

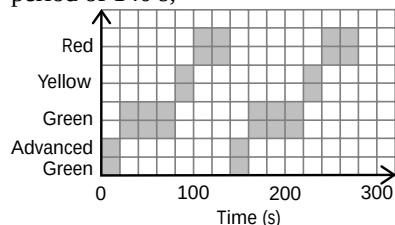
(c) The period represents one lap around the track. No speed was given, therefore the length of the period cannot be found.

13. (a) The graph is self-replicating so it represents a periodic relation: the traffic light goes from one colour to the next following the same pattern of colour and the same time interval for each colour.

(b) One cycle: long green light(60 s) changes to yellow for a short time(20 s) and then turns red(40 s)

(c) period = $60 + 20 + 40 = 120$ s or 2 min

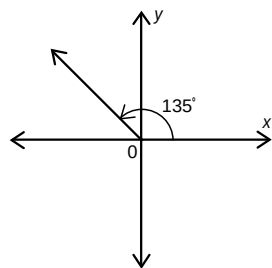
(d) An advanced green adds 20 s to the period, to give a period of 140 s;



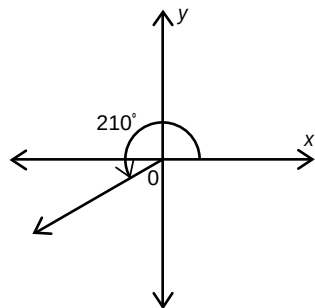
14. Answers will vary. *Example:* A function such that the range of the independent variable can be separated into equal subintervals so that the graph of the function is the same in each subinterval. (The length of the smallest equal subinterval is the period of the function). The rising and setting of the sun is a example of a periodic function. Since the sun rises and sets at approximately the same time everyday (i.e the cycle repeats every day), this is a self-replicating function.

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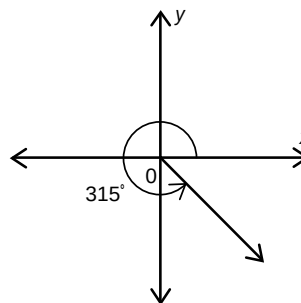
1. (a)



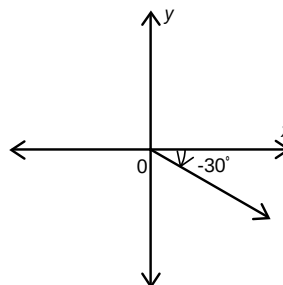
(b)



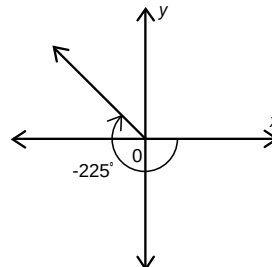
(c)



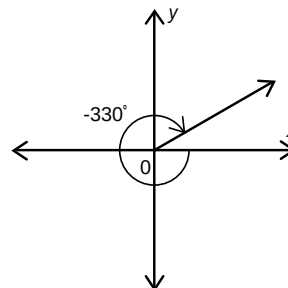
(d)



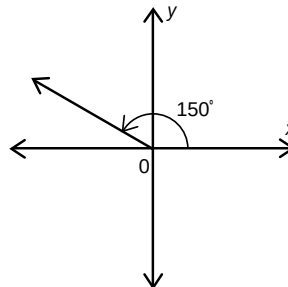
(e)



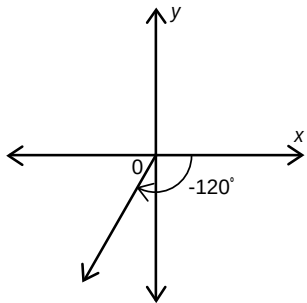
(f)



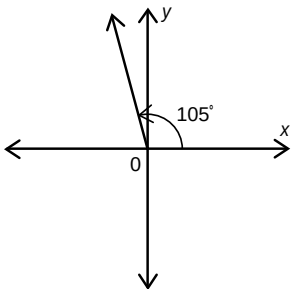
(g)



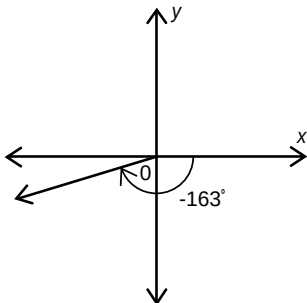
(h)



