

$$\begin{aligned}
\text{(c)} \quad & -3x(x^2 - 2x - 4) - x(x^2 - 3x - 2) \\
& = -3x^3 + 6x^2 + 12x - x^3 + 3x^2 + 2x \\
& = -4x^3 + 9x^2 + 14x \\
\text{(d)} \quad & 4x^2(2x^3 - x + 3) - x^3(3x^3 + 5x - 1) \\
& = 8x^5 - 4x^3 + 12x^2 - 3x^6 - 5x^4 + x^3 \\
& = -3x^6 + 8x^5 - 5x^4 - 3x^3 + 12x^2 \\
\text{(e)} \quad & x[2x - (3x - 4)] + 4x[2x - (3 - x)] \\
& = x[2x - 3x + 4] + 4x[2x - 3 + x] \\
& = x[-x + 4] + 4x[3x - 3] \\
& = -x^2 + 4x + 12x^2 - 12x \\
& = 11x^2 - 8x \\
\text{(f)} \quad & x[2x - (5x - 4)] + 4x[x - (6 - x)] \\
& = x[2x - 5x + 4] + 4x[x - 6 + x] \\
& = x[-3x + 4] + 4x[2x - 6] \\
& = -3x^2 + 4x + 8x^2 - 24x \\
& = 5x^2 - 20x
\end{aligned}$$

$$\begin{aligned}
60. \text{ (a)} \quad & (x^2 + 4)(x^2 - 2x + 3) \\
& = x^4 - 2x^3 + 3x^2 + 4x^2 - 8x + 12 \\
& = x^4 - 2x^3 + 7x^2 - 8x + 12 \\
\text{(b)} \quad & 3x(3x^2 - 1)^2 - 2x(1 - x^2)^2 \\
& = 3x(9x^4 - 6x^2 + 1) - 2x(1 - 2x^2 + x^4) \\
& = 27x^5 - 18x^3 + 3x - 2x + 4x^3 - 2x^5 \\
& = 25x^5 - 14x^3 + x \\
\text{(c)} \quad & (t - 2)(3t - 1)(4t + 3) - (2 - t)^2(t + 5) \\
& = (t - 2)(12t^2 + 5t - 3) - (4 - 4t + t^2)(t + 5) \\
& = 12t^3 + 5t^2 - 3t - 24t^2 - 10t + 6 \\
& \quad - (4t + 20 - 4t^2 - 20t + t^3 + 5t^2) \\
& = 12t^3 - 19t^2 - 13t + 6 - (t^3 + t^2 - 16t + 20) \\
& = 12t^3 - 19t^2 - 13t + 6 - t^3 - t^2 + 16t - 20 \\
& = 11t^3 - 20t^2 + 3t - 14 \\
\text{(d)} \quad & (x - 3)^2(x + 1)^2 - (x + 2)^2(x - 4)^2 \\
& = (x^2 - 6x + 9)(x^2 + 2x + 1) - (x^2 + 4x + 4)(x^2 - 8x + 16) \\
& = x^4 + 2x^3 + x^2 - 6x^3 - 12x^2 - 6x + 9x^2 + 18x + 9 \\
& \quad - (x^4 - 8x^3 + 16x^2 + 4x^3 - 32x^2 + 64x + 4x^2 - 32x + 64) \\
& = x^4 - 4x^3 - 2x^2 + 12x + 9 - (x^4 - 4x^3 - 12x^2 + 32x + 64) \\
& = x^4 - 4x^3 - 2x^2 + 12x + 9 - x^4 + 4x^3 + 12x^2 - 32x - 64 \\
& = 10x^2 - 20x - 55 \\
\text{(e)} \quad & (3x - 2)(x^3 + 4x^2 - 2x - 1) \\
& = 3x^4 + 12x^3 - 6x^2 - 3x - 2x^3 - 8x^2 + 4x + 2 \\
& = 3x^4 + 10x^3 - 14x^2 + x + 2 \\
\text{(f)} \quad & (a^2 + 4a - 3)(3a^2 - 2a + 4) \\
& = 3a^4 - 2a^3 + 4a^2 + 12a^3 - 8a^2 + 16a - 9a^2 + 6a - 12 \\
& = 3a^4 + 10a^3 - 13a^2 + 22a - 12 \\
\text{(g)} \quad & (x^2 - 3x + 2)^2 \\
& = (x^2 - 3x + 2)(x^2 - 3x + 2) \\
& = x^4 - 3x^3 + 2x^2 - 3x^3 + 9x^2 - 6x + 2x^2 - 6x + 4 \\
& = x^4 - 6x^3 + 13x^2 - 12x + 4 \\
\text{(h)} \quad & (x - 4)^2(x + 1)^2 \\
& = (x^2 - 8x + 16)(x^2 + 2x + 1) \\
& = x^4 + 2x^3 + x^2 - 8x^3 - 16x^2 - 8x + 16x^2 + 32x + 16 \\
& = x^4 - 6x^3 + x^2 + 24x + 16
\end{aligned}$$

