

23. (a) $t = (1.66 \times 10^{-4})(6370 + 450)^{1.5} = 93.49$ min
 OR 93 min 29.7 s.
 $\therefore$ a satellite situated 450 km above the Earth takes 93 min and 29.7 s to orbit the Earth once.

(b)

$$24(60) = (1.66 \times 10^{-4})(6370 + h)^{1.5}$$

$$\frac{1440}{1.66 \times 10^{-4}} = (6370 + h)^{1.5}$$

$$8\,674\,698.795 = (6370 + h)^{1.5}$$

$$(8\,674\,698.795)^{\frac{1}{1.5}} = 6370 + h$$

$$h = (8\,674\,698.795)^{\frac{1}{1.5}} - 6370$$

$$h = 35\,848.51 \text{ km}$$

$\therefore$ the television satellite must be 35 848.51 km above the Earth's surface.

Practise, Apply, Solve 1.10, page 85

1. (a) $(x^4)(x^3) = x^{4+3} = x^7$

(b) $(c^4)^5 = c^{(4)(5)} = c^{20}$

(c) $x^6 \div x^3 = x^{6-3} = x^3$

(d) $(ab)^3 = a^3b^3$

(e) $(d^4)(d^2)(d^{-3}) = d^{4+2-3} = d^3$

(f) $b^{-3} \div b^2 = b^{-3-2} = b^{-5} = \frac{1}{b^5}$

(g) $(2x^3)^2 = 2^2x^{3(2)} = 4x^6$

(h) $\left(\frac{2x}{-3y^2}\right)^3 = \frac{(2x)^3}{(-3y^2)^3} = \frac{2^3x^3}{(-3)^3y^{2 \times 3}} = \frac{8x^3}{-27y^6}$

2. (a) $2^4 \times 2^5 = 2^{4+5} = 2^9$

(b) $2^n \times 2^m = 2^{n+m}$

(c) $(2^4)^x = 2^{4x}$

(d) $4^3 = (2^2)^3 = 2^6$

(e) $8^n = (2^3)^n = 2^{3n}$

(f) $2^6 \div 2^{-2} = 2^{6+2} = 2^8$

(g) $4^4 \div 2^m = (2^2)^4 \div 2^m = 2^{8-m}$

(h) $16^{3x-1} = (2^4)^{3x-1} = 2^{12x-4}$

3. (a) $3^5 = 243$

(b) $7^0 = 1$

(c) $8^{-2} = \frac{1}{64}$

(d) $(-3)^4 = (-1)^43^4 = 81$

(e) $-16^{\frac{-3}{4}} = (-1)(\sqrt[4]{16})^{-3} = \frac{-1}{8}$

(f) $16^{\frac{1}{4}} = \sqrt[4]{16} = 2$

(g) $32^{\frac{2}{5}} = (\sqrt[5]{32})^2 = 2^2 = 4$

(h) $4^{\frac{3}{2}} = (\sqrt{4})^3 = 2^3 = 8$

(i) $125^{\frac{-1}{3}} = (\sqrt[3]{125})^{-1} = \frac{1}{5}$

(j) $100^{\frac{-1}{2}} = \frac{1}{\sqrt{100}} = \frac{1}{10}$

(k) $-27^{\frac{1}{3}} = -\sqrt[3]{27} = -3$

(l) $256^{\frac{3}{4}} = (16^2)^{\frac{3}{4}} = 16^{\frac{3}{2}} = (\sqrt{16})^3 = 4^3 = 64$

(m) $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$

(n) $128^{\frac{-3}{7}} = (\sqrt[7]{128})^{-3} = 2^{-3} = \frac{1}{8}$

(o) $64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = 4^2 = 16$

4. (a) $(x^{-3})^2 = x^{-6} = \frac{1}{x^6}$

(b) $\sqrt[3]{x} = x^{\frac{1}{3}}$

(c) $\sqrt[4]{c^5} = c^{\frac{5}{4}}$

(d) $\sqrt[3]{a^{-2}} = a^{\frac{-2}{3}} = \frac{1}{a^{\frac{2}{3}}}$

(e) $(\sqrt{c^{-3}})^2 = \left(c^{\frac{-3}{2}}\right)^2 = c^{-3} = \frac{1}{c^3}$