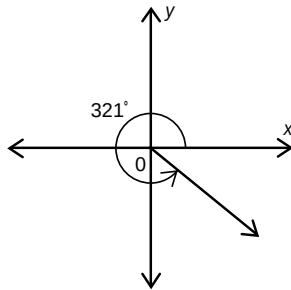
(i)



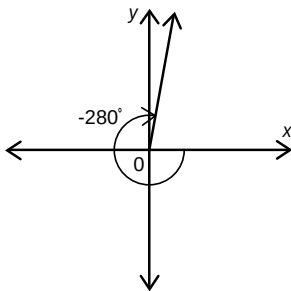
(j)



(k)



(l)



2. (a) $180^\circ - 135^\circ = 45^\circ$

(b) $210^\circ - 180^\circ = 30^\circ$

(c) $360^\circ - 315^\circ = 45^\circ$

(d) $|-30^\circ| = 30^\circ$

(e) $|-225^\circ - (-180^\circ)| = 45^\circ$

(f) $|-360^\circ - (-330^\circ)| = 30^\circ$

(g) $180^\circ - 150^\circ = 30^\circ$

(h) $|-180^\circ - (-120^\circ)| = 60^\circ$

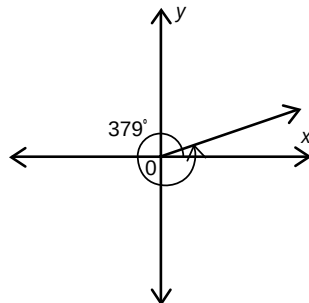
(i) $180^\circ - 105^\circ = 75^\circ$

(j) $|-180^\circ - (-163^\circ)| = 17^\circ$

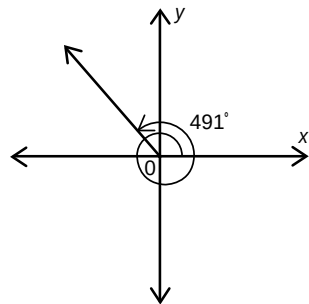
(k) $360^\circ - 321^\circ = 39^\circ$

(l) $|-360^\circ - (-280^\circ)| = 80^\circ$

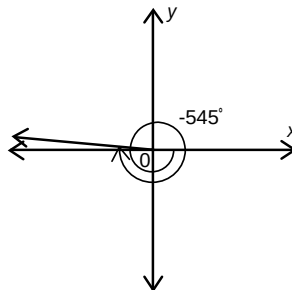
3. (a)



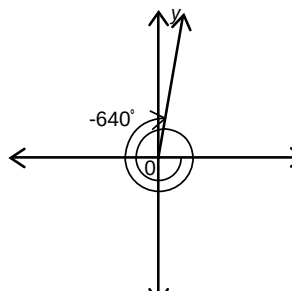
(b)



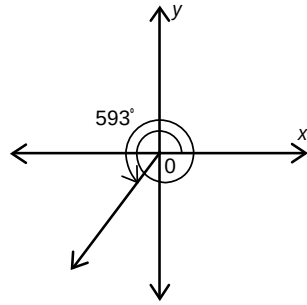
(c)



(d)



(e)



4. (a) coterminal; $360^\circ + 23^\circ = 383^\circ$
 (b) not coterminal; $360^\circ + 41^\circ \neq 421^\circ$
 (c) coterminal; $360^\circ - 50^\circ = 310^\circ$
 (d) coterminal; $360^\circ + 38^\circ = 398^\circ$
 (e) not coterminal; $360^\circ - 19^\circ \neq 390^\circ$
 (f) coterminal; $360^\circ - 41^\circ = 319^\circ$
 (g) not coterminal; $28^\circ - 360^\circ \neq -232^\circ$
 (h) coterminal; $-360^\circ - 05^\circ = -465^\circ$
 (i) coterminal; $360^\circ - 123^\circ = 237^\circ$
 (j) not coterminal; $-360^\circ - (-190)^\circ \neq 180^\circ$

