)}{x(1+x)} \div \frac{y(y-1)(y+1)}{x^2}, \\ x \neq 0, -1, y \neq -1 \\ = \frac{(1-x)(1+x)}{(1+y)} \times \frac{(1-y)(1+y)}{x(1+x)} \times \frac{x^2}{y(y-1)(y+1)}, \\ x \neq 0, -1, y \neq 0, \pm 1 \\ = \frac{-x(1-x)}{y(y+1)} \\ = \frac{x(x-1)}{y(y+1)} \text{ or } \frac{x^2-x}{y^2+y}, \quad x \neq 0, -1, y \neq 0, \pm 1 \end{aligned}$$

$$\begin{aligned} (d) \quad \frac{x^2-y^2}{4x^2-y^2} \times \frac{4x^2+8xy+3y^2}{x+y} \div \frac{2x+3y}{2x-y} \\ = \frac{(x-y)(x+y)}{(2x-y)(2x+y)} \times \frac{(2x+y)(2x+3y)}{x+y} \div \frac{2x+3y}{2x-y}, \\ x \neq \pm \frac{y}{2}, -y \\ = \frac{(x-y)(x+y)}{(2x-y)(2x+y)} \times \frac{(2x+y)(2x+3y)}{(x+y)} \times \frac{(2x-y)}{(2x+3y)}, \\ x \neq \pm \frac{y}{2}, -y, -\frac{3y}{2} \\ = \frac{x-y}{1} \\ = x-y, \quad x \neq \pm \frac{y}{2}, -y, -\frac{3y}{2} \end{aligned}$$

$$\begin{aligned} (e) \quad \frac{a^2+5ab+6b^2}{a^2-5ab+6b^2} \times \frac{a^2-2ab-3b^2}{a^2+ab-2b^2} \div \frac{a^2+4ab+3b^2}{a^2-ab-2b^2} \\ = \frac{(a+2b)(a+3b)}{(a-2b)(a-3b)} \times \frac{(a-3b)(a+b)}{(a+2b)(a-b)} \div \frac{(a+b)(a+3b)}{(a-2b)(a+b)}, \\ a \neq \pm b, \pm 2b, 3b \\ = \frac{(a+2b)(a+3b)}{(a-2b)(a-3b)} \times \frac{(a-3b)(a+b)}{(a+2b)(a-b)} \times \frac{(a-2b)(a+b)}{(a+b)(a+3b)}, \\ a \neq \pm b, \pm 2b, \pm 3b \\ = \frac{a+b}{a-b}, \quad a \neq \pm b, \pm 2b, \pm 3b \end{aligned}$$

44. Use the product property of complex conjugates, ie. $z\bar{z}$ is always a real number.

45. To divide two complex numbers, multiply the numerator and denominator by the conjugate of the denominator.

$$\begin{aligned} \text{eg. } \frac{2+3i}{1-4i} &= \frac{2+3i}{1-4i} \times \frac{1+4i}{1+4i} \\ &= \frac{2+11i+12i^2}{1-16i^2} \\ &= \frac{2+11i+12(-1)}{1-16(-1)} \\ &= \frac{-10+11i}{17} \\ &= -\frac{10}{17} + \frac{11}{17}i \end{aligned}$$

46. (a) For $2-i$, i. conjugate is $2+i$,

ii. reciprocal is $\frac{1}{2-i}$

$$\begin{aligned} \text{iii. } \frac{1}{2-i} \times \frac{2+i}{2+i} &= \frac{2+i}{4-i^2} \\ &= \frac{2}{5} + \frac{1}{5}i \end{aligned}$$

(b) For $-1+i$, i. conjugate is $-1-i$

ii. reciprocal is $\frac{1}{-1+i}$

iii.

$$\frac{1}{-1+i} \times \frac{-1-i}{-1-i} = \frac{-1-i}{1-i^2}$$

$$= -\frac{1}{2} - \frac{1}{2}i$$

(c) For $\sqrt{2} - \sqrt{2}i$, i. conjugate is $\sqrt{2} + \sqrt{2}i$

ii. reciprocal is $\frac{1}{\sqrt{2} - \sqrt{2}i}$

iii.

$$\frac{1}{\sqrt{2} - \sqrt{2}i} \times \frac{\sqrt{2} + \sqrt{2}i}{\sqrt{2} + \sqrt{2}i} = \frac{\sqrt{2} + \sqrt{2}i}{2 - 2i^2}$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4}i$$

(d) For $-3 + \sqrt{3}i$, i. conjugate is $-3 - \sqrt{3}i$

ii. reciprocal is $\frac{1}{-3 + \sqrt{3}i}$

iii.

$$\frac{1}{-3 + \sqrt{3}i} \times \frac{-3 - \sqrt{3}i}{-3 - \sqrt{3}i} = \frac{-3 - \sqrt{3}i}{9 - 3i^2}$$

$$= -\frac{3}{12} - \frac{\sqrt{3}}{12}i$$

$$= -\frac{1}{4} - \frac{\sqrt{3}}{12}i$$

(e) For $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$, i. conjugate is $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$

ii. reciprocal is $\frac{1}{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}$

iii.