61. If the triangle contains a right angle, its sides will obey the Pythagorean Theorem.

Squaring each side:

$$\begin{aligned}
\left[\frac{1}{2}(n^2 + 1)\right]^2 &= \frac{1}{4}(n^4 + 2n^2 + 1) \\
\left[\frac{1}{2}(n^2 - 1)\right]^2 &= \frac{1}{4}(n^4 - 2n^2 + 1) \\
n^2 &= n^2
\end{aligned}$$

$$\begin{aligned}
\text{But } \frac{1}{4}(n^4 - 2n^2 + 1) + n^2 &= \frac{1}{4}n^4 - \frac{1}{2}n^2 + \frac{1}{4} + n^2 \\
&= \frac{1}{4}n^4 + \frac{1}{2}n^2 + \frac{1}{4} \\
&= \frac{1}{4}(n^4 + 2n^2 + 1)
\end{aligned}$$

and we have shown that $\left[\frac{1}{2}(n^2 - 1)\right]^2 + [n]^2 = \left[\frac{1}{2}(n^2 + 1)\right]^2$.

The sides obey the Pythagorean Theorem. The triangle with these sides has a right angle.

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1. (a) $z = -3 + 2i$, $w = 2 - i$

$$\begin{aligned}
z + w &= -3 + 2i + 2 - i \\
&= -1 + i
\end{aligned}$$

$$\begin{aligned}
z - w &= -3 + 2i - (2 - i) \\
&= -3 + 2i - 2 + i \\
&= -5 + 3i
\end{aligned}$$

$$z\bar{z} = (-3 + 2i)(-3 - 2i)$$

$$\begin{aligned}
&= 9 - 4i^2 \\
&= 9 - 4(-1) \\
&= 13
\end{aligned}$$

$$\begin{aligned}
\frac{z}{2w} &= \frac{-3 + 2i}{4 - 2i} \\
&= \frac{-3 + 2i}{4 - 2i} \times \frac{4 + 2i}{4 + 2i} \\
&= \frac{(-3 + 2i)(4 + 2i)}{16 - 4i^2} \\
&= \frac{-12 - 6i + 8i + 4i^2}{16 + 4} \\
&= \frac{-12 + 2i - 4}{20} \\
&= \frac{-16 + 2i}{20} \\
&= \frac{2(-8 + i)}{2(10)} \\
&= -\frac{4}{5} + \frac{1}{10}i
\end{aligned}$$

(b) $f(x) = -2x^2 - 3x + 1$

$$\begin{aligned}
f(2 - 5i) &= -2(2 - 5i)^2 - 3(2 - 5i) + 1 \\
&= -2(4 - 20i + 25i^2) - 6 + 15i + 1 \\
&= -2(4 - 20i - 25) - 5 + 15i \\
&= -2(-21 - 20i) - 5 + 15i \\
&= 42 + 40i - 5 + 15i \\
&= 37 + 55i
\end{aligned}$$