5. (a) $132^\circ + 360^\circ = 492^\circ$, $132^\circ + 720^\circ = 852^\circ$
 (b) $275^\circ + 360^\circ = 635^\circ$, $275^\circ + 720^\circ = 995^\circ$
 (c) $305^\circ + 360^\circ = 665^\circ$, $305^\circ + 720^\circ = 1025^\circ$
 (d) $73^\circ + 360^\circ = 433^\circ$, $73^\circ + 720^\circ = 793^\circ$
 (e) $270^\circ + 360^\circ = 630^\circ$, $270^\circ + 720^\circ = 990^\circ$

6. (a) $-53^\circ - 360^\circ = -413^\circ$, $-53^\circ - 720^\circ = -773^\circ$
 (b) $-138^\circ - 360^\circ = -498^\circ$, $-138^\circ - 720^\circ = -858^\circ$
 (c) $-299^\circ - 360^\circ = -659^\circ$, $-299^\circ - 720^\circ = -1019^\circ$
 (d) $-180^\circ - 360^\circ = -540^\circ$, $-180^\circ - 720^\circ = -900^\circ$
 (e) $-192^\circ - 360^\circ = -552^\circ$, $-192^\circ - 720^\circ = -912^\circ$

7. (a) vi.
 (b) i.
 (c) ii.
 (d) v.
 (e) iii.
 (f) iv.
 (g) viii.
 (h) vii.

8. (a) $360^\circ - 187^\circ = 173^\circ$
 (b) $410^\circ - 360^\circ = 50^\circ$
 (c) $360^\circ - 67^\circ = 293^\circ$
 (d) $905^\circ - 720^\circ = 185^\circ$
 (e) $360^\circ - 282^\circ = 78^\circ$
 (f) $1080^\circ - 730^\circ = 350^\circ$
 (g) 135°
 (h) $1249^\circ - 1080^\circ = 169^\circ$

9. (a) $180^\circ - 24^\circ = 156^\circ$
 (b) $180^\circ + 35^\circ = 215^\circ$
 (c) $360^\circ - 19^\circ = 341^\circ$
 (d) 63°

10. (a) $51^\circ + 360^\circ n$, $n = 4, 5, 6$

$$51^\circ + 1440^\circ = 1491^\circ$$

$$51^\circ + 1800^\circ = 1851^\circ$$

$$51^\circ + 2160^\circ = 2211^\circ$$

- (b) $-71^\circ + 360^\circ n$, $n = -1, 0, 1, 2$

$$-71^\circ - 360^\circ = -431^\circ$$

$$-71^\circ + 0 = -71^\circ$$

$$-71^\circ + 360^\circ = 289^\circ$$

$$-71^\circ + 720^\circ = 649^\circ$$

- (c) $-123^\circ + 360^\circ n$, $n = -2, -1, 0$

$$-123^\circ - 720^\circ = -843^\circ$$

$$-123^\circ - 360^\circ = -483^\circ$$

$$-123^\circ + 0^\circ = -123^\circ$$

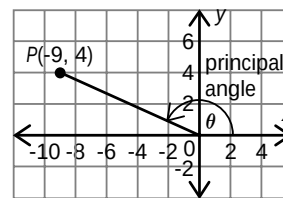
- (d) $195^\circ + 360^\circ n$, $n = 5, 6, 7$

$$195^\circ + 1800^\circ = 1995^\circ$$

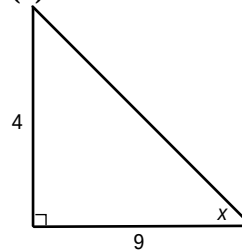
$$195^\circ + 2160^\circ = 2355^\circ$$

$$195^\circ + 2520^\circ = 2715^\circ$$

11. (a)



- (b)



Examine the triangle.

Then:

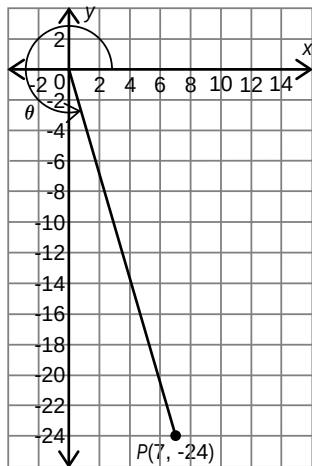
$$\tan x = \frac{4}{9}$$

$$x = \tan^{-1} \left(\frac{4}{9} \right)$$

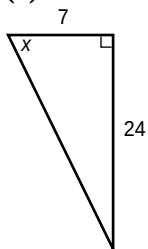
$$x \doteq 24^\circ$$

- (c) Then $\theta = 180^\circ - 24^\circ = 156^\circ$

12. (a)



(b)



Examine the triangle.