$$\frac{1}{-\frac{1}{2} + \frac{\sqrt{3}}{2}i} \times \frac{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}{-\frac{1}{2} - \frac{\sqrt{3}}{2}i} = \frac{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}{\frac{1}{4} - \frac{3}{4}i^2}$$

$$= -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

47. (a)

$$\frac{4}{1-i} \times \frac{1+i}{1+i} = \frac{4+4i}{1-i^2}$$

$$= \frac{4+4i}{1-(-1)}$$

$$= 2+2i$$

(b)

$$\frac{5}{2+i} \times \frac{2-i}{2-i} = \frac{10-5i}{4-i^2}$$

$$= \frac{10-5i}{5}$$

$$= 2-i$$

(c)

$$\frac{6i}{1+i} \times \frac{1-i}{1-i} = \frac{6i-6i^2}{1-i^2}$$

$$= \frac{6i+6}{2}$$

$$= 3+3i$$

(d)

$$\frac{8}{1-3i} \times \frac{1+3i}{1+3i} = \frac{8+24i}{1-9i^2}$$

$$= \frac{8+24i}{10}$$

$$= \frac{4}{5} + \frac{12}{5}i$$

(e)

$$\frac{3-i}{2+i} \times \frac{2-i}{2-i} = \frac{6-5i+i^2}{4-i^2}$$

$$= \frac{6-5i+(-1)}{4-(-1)}$$

$$= \frac{5-5i}{5}$$

$$= 1-i$$

(f)

$$\frac{5i-2}{2+3i} \times \frac{2-3i}{2-3i} = \frac{16i-4-15i^2}{4-9i^2}$$

$$= \frac{11+16i}{13}$$

$$= \frac{11}{13} + \frac{16}{13}i$$

(g)

$$\frac{5-i}{i} \times \frac{-i}{-i} = \frac{-5i+i^2}{-i^2}$$

$$= -1-5i$$

(h)

$$\frac{-3+4i}{-i} \times \frac{i}{i} = \frac{-3i+4i^2}{-i^2}$$

$$= -4-3i$$

(i)

$$\frac{a+bi}{a-bi} \times \frac{a+bi}{a+bi} = \frac{a^2+2abi+b^2i^2}{a^2-b^2i^2}$$

$$= \frac{a^2-b^2+2abi}{a^2+b^2}$$

$$= \frac{a^2-b^2}{a^2+b^2} + \frac{2ab}{a^2+b^2}i$$

(j)

$$\frac{(1-i)(3+2i)}{1+i} = \frac{3-i-2i^2}{1+i}$$

$$= \frac{5-i}{1+i}$$

$$= \frac{5-i}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{5-6i+i^2}{1-i^2}$$

$$= \frac{4-6i}{2}$$

$$= 2-3i$$

(k)

$$\begin{aligned}
\frac{(4-i)(2+3i)}{(2-i)^2} &= \frac{8+10i-3i^2}{4-4i+i^2} \\
&= \frac{11+10i}{3-4i} \times \frac{3+4i}{3+4i} \\
&= \frac{33+74i+40i^2}{9-16i^2} \\
&= \frac{-7+74i}{25} \\
&= -\frac{7}{25} + \frac{74}{25}i
\end{aligned}$$

48. (a)

$$\begin{aligned}
z &= \sqrt{3} + i \\
z + \frac{9}{z} &= \sqrt{3} + i + \frac{9}{\sqrt{3} + i} \times \frac{\sqrt{3} - i}{\sqrt{3} - i} \\
&= \sqrt{3} + i + \frac{9\sqrt{3} - 9i}{3 - i^2} \\
&= \sqrt{3} + i + \frac{9\sqrt{3}}{4} - \frac{9}{4}i \\
&= \sqrt{3} + \frac{9\sqrt{3}}{4} + i - \frac{9}{4}i \\
&= \frac{4\sqrt{3} + 9\sqrt{3}}{4} + \frac{4i - 9i}{4} \\
&= \frac{13\sqrt{3}}{4} - \frac{5}{4}i
\end{aligned}$$

(b)

$$\begin{aligned}
z &= x + yi \\
z + \frac{1}{z} &= x + yi + \frac{1}{x + yi} \times \frac{x - yi}{x - yi} \\
&= x + yi + \frac{x - yi}{x^2 - y^2i^2} \\
&= x + yi + \frac{x}{x^2 + y^2} - \frac{yi}{x^2 + y^2} \\
&= x + \frac{x}{x^2 + y^2} + yi - \frac{yi}{x^2 + y^2} \\
&= \frac{x^3 + xy^2 + x}{x^2 + y^2} + \frac{x^2y + y^3 - y}{x^2 + y^2}i
\end{aligned}$$