2. (a)

$$\frac{28x^3y}{-21xy^2} = \frac{(7xy)(4x^2)}{(7xy)(-3y)}, x \neq 0, y \neq 0$$

$$= -\frac{4x^2}{3y}, x \neq 0, y \neq 0$$

(b)

$$\frac{7}{x^2 - 4} - \frac{3}{x^2 + 5x + 6}$$

$$= \frac{7}{(x-2)(x+2)} - \frac{3}{(x+2)(x+3)}, x \neq \pm 2, -3$$

$$= \frac{7}{(x-2)(x+2)} \times \frac{(x+3)}{(x+3)} - \frac{3}{(x+2)(x+3)} \times \frac{(x-2)}{(x-2)}$$

$$= \frac{7x+21}{(x-2)(x+2)(x+3)} - \frac{(3x-6)}{(x+2)(x+3)(x-2)}$$

$$= \frac{7x+21-3x+6}{(x-2)(x+2)(x+3)}$$

$$= \frac{4x+27}{(x-2)(x+2)(x+3)}, x \neq \pm 2, -3$$

(c)

$$\frac{a^2 - b^2}{a} \times \frac{3ab}{a^2 + 4ab + 3b^2} \div \frac{a^2 - ab}{a + 3b}$$

$$= \frac{(a-b)(a+b)}{a} \times \frac{3ab}{(a+3b)(a+b)} \div \frac{a(a-b)}{a+3b}, a \neq 0, -b, -3b$$

$$= \frac{(a-b)(a+b)}{a} \times \frac{3ab}{(a+3b)(a+b)} \times \frac{a+3b}{a(a-b)}, a \neq 0, \pm b, -3b$$

$$= \frac{3b}{a}, a \neq 0, \pm b, -3b$$

(d)

$$(x-y)^3 + (x+y)^3 + 3(x+y)(x-y)^2 + 3(x-y)(x+y)^2$$

$$= (x-y)^2[x-y+3x+3y] + (x+y)^2[x+y+3x-3y]$$

$$= (x-y)^2[4x+2y] + (x+y)^2[4x-2y]$$

$$= (x^2 - 2xy + y^2)(4x+2y) + (x^2 + 2xy + y^2)(4x-2y)$$

$$= 4x^3 + 2x^2y - 8xy^2 - 4xy^2 + 4xy^2 + 2y^3 + 4x^3 - 2x^2y$$

$$+ 8x^2y - 4xy^2 + 4xy^2 - 2y^3$$

$$= 8x^3$$

3. (a) (i)

$$f(x) = -3x^2 + 8x$$

$$= -3x\left(x - \frac{8}{3}\right)$$

$$x\text{-coordinate of vertex is } \frac{-b}{2a} = \frac{-8}{2(-3)} = \frac{4}{3}.$$

y-coordinate of vertex is

$$f\left(\frac{4}{3}\right) = -3\left(\frac{4}{3}\right)^2 + 8\left(\frac{4}{3}\right)$$

$$= -\frac{16}{3} + \frac{32}{3}$$

$$= \frac{16}{3}$$

$$\text{Vertex has coordinates } \left(\frac{4}{3}, \frac{16}{3}\right).$$

Since $a < 0$, the function has a maximum value.

(ii) $g(x) = (x-1)(x+5)$

$$x\text{-coordinate of vertex is } \frac{1+(-5)}{2} = -2.$$

$$y\text{-coordinate of vertex is } g(-2) = (-2-1)(-2+5) = -9.$$

Vertex has coordinates $(-2, -9)$.

Since $a > 0$, $g(x)$ has a minimum value.

(iii) $h(t) = -4.9t^2 + 24.5t + 6$

$$x\text{-coordinate of vertex is } \frac{-b}{2a} = \frac{-24.5}{2(-4.9)} = 2.5.$$

$$y\text{-coordinate of vertex is } h(2.5) = -4.9(2.5)^2 + 24.5(2.5) + 6$$

$$= 36.625$$

Vertex has coordinates $(2.5, 36.625)$.

Since $a < 0$, $h(t)$ has a maximum value.

(b) Let x be the number of \$2 increases in rental price. Then rental price is $48 + 2x$, and number of cars rented is $150 - 4x$.

Revenue from car rental $R(x) = (150 - 4x)(48 + 2x)$.