5. (a) $\frac{(xy)^4}{xy} = (xy)^3 = x^3y^3$

(b) $\frac{3(ab)^4}{(-a^2)^2} = \frac{3(a^2)^2b^4}{(-1)^2(a^2)^2} = 3b^4$

(c) $(a^3b)^2\left(\frac{-a}{b}\right)^3 = \frac{a^3b^2(-1)^3a^3}{b^3} = \frac{-a^6}{b}$

(d) $(-d^3)^4\left(\frac{c}{d}\right)^6 = \frac{(-1)^4d^{12}c^6}{d^6} = c^6d^6$

(e) $\left(\frac{-1}{b}\right)^2(a^3b)^2 = \left(\frac{1}{b^2}\right)a^6b^2 = a^6$

(f) $\frac{(a^4b^2)^3}{(a^2b^2)^2} = \frac{a^{12}b^6}{a^4b^4} = a^8b^2$

(g) $\left(\frac{x}{-y}\right)^3(-xy)^4 = \frac{(-1)^3x^3}{y^3}(-1)^4x^4y^4 = -x^7y$

(h) $\left(\frac{x^3}{y^4}\right)^4\left(\frac{y^2}{x^4}\right)^3 = \left(\frac{x^{12}}{y^{16}}\right)\left(\frac{y^6}{x^{12}}\right) = \frac{1}{y^{10}}$

6. (a) $49^{\frac{1}{2}} + 16^{\frac{1}{4}} = \sqrt{49} + \sqrt[4]{16} = 7 + 2 = 9$

(b) $27^{\frac{2}{3}} - 81^{\frac{3}{4}} = (\sqrt[3]{27})^2 - (\sqrt[4]{81})^3 = 3^2 - 3^3 = 9 - 27 = -18$

(c) $16^{\frac{3}{4}} + 16^{\frac{3}{4}} - 81^{\frac{-1}{4}} = 2(2^4)^{\frac{3}{4}} - (3^4)^{\frac{-1}{4}} = 2(8) - 3^{-1} = 15\frac{2}{3}$

(d) $128^{\frac{-5}{7}} - 16^{-0.75} = (2^7)^{\frac{-5}{7}} - (2^4)^{\frac{-3}{4}} = 2^{-5} - 2^{-3} = \frac{1}{32} - \frac{1}{8} = \frac{-3}{32}$

$$\begin{aligned} \text{(e)} \quad & 16^{\frac{3}{2}} + 16^{-0.5} + 8 - 27^{\frac{2}{3}} = (4^2)^{\frac{3}{2}} + (4^2)^{\frac{-1}{2}} + 8 - (3^3)^{\frac{2}{3}} \\ & = 64 + \frac{1}{4} + 8 - 9 = \frac{253}{4} \\ \text{(f)} \quad & 81^{\frac{1}{2}} + \sqrt[3]{8} - 32^{\frac{4}{5}} + 16^{\frac{3}{4}} = 9 + 2 - 2^4 + 2^3 = 11 - 16 + 8 = 3 \\ \text{(g)} \quad & 9^{\frac{1}{2}} - \sqrt[4]{16} + 16^{\frac{1}{2}} - 2(2^{-3}) = 3 - 2 + 4 - 2\left(\frac{1}{8}\right) = \frac{19}{4} \\ \text{(h)} \quad & \left(\frac{1}{8}\right)^{\frac{1}{3}} - \sqrt[3]{\frac{27}{125}} + 4\left(8^{\frac{-2}{3}}\right) = (2^{-3})^{\frac{1}{3}} - \frac{27^{\frac{1}{3}}}{125^{\frac{1}{3}}} + (2^3)(2^3)^{\frac{-2}{3}} \\ & = 2^{-1} - \frac{3}{5} + 1 = \frac{9}{10} \end{aligned}$$

$$7. \text{(a)} \quad 2^3 \times 4^{-2} \div 2^2 = 8\left(\frac{1}{16}\right) \div 4 = \frac{2}{16} = \frac{1}{8}$$