49. (a)

$$\begin{aligned}
(4 - 3i) &= z(1 + i) \\
z &= \frac{4 - 3i}{1 + i} \\
&= \frac{4 - 3i}{1 + i} \times \frac{1 - i}{1 - i} \\
&= \frac{4 - 7i + 3i^2}{1 - i^2} \\
&= \frac{1 - 7i}{2} \\
z &= \frac{1}{2} - \frac{7}{2}i \text{ or } 0.5 - 3.5i
\end{aligned}$$

(b)

$$\begin{aligned}
6z - z(4 - i) &= -2 + 6i \\
6z - 4z + zi &= -2 + 6i \\
2z + zi &= -2 + 6i \\
z(2 + i) &= -2 + 6i \\
z &= \frac{-2 + 6i}{2 + i} \\
&= \frac{-2 + 6i}{2 + i} \times \frac{2 - i}{2 - i} \\
&= \frac{-4 + 14i - 6i^2}{4 - i^2} \\
&= \frac{2 + 14i}{5} \\
&= \frac{2}{5} + \frac{14}{5}i \text{ or } 0.4 + 2.8i
\end{aligned}$$

50. If the LCD is not the product of the denominators, the denominators must have a common factor.

For example, if you need to add $\frac{2x}{x^2 - 4}$ to $\frac{3}{x^2 + 3x - 10}$ the first step is to factor both denominators, and state restrictions.

$$\frac{2x}{(x-2)(x+2)} + \frac{3}{(x-2)(x+5)}, \quad x \neq \pm 2, -5$$

The LCD is $(x-2)(x+2)(x+5)$. We only need to include one $(x-2)$ in the common denominator.

Then continue with the sum using the LCD.

$$\begin{aligned}
&\frac{2x}{(x-2)(x+2)} \times \frac{(x+5)}{(x+5)} + \frac{3}{(x-2)(x+5)} \times \frac{(x+2)}{(x+2)} \\
&= \frac{2x(x+5)}{(x-2)(x+2)(x+5)} + \frac{3(x+2)}{(x-2)(x+5)(x+2)}
\end{aligned}$$

Now that both expressions have the same denominator, add the numerators

$$\begin{aligned}
&\frac{2x^2 + 10x + 3x + 6}{(x-2)(x+2)(x+5)} \\
&\text{and simplify the numerator} \\
&= \frac{2x^2 + 13x + 6}{(x-2)(x+2)(x+5)}
\end{aligned}$$

51. When you reduce, you are dividing by common factors.

Since you cannot divide by zero, you must make sure the factors you divide by will never have the value zero. For example, if you are dividing by a factor of $(x-3)$, you must make sure x can never take the value 3, otherwise you will be dividing by zero.

52. (a)

$$\begin{aligned}
&\frac{4}{5x} - \frac{2}{3x}, \quad x \neq 0 \\
&= \frac{4}{5x} \times \frac{3}{3} - \frac{2}{3x} \times \frac{5}{5} \\
&= \frac{12}{15x} - \frac{10}{15x} \\
&= \frac{2}{15x}, \quad x \neq 0
\end{aligned}$$

$$(b) \quad \frac{3}{ab^2} + \frac{5}{a^2b}, a \neq 0, b \neq 0$$

$$= \frac{3}{ab^2} \times \frac{a}{a} + \frac{5}{a^2b} \times \frac{b}{b}$$

$$= \frac{3a}{a^2b^2} + \frac{5b}{a^2b^2}, a \neq 0, b \neq 0$$

$$(c) \quad \frac{5}{x+1} - \frac{2}{x-1} = \frac{5}{x+1} \times \frac{x-1}{x-1} - \frac{2}{x-1} \times \frac{x+1}{x+1}, x \neq \pm 1$$

$$= \frac{5(x-1)}{(x+1)(x-1)} - \frac{2(x+1)}{(x-1)(x+1)}$$

$$= \frac{5x-5-2x-2}{(x+1)(x-1)}$$

$$= \frac{3x-7}{(x+1)(x-1)} \text{ or } \frac{3x-7}{x^2-1}, x \neq \pm 1$$

$$(d) \quad \frac{4x}{(x+3)^2} - \frac{3}{(x+3)} = \frac{4x}{(x+3)^2} - \frac{3}{(x+3)} \times \frac{(x+3)}{(x+3)}, x \neq -3$$

$$= \frac{4x}{(x+3)^2} - \frac{3(x+3)}{(x+3)^2}$$

$$= \frac{4x-3x-9}{(x+3)^2}$$

$$= \frac{x-9}{(x+3)^2} \text{ or } \frac{x-9}{x^2+6x+9}, x \neq -3$$

$$(e) \quad \frac{1}{x^2+3x-4} + \frac{1}{x^2+x-12}$$

$$= \frac{1}{(x+4)(x-1)} + \frac{1}{(x+4)(x-3)}, x \neq -4, 1, 3$$

$$= \frac{1}{(x+4)(x-1)} \times \frac{(x-3)}{(x-3)} + \frac{1}{(x+4)(x-3)} \times \frac{(x-1)}{(x-1)}$$