Maintenance costs $C(x) = 5(150 - 4x)$.

$$\text{Profit } P(x) = R(x) - C(x)$$

$$= (150 - 4x)(48 + 2x) - 5(150 - 4x)$$

$$= (150 - 4x)(48 + 2x - 5)$$

$$P(x) = (150 - 4x)(43 + 2x)$$

$$= 6450 + 128x - 8x^2$$

$$= -8(x^2 - 16x) + 6450$$

$$= -8(x^2 - 16x + 64 - 64) + 6450$$

$$= -8(x^2 - 16x + 64) + 512 + 6450$$

$$= -8(x - 8)^2 + 6962$$

Profit is maximized when $x = 8$. A rental charge of $\$48 + \$2(8) = \$64$ will maximize profit.

OR with $P(x)$ in factored form:

$$P(x) = (150 - 4x)(43 + 2x)$$

$$x\text{-coordinate of vertex is } \frac{1}{2}\left[\frac{150}{4} + \left(-\frac{43}{2}\right)\right] = 8.$$

$$4. s(t) = s_0 + v_0t - \frac{1}{2}gt^2$$

Assume that Romeo lets go of the locket at a height of 1 metre above the ground. So $s_0 = 1$, and $s(t) = 1 + v_0t - 4.9t^2$.

The locket must reach a height of at least 9 metres.

$$\text{If } s(t) = 9, 9 = 1 + v_0t - 4.9t^2$$

$$0 = -4.9t^2 + v_0t - 8$$

This equation must have at least one root.

$$\text{so } b^2 - 4ac \geq 0$$

$$v_0^2 - 4(-4.9)(-8) \geq 0$$

$$v_0^2 - 156.8 \geq 0$$

$$v_0^2 \geq 156.8$$

Assume Romeo throws the locket up (not down!) ie. assume $v_0 > 0$. Then $v_0 \geq 12.5$ (correct to 1 decimal place). Romeo must give the locket an initial velocity of at least 12.5 m/s for it to reach Juliet.

5. (a) $kx^2 + k = 8x - 2kx$

$$kx^2 - 8x + 2kx + k = 0$$

$$kx^2 + (2k - 8)x + k = 0$$

$$\begin{aligned} b^2 - 4ac &= (2k - 8)^2 - 4(k)(k) \\ &= 4k^2 - 32k + 64 - 4k^2 \\ &= 64 - 32k \end{aligned}$$

(i) for two real roots, $b^2 - 4ac > 0$

$$64 - 32k > 0$$

$$64 > 32k$$

$$2 > k$$

Equation has two real roots if $k < 2$.

(ii) for one real root, $b^2 - 4ac = 0$

$$64 - 32k = 0$$

$$64 = 32k$$

$$2 = k$$

Equation has one real root if $k = 2$.

(iii) for no real roots, $b^2 - 4ac < 0$

$$64 - 32k < 0$$

$$64 < 32k$$

$$2 < k$$

Equation has no real roots if $k > 2$.

(b) If $1 - 2i$ is one root, the other root is $1 + 2i$.

$$\text{Equation is } (x - (1 - 2i))(x - (1 + 2i)) = 0$$

$$(x - 1 + 2i)(x - 1 - 2i) = 0$$

$$((x - 1) + 2i)((x - 1) - 2i) = 0$$

$$(x - 1)^2 - 4i^2 = 0$$

$$x^2 - 2x + 1 + 4 = 0$$

$$x^2 - 2x + 5 = 0$$

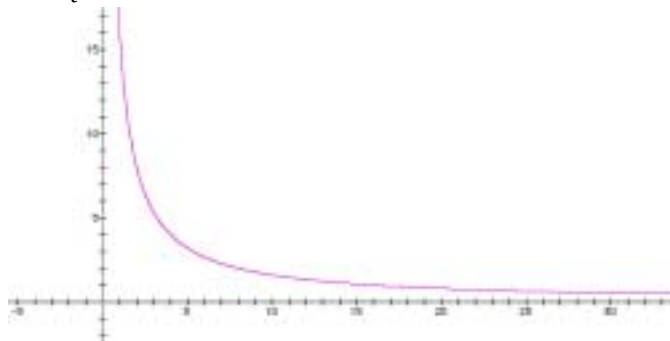
6.

	Distance (km)	Speed (km/h)	Time (h)
walk	16	3	5. $\dot{3}$
jog	16	4	4
run	16	6	2. $\dot{6}$
cycle	16	10	1.6

If s represents speed and t represents time,

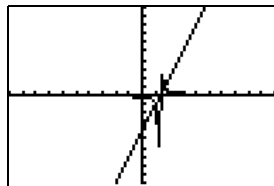
$$st = 16$$

$$s = \frac{16}{t}$$



The relationship between speed and time is an example of an inverse variation.