$$\text{(b)} \quad (2^2 \times 3)^{-1} = (4 \times 3)^{-1} = \frac{1}{12}$$

$$\text{(c)} \quad \left(\frac{3^{-1}}{2^{-1}}\right)^{-2} = \left(\frac{\frac{1}{3}}{\frac{1}{2}}\right)^{-2} = \left(\frac{\frac{1}{2}}{\frac{1}{3}}\right)^2 = \frac{9}{4}$$

$$\text{(d)} \quad 4^{-1}(4^2 + 4^0) = \frac{1}{4}(16 + 1) = \frac{17}{4}$$

$$\text{(e)} \quad \frac{2^5}{3^{-2}} \times \frac{3^{-1}}{2^4} = 2^1 3^{-1+2} = (2)(3) = 6$$

$$\text{(f)} \quad (5^0 + 5^2)^{-1} = (1 + 25)^{-1} = \frac{1}{26}$$

$$\text{(g)} \quad \frac{3^{-2} \times 2^{-3}}{3^{-1} \times 2^{-2}} = \frac{3^1 \times 2^2}{3^2 \times 2^3} = \frac{1}{3^1 \times 2^1} = \frac{1}{6}$$

$$\text{(h)} \quad \frac{4^{-2} + 3^{-1}}{3^{-2} + 2^{-3}} = \frac{\frac{1}{16} + \frac{1}{3}}{\frac{1}{9} + \frac{1}{8}} = \frac{\frac{19}{48}}{\frac{17}{72}} = \frac{57}{34}$$

$$\text{(i)} \quad \frac{5^{-1} - 2^{-2}}{5^{-1} + 2^{-2}} = \frac{\frac{1}{5} - \frac{1}{4}}{\frac{1}{5} + \frac{1}{4}} = \frac{\frac{-1}{20}}{\frac{9}{20}} = \frac{-1}{9}$$

$$8. \text{(a)} \quad \frac{x}{y^2} = xy^{-2}$$

$$\text{(b)} \quad \frac{m^3}{n^2} = m^3 n^{-2}$$

$$\text{(c)} \quad \frac{2x^3}{y^5 z^2} = 2x^3 y^{-5} z^{-2}$$

$$\text{(d)} \quad \frac{10a^2 b}{2c^2} = 5a^2 bc^{-2}$$

$$\text{(e)} \quad \frac{30m^2}{-6n^4} = -5m^2 n^{-4}$$

$$\text{(f)} \quad \frac{1}{x^{-2}} = x^2$$

$$\text{(g)} \quad \frac{m^{-5}}{n^{-4}} = m^{-5} n^4$$

$$\text{(h)} \quad \frac{8x^3 y^{-2}}{4z^{-3}} = 2x^3 y^{-2} z^3$$

$$9. \text{(a)} \quad a^4 b^{-3} = \frac{a^4}{b^3}$$

$$\text{(b)} \quad \frac{4a^{-2} b^3}{c^{-2}} = \frac{4b^3 c^2}{a^2}$$

$$\text{(c)} \quad \frac{5a^{-2} b^{-3}}{ab^{-2}} = \frac{5}{a^3 b}$$

$$\text{(d)} \quad \frac{2a^2 b^{-3}}{3bc^{-4}} = \frac{2a^2 c^4}{3b^4}$$

$$\text{(e)} \quad \frac{(4a)^{-2}}{(3b)^{-1}} = \frac{3b}{16a^2}$$

$$10. \text{(a)} \quad ab^c = (1)(3)^2 = 9$$

$$\text{(b)} \quad a^c b^c = 1^2 3^2 = 9$$

$$\text{(c)} \quad (ab)^{-c} = (3)^{-2} = \frac{1}{9}$$

$$\text{(d)} \quad (b \div c)^{-a} = \left(\frac{c}{b}\right)^a = \frac{2}{3}$$

$$\text{(e)} \quad (-a \div b)^{-c} = \left(\frac{-b}{a}\right)^c = (-3)^2 = 9$$

$$\text{(f)} \quad (a^{-1} b^{-2})^c = \left(\frac{1}{ab^2}\right)^c = \left(\frac{1}{3^2}\right)^2 = \frac{1}{81}$$