$$= \frac{x-3}{(x+4)(x-1)(x-3)} + \frac{(x-1)}{(x+4)(x-1)(x-3)}$$

$$= \frac{x-3+x-1}{(x+4)(x-1)(x-3)}$$

$$= \frac{2x-4}{(x+4)(x-1)(x-3)}, x \neq -4, 1, 3$$

$$(f) \quad \frac{1}{x^2-5x+6} - \frac{1}{x^2-9}$$

$$= \frac{1}{(x-2)(x-3)} - \frac{1}{(x-3)(x+3)}, x \neq 2, \pm 3$$

$$= \frac{1}{(x-2)(x-3)} \times \frac{(x+3)}{(x+3)} - \frac{1}{(x-3)(x+3)} \times \frac{(x-2)}{(x-2)}$$

$$= \frac{x+3}{(x-2)(x-3)(x+3)} - \frac{(x-2)}{(x-3)(x+3)(x-2)}$$

$$= \frac{x+3-x-2}{(x-2)(x-3)(x+3)}$$

$$= \frac{5}{(x-2)(x-3)(x+3)}, x \neq 2, \pm 3$$

$$(g) \quad \frac{x^2-4y^2}{x^2+2xy} + \frac{x^2-2xy+4y^2}{2xy}$$

$$= \frac{(x-2y)(x+2y)}{x(x+2y)} + \frac{x^2-2xy+4y^2}{2xy}, x \neq 0, -2y, y \neq 0$$

$$= \frac{x-2y}{x} \times \frac{2y}{2y} + \frac{x^2-2xy+4y^2}{2xy}$$

$$= \frac{2y(x-2y)}{2xy} + \frac{x^2-2xy+4y^2}{2xy}$$

$$= \frac{2xy-4y^2+x^2-2xy+4y^2}{2xy}$$

$$= \frac{x^2}{2xy}$$

$$= \frac{x}{2y}, x \neq 0, -2y, y \neq 0$$

$$(h) \quad \frac{1}{x^2+2x-3} + \frac{1}{x^2-1} - \frac{2}{x^2+4x+3}$$

$$= \frac{1}{(x+3)(x-1)} + \frac{1}{(x+1)(x-1)} - \frac{2}{(x+1)(x+3)}, x \neq \pm 1, -3$$

$$= \frac{1}{(x+3)(x-1)} \times \frac{(x+1)}{(x+1)} + \frac{1}{(x+1)(x-1)} \times \frac{(x+3)}{(x+3)}$$

$$- \frac{2}{(x+1)(x+3)} \times \frac{(x-1)}{(x-1)}$$

$$= \frac{x+1}{(x+3)(x-1)(x+1)} + \frac{x+3}{(x+3)(x-1)(x+1)}$$

$$- \frac{2(x-1)}{(x+3)(x-1)(x+1)}$$

$$= \frac{x+1+x+3-2x+2}{(x+3)(x-1)(x+1)}$$

$$= \frac{6}{(x+3)(x-1)(x+1)}, x \neq \pm 1, -3$$

$$(i) \quad \frac{x^2-1}{(x+1)^2} - \frac{x^2-5x+6}{x^2-4x+4} + \frac{x+3}{x^2+4x+3}$$

$$= \frac{(x-1)(x+1)}{(x+1)^2} - \frac{(x-2)(x-3)}{(x-2)^2} + \frac{x+3}{(x+1)(x+3)},$$

$$x \neq -3, -1, 2$$

$$= \frac{(x-1)}{(x+1)} \times \frac{(x-2)}{(x-2)} - \frac{(x-3)}{(x-2)} \times \frac{(x+1)}{(x+1)} + \frac{1}{(x+1)} \times \frac{(x-2)}{(x-2)}$$

$$= \frac{(x-1)(x-2)}{(x+1)(x-2)} - \frac{(x-3)(x+1)}{(x+1)(x-2)} + \frac{(x-2)}{(x+1)(x-2)}$$

$$= \frac{x^2-3x+2-(x^2-2x-3)+x-2}{(x+1)(x-2)}$$

$$= \frac{x^2-3x+2-x^2+2x+3+x-2}{(x+1)(x-2)}$$

$$= \frac{3}{(x+1)(x-2)} \text{ or } \frac{3}{x^2-x-2}, x \neq -3, -1, 2$$

(j)

$$\begin{aligned}
& \frac{a-b}{ab} - \frac{a-2b}{ab+b^2} - \frac{2a+b}{a^2-ab} \\
&= \frac{a-b}{ab} - \frac{a-2b}{b(a+b)} - \frac{2a+b}{a(a-b)}, a \neq 0, \pm b, b \neq 0 \\
&= \frac{a-b}{ab} \times \frac{(a+b)(a-b)}{(a+b)(a-b)} - \frac{a-2b}{b(a+b)} \times \frac{a(a-b)}{a(a-b)} - \frac{2a+b}{a(a-b)} \times \frac{b(a+b)}{b(a+b)} \\
&= \frac{(a^2-b^2)(a-b)}{ab(a+b)(a-b)} - \frac{(a-2b)(a^2-ab)}{ab(a+b)(a-b)} - \frac{(2a+b)(ab+b^2)}{ab(a+b)(a-b)} \\
&= \frac{a^3-a^2b-ab^2+b^3-(a^3-a^2b-2a^2b+2ab^2)-(2a^2b+3ab^2+b^3)}{ab(a+b)(a-b)} \\
&= \frac{-6ab^2}{ab(a+b)(a-b)} \\
&= \frac{-6b}{(a+b)(a-b)} \text{ or } -\frac{6b}{a^2-b^2}, a \neq 0, \pm b, b \neq 0
\end{aligned}$$