7.



For reciprocal function, the domain is all real numbers except $\frac{4}{3}$. The range is all real numbers except 0.

The properties of functions and their reciprocals that help with sketching the graph of the reciprocal functions are:

- at each zero of $f(x)$, the graph of $y = \frac{1}{f(x)}$ has a vertical asymptote;
- the graph of $y = \frac{1}{f(x)}$ always passes through the points where $f(x) = 1$ or $f(x) = -1$;
- when $f(x)$ is close to 1, $\frac{1}{f(x)}$ is also close to 1;
- when $f(x)$ is close to -1 , $\frac{1}{f(x)}$ is also close to -1 ;
- both graphs occur in the same quadrants;
- when $f(x)$ is positive, $\frac{1}{f(x)}$ is positive;
- when $f(x)$ is negative, $\frac{1}{f(x)}$ is negative;
- as $f(x)$ gets larger, $\frac{1}{f(x)}$ gets smaller;
- as $f(x)$ gets smaller, $\frac{1}{f(x)}$ gets larger.

8.

$$\begin{aligned} \frac{(t^3 - 4t^2 - 5t)a}{22t^2 - 110t} &= \frac{t(t^2 - 4t - 5)a}{22t(t - 5)}, t \neq 0, 5 \\ &= \frac{t(t - 5)(t + 1)a}{22t(t - 5)} \\ &= \frac{(t + 1)a}{22}, t \neq 0, 5 \end{aligned}$$

(a) For a one-year-old child, $t = 1$,

$$\text{dose} = \frac{(1 + 1)}{22}(200 \text{ mg}) \doteq 18.2 \text{ mg}$$

(b) For a five-year-old child, we cannot use $t = 5$ since this was a restriction on t . Instead, we can use $t = 5.001$.

$$\text{dose} = \frac{(5.001 + 1)}{22}(200 \text{ mg}) \doteq 54.6 \text{ mg}$$

9. Let Jovanna's speed be x km/h.

	Distance (km)	Speed (km/h)	Time (h)
Marissa	200	$x + 5$	$\frac{200}{x + 5}$
Jovanna	200	x	$\frac{200}{x}$

Since the race finished in a tie, the difference between the times taken for each girl to finish the race is $\frac{1}{2}$ hour, the time

it took to repair Marissa's flat tire.

$$\frac{200}{x} - \frac{200}{x+5} = \frac{1}{2}, \quad x \neq 0, 5$$

Multiply through by $2x(x+5)$ to clear all fractions.

$$\frac{200}{x} \times 2x(x+5) - \frac{200}{x+5} \times 2x(x+5) = \frac{1}{2} \times 2x(x+5)$$

$$400(x+5) - 400x = x(x+5)$$

$$400x + 2000 - 400x = x^2 + 5x$$

$$x^2 + 5x - 2000 = 0$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-2000)}}{2}$$

$$= \frac{-5 \pm \sqrt{8025}}{2}$$

$$x \doteq -47.3 \text{ or } 42.3$$

We reject the negative root in this context. Speed is positive.

Jovanna's speed is 42.3 km/h and Marissa's speed is 47.3 km/h.

10. In a right triangle, the lengths of the sides are related as stated in the Pythagorean Theorem. The sum of the squares of the two shorter sides is equal to the square of the hypotenuse.

If we square the sides of this triangle,

$$(n^2 + 1)^2 = n^4 + 2n^2 + 1$$

$$(n^2 - 1)^2 = n^4 - 2n^2 + 1$$

$$(2n)^2 = 4n^2$$

When we add the squares of the last two sides listed,

$$\begin{aligned} (n^2 - 1)^2 + (2n)^2 &= n^4 - 2n^2 + 1 + 4n^2 \\ &= n^4 + 2n^2 + 1 \end{aligned}$$

This is the square of the first side listed.

Therefore, $(n^2 - 1)^2 + (2n)^2 = (n^2 + 1)^2$

The sides are related as stated in the Pythagorean Theorem.

The triangle with these sides must be right-angled.