$$\text{(g)} \quad (a^b b^a)^c = [(1)^3 (3)^1]^2 = 9$$

$$\text{(h)} \quad [(b)^{-a}]^{-c} = \left(\frac{1}{3}\right)^{-2} = 3^2 = 9$$

$$11. \text{(a)}$$

$$n^3 = 64$$

$$n = 64^{\frac{1}{3}}$$

$$n = 4$$

$$\text{(b)}$$

$$c^3 = 512$$

$$c = 512^{\frac{1}{3}}$$

$$c = 8$$

$$\text{(c)}$$

$$d^4 = 625$$

$$d = 625^{\frac{1}{4}}$$

$$d = 5$$

$$\text{(d)}$$

$$(n^2)^5 \div n^5 = 243$$

$$n^{10-5} = 243$$

$$n^5 = 243$$

$$n = 243^{\frac{1}{5}}$$

$$n = 3$$

$$\text{(e)}$$

$$x^{15} \div x^2 = 12^{13}$$

$$x^{13} = 12^{13}$$

$$x = 12$$

$$\text{(f)}$$

$$(t^2)^2 \times t^0 = 10\,000$$

$$t^4 = 10^4$$

$$t = 10$$

$$(g) \quad m^3 \times m^2 = 3125$$

$$m^5 = 3125$$

$$m = 3125^{\frac{1}{5}}$$

$$m = 5$$

$$(h) \quad \left(\frac{m^7}{m^6}\right)^2 = 196$$

$$(m)^2 = 14^2$$

$$m = 14$$

$$(i) \quad n^{10} \div n^8 = 576$$

$$n^2 = 576$$

$$n = 24$$

$$(j) \quad (x^8)(x^2) \div (x^7) = 1$$

$$x^{16-7} = 1$$

$$x^9 = 1$$

$$x = 1$$

$$(k) \quad (p \times p^2)^3 = 3^9$$

$$(p^3)^3 = 3^9$$

$$p^9 = 3^9$$

$$p = 3$$

$$(l) \quad (x^2)^2 \div x^3 = 15$$

$$x^{4-3} = 15$$

$$x = 15$$

$$12. (a) \quad \text{LS} = 9^{\frac{1}{2}} + 4^{\frac{1}{2}} = 3 + 2 = 5, \text{RS} = (9 + 4)^{\frac{1}{2}} = 13^{\frac{1}{2}} \doteq 3.606$$

$\therefore \text{LS} \neq \text{RS}$, so the expression is **false**.

$$(b) \quad \text{LS} = \left(9^{\frac{1}{2}}\right)\left(4^{\frac{1}{2}}\right) = (3)(2) = 6$$

$$\text{RS} = (9 \times 4)^{\frac{1}{2}} = 36^{\frac{1}{2}} = 6$$

$\therefore \text{LS} = \text{RS}$, so the expression is **true**.

$$(c) \quad \text{LS} = \left(\frac{1}{a} + \frac{1}{b}\right)^{-1} = \left(\frac{b+a}{ab}\right)^{-1} = \left(\frac{ab}{a+b}\right)^1 = \frac{ab}{a+b} \neq \text{RS}$$

$\therefore \text{LS} \neq \text{RS}$, so the expression is **false**.

$$(d) \quad \text{LS} = \left(\frac{1}{a} \times \frac{1}{b}\right)^{-1} = \left(\frac{1}{ab}\right)^{-1} = \left(\frac{ab}{1}\right)^1 = ab = \text{RS}$$

$\therefore \text{LS} = \text{RS}$, so the expression is **true**.

$$(e) \quad \text{LS} = \left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right)^6 \neq \text{RS}$$

$\therefore \text{LS} \neq \text{RS}$, so the expression is **false**.

$$(f) \quad \text{LS} = \left[\left(x^{\frac{1}{3}}\right)\left(y^{\frac{1}{3}}\right)\right]^6 = \left(x^{\frac{6}{3}}\right)\left(y^{\frac{6}{3}}\right) = x^2 y^2 = \text{RS}$$

$\therefore \text{LS} = \text{RS}$, so the expression is **true**.