53. (a)

$$\begin{aligned}
f(x) &= 4x - 12 + \frac{9}{x+2}, x \neq -2 \\
&= \frac{4x-12}{1} \times \frac{(x+2)}{(x+2)} + \frac{9}{x+2} \\
&= \frac{(4x-12)(x+2)}{(x+2)} + \frac{9}{(x+2)} \\
&= \frac{4x^2-4x-24+9}{(x+2)} \\
&= \frac{4x^2-4x-15}{(x+2)} \\
&= \frac{(2x-5)(2x+3)}{(x+2)}, x \neq -2
\end{aligned}$$

$$\text{Zeros are } x = \frac{5}{2}, -\frac{3}{2} \text{ or } x = 2.5, -1.5$$

(b)

$$\begin{aligned}
f(x) &= 9x - 15 + \frac{40}{x+3}, x \neq -3 \\
&= \frac{9x-15}{1} \times \frac{(x+3)}{(x+3)} + \frac{40}{x+3} \\
&= \frac{(9x-15)(x+3)}{(x+3)} + \frac{40}{(x+3)} \\
&= \frac{9x^2+12x-45+40}{(x+3)} \\
&= \frac{9x^2+12x-5}{(x+3)} \\
&= \frac{(3x+5)(3x-1)}{(x+3)}, x \neq -3
\end{aligned}$$

$$\text{Zeros are } x = -\frac{5}{3}, \frac{1}{3} \text{ or } x = -1.\dot{6}, 0.\dot{3}$$

54. (a)

$$\begin{aligned}
& \frac{3}{x-5} + \frac{2x}{x-3} = 5, x \neq 3, 5 \\
& \frac{3}{(x-5)} \times \frac{(x-3)}{(x-3)} + \frac{2x}{(x-3)} \times \frac{(x-5)}{(x-5)} = 5 \\
& \frac{3x-9}{(x-5)(x-3)} + \frac{2x^2-10x}{(x-3)(x-5)} = 5 \\
& \frac{3x-9+2x^2-10x}{(x-5)(x-3)} = 5 \\
& (x-5)(x-3) \times \frac{(2x^2-7x-9)}{(x-5)(x-3)} = 5(x-5)(x-3) \\
& 2x^2-7x-9 = 5(x^2-8x+15) \\
& 2x^2-7x-9 = 5x^2-40x+75 \\
& 3x^2-33x+84 = 0 \\
& 3(x^2-11x+28) = 0 \\
& 3(x-4)(x-7) = 0 \\
& x = 4 \text{ or } 7
\end{aligned}$$

$$\text{(b)} \quad \frac{3x}{x-1} + 4 = \frac{x}{x+1}, x \neq \pm 1$$

Multiply through by $(x-1)(x+1)$ to clear all denominators.

$$\begin{aligned}
& \frac{3x}{x-1} \times (x-1)(x+1) + 4(x-1)(x+1) = \frac{x}{x+1} \times (x-1)(x+1), \\
& x \neq \pm 1
\end{aligned}$$

$$\begin{aligned}
& 3x(x+1) + 4(x^2-1) = x(x-1) \\
& 3x^2+3x+4x^2-4 = x^2-x \\
& 6x^2+4x-4 = 0 \\
& 2(3x^2+2x-2) = 0 \\
& 3x^2+2x-2 = 0 \\
& x = \frac{-2 \pm \sqrt{2^2-4(3)(-2)}}{2(3)} \\
& = \frac{-2 \pm \sqrt{28}}{6} \\
& = \frac{-2 \pm 2\sqrt{7}}{6} \\
& = \frac{-1 \pm \sqrt{7}}{3} \\
& x = \frac{-1+\sqrt{7}}{3} \text{ or } \frac{-1-\sqrt{7}}{3}
\end{aligned}$$

$$\text{(c)} \quad \frac{1}{1-x} - \frac{1}{1+x} = \frac{4x}{1+x^2}, x \neq \pm 1$$

Work with the left side only, first.

$$\begin{aligned}
& \frac{1}{1-x} \times \frac{1+x}{1+x} - \frac{1}{1+x} \times \frac{1-x}{1-x} = \frac{4x}{1+x^2} \\
& \frac{1+x}{1-x^2} - \frac{1-x}{1-x^2} = \frac{4x}{1+x^2} \\
& \frac{1+x-1+x}{1-x^2} = \frac{4x}{1+x^2} \\
& \frac{2x}{1-x^2} = \frac{4x}{1+x^2}
\end{aligned}$$

Cross multiply $2x(1+x^2) = 4x(1-x^2)$

$$2x + 2x^3 = 4x - 4x^3$$

$$2x + 2x^3 - 4x + 4x^3 = 0$$

$$6x^3 - 2x = 0$$

$$2x(3x^2 - 1) = 0$$

Either $2x = 0$ or $3x^2 - 1 = 0$

$$x = 0 \text{ or } 3x^2 = 1$$

$$x^2 = \frac{1}{3}$$

$$x = \pm \frac{1}{\sqrt{3}}$$

Solutions are $x = 0, \pm \frac{1}{\sqrt{3}}$

(d)

$$\frac{5}{x+2} + \frac{x(x+3)}{x^2-4} = \frac{x}{x-2}$$

$$\frac{5}{(x+2)} + \frac{x(x+3)}{(x-2)(x+2)} = \frac{x}{x-2}, x \neq \pm 2$$

$$\frac{5}{(x+2)} \times \frac{(x-2)}{(x-2)} + \frac{x(x+3)}{(x-2)(x+2)} = \frac{x}{x-2} \times \frac{(x+2)}{(x+2)}$$

$$\frac{5(x-2)}{(x+2)(x-2)} + \frac{x(x+3)}{(x-2)(x+2)} = \frac{x(x+2)}{(x-2)(x+2)}$$

Multiply through by $(x+2)(x-2)$ to clear all denominators.

$$5(x-2) + x(x+3) = x(x+2)$$

$$5x - 10 + x^2 + 3x = x^2 + 2x$$

$$6x - 10 = 0$$

$$6x = 10$$

$$x = \frac{5}{3}$$

55. Let Tom's speed be x m/s.

	Distance (m)	Speed (m/s)	Time (s)
Genna	650	$x + 5$	$\frac{650}{x+5}$
Tom	650	x	$\frac{650}{x}$

Genna's time to complete the course is 6.5 s less than Tom's time.

$$\text{So } \frac{650}{x} - \frac{650}{x+5} = 6.5$$

$$\frac{650}{x} \times \frac{x+5}{x+5} - \frac{650}{x+5} \times \frac{x}{x} = \frac{13}{2}$$

$$\frac{650x + 3250}{x(x+5)} - \frac{650x}{x(x+5)} = \frac{13}{2}$$

$$\frac{3250}{x(x+5)} = \frac{13}{2}$$

Cross multiply $3250 \times 2 = 13x(x+5)$

Divide both sides by 13 $250 \times 2 = x(x+5)$

$$500 = x^2 + 5x$$

$$0 = x^2 + 5x - 500$$

$$0 = (x+25)(x-20)$$

$$x = -25 \text{ or } 20$$

Reject the negative root in this context.

Tom's speed is 20 m/s or 72 km/h.

Genna's speed is 25 m/s or 90 km/h.

(b) Tom's time for the lap is $\frac{650}{20} = 32.5$ seconds.

Genna's time for the lap is $\frac{650}{25} = 26$ seconds.

56. (i) $3x^2 + 2x - 1$ is a trinomial. It has degree 2.

(ii) $2 + 3x^2 - 5x^4 + x^5$ is a polynomial with degree 5.

(iii) $4x^3 + 4$ is a binomial with degree 3.

In a polynomial, the exponents must be whole numbers. The degree of a polynomial is the largest exponent for any term.

57. To add polynomials, add like terms.

$$\text{eg. } (3x^2 + 2x - 1) + (2 + 3x^2 - 5x^4 + x^5)$$

$$= 3x^2 + 2x - 1 + 2 + 3x^2 - 5x^4 + x^5$$

$$= 3x^2 + 3x^2 + 2x - 1 + 2 - 5x^4 + x^5$$

$$= 6x^2 + 2x + 1 - 5x^4 + x^5$$

$$= 1 + 2x + 6x^2 - 5x^4 + x^5 \text{ written with exponents in ascending order}$$

To subtract polynomials, subtract like terms.

$$(3x^2 + 2x - 1) - (2 + 3x^2 - 5x^4 + x^5)$$

$$= 3x^2 + 2x - 1 - 2 - 3x^2 + 5x^4 - x^5$$

$$= 3x^2 - 3x^2 + 2x - 1 - 2 + 5x^4 - x^5$$

$$= 2x - 3 + 5x^4 - x^5$$

$$= -3 + 2x + 5x^4 - x^5$$

To multiply polynomials, use the distributive law.

$$(3x^2 + 2x - 1)(2 + 3x^2 - 5x^4 + x^5)$$

$$= 6x^2 + 9x^4 - 15x^6 + 3x^7 + 4x + 6x^3 - 10x^5 + 2x^6 - 2 - 3x^2 + 5x^4 - x^5$$

Combine like terms.