$$13. (a) \quad \sqrt{x^2} = x$$

$$(b) \quad \sqrt[3]{c^3} = c$$

$$(c) \quad \sqrt[3]{z^9} = (z^9)^{\frac{1}{3}} = z^3$$

$$(d) \quad \sqrt[4]{d^8} = d^{\frac{8}{4}} = d^2$$

$$(e) \quad \left(\frac{2}{3}\right)^{\frac{5}{3}}\left(\frac{2}{3}\right)^{\frac{1}{3}} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$(f) \quad \left(4^{\frac{1}{3}}\right)^{\frac{6}{4}} = 4^{\frac{1}{2}} = 2$$

$$(g) \quad \left(2x^{\frac{-1}{3}}y^{\frac{3}{4}}\right)^2 = 4x^{\frac{-2}{3}}y^{\frac{3}{2}} = \frac{4y^{\frac{3}{2}}}{x^{\frac{2}{3}}}$$

$$(h) \quad \left(-3u^{\frac{3}{5}}v^{\frac{-1}{5}}\right)^2 = 9u^{\frac{6}{5}}v^{\frac{-2}{5}} = \frac{9u^{\frac{6}{5}}}{v^{\frac{2}{5}}}$$

$$(i) \quad \frac{18y^{\frac{4}{3}}z^{\frac{-1}{3}}}{24y^{\frac{-2}{3}}z} = \frac{3}{4}y^2z^{\frac{-4}{3}} = \frac{3y^2}{4z^{\frac{4}{3}}}$$

$$(j) \quad \frac{a^{\frac{3}{4}} \times a^{\frac{1}{2}}}{a^{\frac{3}{2}}} = a^{\frac{3}{4} + \frac{2}{4} - \frac{6}{4}} = a^{\frac{-1}{4}} = \frac{1}{a^{\frac{1}{4}}}$$

$$(k) \quad \left(c^{\frac{3}{2}}\right)^{\frac{2}{3}} = c^{\frac{1}{2}} = \sqrt{c}$$

$$(l) \quad \left(k^{\frac{-1}{2}}\right)^{\frac{2}{5}} = k^{\frac{-1}{5}} = \frac{1}{k^{\frac{1}{5}}}$$

$$14. (a) \quad \frac{x^{\frac{1}{2}} \times x^{\frac{2}{3}}}{x^{\frac{1}{4}}} = x^{\frac{1}{2} + \frac{2}{3} - \frac{1}{4}} = x^{\frac{11}{12}}$$

$$(b) \quad \frac{x^{\frac{5}{6}} \times x^{\frac{2}{3}}}{x^{\frac{1}{2}}} = x^{\frac{5}{6} + \frac{4}{6} - \frac{3}{6}} = x$$

$$(c) \quad \left(y^{\frac{1}{2}}\right)^2 \div (16y^6)^{\frac{1}{2}} = y \div 4y^3 = \frac{y}{4y^3} = \frac{1}{4y^2}$$

$$(d) \quad \left(\frac{\sqrt[4]{y^4}}{\sqrt{y^2}}\right)^3 = \left(\frac{y}{y}\right)^3 = 1$$

$$(e) \quad \left(\frac{x^3}{81}\right)\left(\frac{81^{\frac{3}{4}}}{x}\right) = \frac{81^{\frac{3}{4}}x^3}{81x} = 81^{\frac{-1}{4}}x^2 = \frac{x^2}{3}$$

$$(f) \quad \frac{(x^2y^4)^{\frac{1}{2}}(x^4y^2)^{\frac{1}{2}}}{(x^{\frac{1}{2}}y^{\frac{1}{2}})^6} = \frac{(xy^2)(x^2y)}{x^3y^3} = \frac{x^3y^3}{x^3y^3} = 1$$

$$15. (a) (x^2)^{5-r} = x^{10-2r}$$

$$(b) (a^{4+2r})(a^{-3r-5}) = a^{4+2r-3r-5} = a^{-r-1} = \frac{1}{a^{r+1}}$$

$$(c) (b^{2m+3n}) \div (b^{m-n}) = b^{2m+3n-(m-n)} = b^{m+4n}$$

$$(d) x^{3(7-r)} x^r = x^{21-3r+r} = x^{-2r+21}$$

$$(e) (a^{10-p}) \left(\frac{1}{a} \right)^p = a^{10-p-p} = a^{10-2p}$$

$$(f) \left[(3x^4)^{6-m} \right] \left(\frac{1}{x} \right)^m = 3^{6-m} x^{24-4m-m} = 3^{(6-m)} x^{(24-5m)}$$