$$= 3x^2 + 14x^4 - 13x^6 + 3x^7 + 4x + 6x^3 - 11x^5 - 2$$

$$= 3x^7 - 13x^6 - 11x^5 + 14x^4 + 6x^3 + 3x^2 + 4x - 2$$

58. $(x+1)(x^2 + 2x + 3) + (x+2)(x+3)(x-4) - (x^2 + x - 1)^2$

$$= x^3 + 2x^2 + 3x + x^2 + 2x + 3 + (x+2)(x^2 - x - 12)$$

$$- (x^2 + x - 1)(x^2 + x - 1)$$

$$= x^3 + 3x^2 + 5x + 3 + x^3 - x^2 - 12x + 2x^2 - 2x - 24$$

$$- (x^4 + x^3 - x^2 + x^3 + x^2 - x - x^2 - x + 1)$$

$$= 2x^3 + 4x^2 - 9x - 21 - (x^4 + 2x^3 - x^2 - 2x + 1)$$

$$= 2x^3 + 4x^2 - 9x - 21 - x^4 - 2x^3 + x^2 + 2x - 1$$

$$= -x^4 + 5x^2 - 7x - 22 \text{ which has degree 4}$$

59. (a) $(5x^4 - x^2 - 2) - (x^4 - 2x^3 + 3x^2 - 5)$

$$= 5x^4 - x^2 - 2 - x^4 + 2x^3 - 3x^2 + 5$$

$$= 4x^4 + 2x^3 - 4x^2 + 3$$

(b) $(-x^2 + x - 6) + (x^2 - 4x + 1) - (2x^2 - x - 5)$

$$= -x^2 + x - 6 + x^2 - 4x + 1 - 2x^2 + x + 5$$

$$= -2x^2 - 2x$$

$$\begin{aligned}
 \text{(c)} \quad & -3x(x^2 - 2x - 4) - x(x^2 - 3x - 2) \\
 & = -3x^3 + 6x^2 + 12x - x^3 + 3x^2 + 2x \\
 & = -4x^3 + 9x^2 + 14x \\
 \text{(d)} \quad & 4x^2(2x^3 - x + 3) - x^3(3x^3 + 5x - 1) \\
 & = 8x^5 - 4x^3 + 12x^2 - 3x^6 - 5x^4 + x^3 \\
 & = -3x^6 + 8x^5 - 5x^4 - 3x^3 + 12x^2 \\
 \text{(e)} \quad & x[2x - (3x - 4)] + 4x[2x - (3 - x)] \\
 & = x[2x - 3x + 4] + 4x[2x - 3 + x] \\
 & = x[-x + 4] + 4x[3x - 3] \\
 & = -x^2 + 4x + 12x^2 - 12x \\
 & = 11x^2 - 8x \\
 \text{(f)} \quad & x[2x - (5x - 4)] + 4x[x - (6 - x)] \\
 & = x[2x - 5x + 4] + 4x[x - 6 + x] \\
 & = x[-3x + 4] + 4x[2x - 6] \\
 & = -3x^2 + 4x + 8x^2 - 24x \\
 & = 5x^2 - 20x
 \end{aligned}$$

$$\begin{aligned}
 60. \text{ (a)} \quad & (x^2 + 4)(x^2 - 2x + 3) \\
 & = x^4 - 2x^3 + 3x^2 + 4x^2 - 8x + 12 \\
 & = x^4 - 2x^3 + 7x^2 - 8x + 12 \\
 \text{(b)} \quad & 3x(3x^2 - 1)^2 - 2x(1 - x^2)^2 \\
 & = 3x(9x^4 - 6x^2 + 1) - 2x(1 - 2x^2 + x^4) \\
 & = 27x^5 - 18x^3 + 3x - 2x + 4x^3 - 2x^5 \\
 & = 25x^5 - 14x^3 + x \\
 \text{(c)} \quad & (t - 2)(3t - 1)(4t + 3) - (2 - t)^2(t + 5) \\
 & = (t - 2)(12t^2 + 5t - 3) - (4 - 4t + t^2)(t + 5) \\
 & = 12t^3 + 5t^2 - 3t - 24t^2 - 10t + 6 \\
 & \quad - (4t + 20 - 4t^2 - 20t + t^3 + 5t^2) \\
 & = 12t^3 - 19t^2 - 13t + 6 - (t^3 + t^2 - 16t + 20) \\
 & = 12t^3 - 19t^2 - 13t + 6 - t^3 - t^2 + 16t - 20 \\
 & = 11t^3 - 20t^2 + 3t - 14 \\
 \text{(d)} \quad & (x - 3)^2(x + 1)^2 - (x + 2)^2(x - 4)^2 \\
 & = (x^2 - 6x + 9)(x^2 + 2x + 1) - (x^2 + 4x + 4)(x^2 - 8x + 16) \\
 & = x^4 + 2x^3 + x^2 - 6x^3 - 12x^2 - 6x + 9x^2 + 18x + 9 \\
 & \quad - (x^4 - 8x^3 + 16x^2 + 4x^3 - 32x^2 + 64x + 4x^2 - 32x + 64) \\
 & = x^4 - 4x^3 - 2x^2 + 12x + 9 - (x^4 - 4x^3 - 12x^2 + 32x + 64) \\
 & = x^4 - 4x^3 - 2x^2 + 12x + 9 - x^4 + 4x^3 + 12x^2 - 32x - 64 \\
 & = 10x^2 - 20x - 55 \\
 \text{(e)} \quad & (3x - 2)(x^3 + 4x^2 - 2x - 1) \\
 & = 3x^4 + 12x^3 - 6x^2 - 3x - 2x^3 - 8x^2 + 4x + 2 \\
 & = 3x^4 + 10x^3 - 14x^2 + x + 2 \\
 \text{(f)} \quad & (a^2 + 4a - 3)(3a^2 - 2a + 4) \\
 & = 3a^4 - 2a^3 + 4a^2 + 12a^3 - 8a^2 + 16a - 9a^2 + 6a - 12 \\
 & = 3a^4 + 10a^3 - 13a^2 + 22a - 12 \\
 \text{(g)} \quad & (x^2 - 3x + 2)^2 \\
 & = (x^2 - 3x + 2)(x^2 - 3x + 2) \\
 & = x^4 - 3x^3 + 2x^2 - 3x^3 + 9x^2 - 6x + 2x^2 - 6x + 4 \\
 & = x^4 - 6x^3 + 13x^2 - 12x + 4 \\
 \text{(h)} \quad & (x - 4)^2(x + 1)^2 \\
 & = (x^2 - 8x + 16)(x^2 + 2x + 1) \\
 & = x^4 + 2x^3 + x^2 - 8x^3 - 16x^2 - 8x + 16x^2 + 32x + 16 \\
 & = x^4 - 6x^3 + x^2 + 24x + 16
 \end{aligned}$$