$$16. (a) (3x^3)^2 = 9x^6$$

$$(b) (4x^2 y^{-4})^{-2} = (4)^{-2} (x^2)^{-2} (y^{-4})^{-2} = \frac{y^8}{16x^4}$$

$$(c) (16x^{10} y^4)^{\frac{1}{2}} = 4x^5 y^2$$

$$(d) (2x^{-2})^2 (3x^4)^{-3} = \left(\frac{4}{x^4} \right) \left(\frac{1}{27x^{12}} \right) = \frac{4}{27x^{16}}$$

$$(e) \frac{(2x^{-1})^{-2}}{2(y^{-1})^{-2}} = \frac{2^{-2} x^2}{2y^2} = \frac{x^2}{8y^2}$$

$$(f) \left(\frac{3x^2}{y^{-1}} \right)^{-2} \left(\frac{2y^2}{3x} \right)^3 = \left(\frac{y^{-2}}{9x^4} \right) \left(\frac{8y^6}{27x^3} \right) = \frac{8y^4}{243x^7}$$

$$17. \left(\frac{3x^3}{2y^4} \right)^2 \left(\frac{2y^2}{3x^4} \right)^{-3} = \left(\frac{9x^6}{4y^8} \right) \left(\frac{27x^{12}}{8y^6} \right) = \frac{243x^{18}}{32y^{14}}$$

$$18. (a)$$

$$x^3 = 27$$

$$x = 27^{\frac{1}{3}}$$

$$x = 3$$

$$(b)$$

$$x^{\frac{1}{2}} = 5$$

$$\left(x^{\frac{1}{2}} \right)^2 = 5^2$$

$$x = 25$$

$$(c)$$

$$c^{\frac{2}{3}} = 64$$

$$\left(c^{\frac{2}{3}} \right)^{\frac{3}{2}} = 64^{\frac{3}{2}}$$

$$c = 8^3$$

$$c = 512$$

$$(d)$$

$$3a^{\frac{4}{5}} = 48$$

$$a^{\frac{4}{5}} = 16$$

$$\left(a^{\frac{4}{5}} \right)^{\frac{5}{4}} = 16^{\frac{5}{4}}$$

$$a = (2^4)^{\frac{5}{4}}$$

$$a = 2^5$$

$$a = 32$$

$$19. (a)$$

$$\left(\frac{1}{16} \right)^{\frac{1}{4}} - \sqrt[3]{\frac{8}{27}} = \sqrt{x^2}$$

$$\frac{1}{2} - \frac{2}{3} = x$$

$$x = \frac{3}{6} - \frac{4}{6}$$

$$x = \frac{-1}{6}$$

$$(b)$$

$$\sqrt[3]{\frac{1}{8}} - \sqrt[4]{x^4} + 15 = \sqrt[4]{16}$$

$$\frac{1}{2} - x + 15 = 2$$

$$x = \frac{1}{2} + 13$$

$$x = \frac{27}{2}$$

$$20. A: \frac{a^{-2b} a^{-b+2}}{(a^{-2})^b} = a^{-2b-b+2+2b} = a^{-b+2} = 2^3 = 8$$

$$B: \frac{(a^b)^{-3} a^{-(1-2b)}}{(a^{-b})^3} = a^{-3b-1+2b+3b} = a^{2b-1} = 2^{-3} = \frac{1}{8}$$

$\therefore$ A has the greater value.

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$$1. (a) 27 = 3^3$$

$$(b) 81 = 3^4$$

$$(c) \frac{1}{9} = 3^{-2}$$

$$(d) 9^{2x} = (3^2)^{2x} = 3^{4x}$$

$$(e) \left(\frac{1}{27} \right)^x = \left(\frac{1}{3} \right)^{3x} = 3^{-3x}$$

$$2. (a) 3^{2x-5} = 27$$

$$(i) \text{ For } x = 1: 3^{2(1)-5} = 3^{-3} = \frac{1}{27} \neq 27$$

$$(ii) \text{ For } x = 4: 3^{2(4)-5} = 3^3 = 27$$

$\therefore x = 4$ is the solution.