61. If the triangle contains a right angle, its sides will obey the Pythagorean Theorem.

Squaring each side:

$$\begin{aligned}
 \left[\frac{1}{2}(n^2 + 1) \right]^2 &= \frac{1}{4}(n^4 + 2n^2 + 1) \\
 \left[\frac{1}{2}(n^2 - 1) \right]^2 &= \frac{1}{4}(n^4 - 2n^2 + 1) \\
 n^2 &= n^2
 \end{aligned}$$

$$\begin{aligned}
 \text{But } \frac{1}{4}(n^4 - 2n^2 + 1) + n^2 &= \frac{1}{4}n^4 - \frac{1}{2}n^2 + \frac{1}{4} + n^2 \\
 &= \frac{1}{4}n^4 + \frac{1}{2}n^2 + \frac{1}{4} \\
 &= \frac{1}{4}(n^4 + 2n^2 + 1)
 \end{aligned}$$

and we have shown that $\left[\frac{1}{2}(n^2 - 1) \right]^2 + [n]^2 = \left[\frac{1}{2}(n^2 + 1) \right]^2$.

The sides obey the Pythagorean Theorem. The triangle with these sides has a right angle.

Chapter 4 Review Test, page 387

1. (a) $z = -3 + 2i$, $w = 2 - i$

$$z + w = -3 + 2i + 2 - i = -1 + i$$

$$z - w = -3 + 2i - (2 - i) = -3 + 2i - 2 + i = -5 + 3i$$

$$z\bar{z} = (-3 + 2i)(-3 - 2i) = 9 - 4i^2 = 9 - 4(-1) = 9 + 4 = 13$$

$$\frac{z}{2w} = \frac{-3 + 2i}{4 - 2i} = \frac{-3 + 2i}{4 - 2i} \times \frac{4 + 2i}{4 + 2i} = \frac{(-3 + 2i)(4 + 2i)}{16 - 4i^2} = \frac{-12 - 6i + 8i + 4i^2}{16 + 4} = \frac{-12 + 2i - 4}{20} = \frac{-16 + 2i}{20} = \frac{2(-8 + i)}{2(10)} = -\frac{4}{5} + \frac{1}{10}i$$

$$\begin{aligned}
 \frac{z}{2w} &= \frac{-3 + 2i}{4 - 2i} \\
 &= \frac{-3 + 2i}{4 - 2i} \times \frac{4 + 2i}{4 + 2i} \\
 &= \frac{(-3 + 2i)(4 + 2i)}{16 - 4i^2} \\
 &= \frac{-12 - 6i + 8i + 4i^2}{16 + 4} \\
 &= \frac{-12 + 2i - 4}{20} \\
 &= \frac{-16 + 2i}{20} \\
 &= \frac{2(-8 + i)}{2(10)} \\
 &= -\frac{4}{5} + \frac{1}{10}i
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad f(x) &= -2x^2 - 3x + 1 \\
 f(2 - 5i) &= -2(2 - 5i)^2 - 3(2 - 5i) + 1 \\
 &= -2(4 - 20i + 25i^2) - 6 + 15i + 1 \\
 &= -2(4 - 20i - 25) - 5 + 15i \\
 &= -2(-21 - 20i) - 5 + 15i \\
 &= 42 + 40i - 5 + 15i \\
 &= 37 + 55i
 \end{aligned}$$