Then:

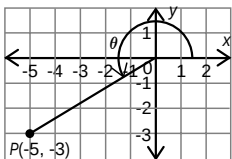
$$\tan x = \frac{24}{7}$$

$$x = \tan^{-1}\left(\frac{24}{7}\right)$$

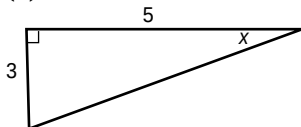
$$x \doteq 74^\circ$$

(c) Then $\theta = 360^\circ - 74^\circ = 286^\circ$

13. (a)



(b)



Examine the triangle

Then:

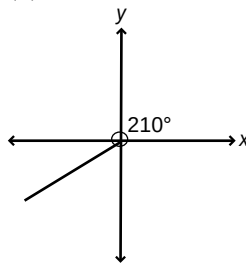
$$\tan x = \frac{3}{5}$$

$$x = \tan^{-1}\left(\frac{3}{5}\right)$$

$$x \doteq 31^\circ$$

(c) Then $\theta = 180^\circ + 31^\circ = 211^\circ$

(d)



The first negative coterminal angle is $-360^\circ + 211^\circ = -149^\circ$

14. The point $(-5, -9)$ forms a right triangle in quadrant III with horizontal side 5 and vertical side 9. The $\angle x$, which is the related acute angle for θ is linked to the sides 5 and 9 by the tangent ratio:

$$\tan x = \frac{9}{5}$$

$$x = \tan^{-1}\left(\frac{9}{5}\right)$$

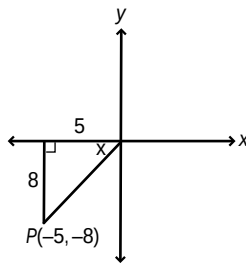
$$x \doteq 61^\circ$$

Once the related acute value is known, then

$\theta = (180^\circ + \text{related acute angle})$ for a quadrant III angle θ .

Therefore, $\theta = 180^\circ + 61^\circ = 241^\circ$

15.



Examine the triangle.

Then:

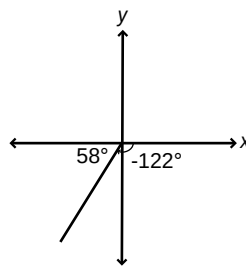
$$\tan x = \frac{8}{5}$$

$$x = \tan^{-1}\left(\frac{8}{5}\right)$$

$$x \doteq 58^\circ$$

So the related acute angle is 58° .

Since $-540^\circ \leq \theta \leq 270^\circ$ then:



$$\theta_1 = -180^\circ + 58^\circ = -122^\circ$$

and the next angle is θ_2 :

$$\theta_2 = -360^\circ - 122^\circ = -482^\circ$$

and the next angle is θ_3 :

$$\theta_3 = -122^\circ + 360^\circ = 238^\circ$$

Since θ_1 , θ_2 and θ_3 are all within the given domain,
 $\theta = -482^\circ, -122^\circ, 238^\circ$ are the values that satisfy the conditions.

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1. (a) iii, $270^\circ \leq \theta \leq 360^\circ$

(b) i, $90^\circ \leq \theta \leq 180^\circ$

(c) iv, $-180^\circ \leq \theta \leq -90^\circ$

(d) ii, $-90^\circ \leq \theta \leq 0^\circ$

2. (a) ii, $-180^\circ \leq \theta \leq 0^\circ$

(b) iii, $-270^\circ \leq \theta \leq -90^\circ$

(c) iv, $90^\circ \leq \theta \leq 360^\circ$

(d) i, $0^\circ \leq \theta \leq 270^\circ$

3. (a) i, $-180^\circ \leq \theta \leq -90^\circ$

(b) ii, $-270^\circ \leq \theta \leq -180^\circ$

(c) iii, $0^\circ \leq \theta \leq 90^\circ$

(d) iv, $90^\circ \leq \theta \leq 270^\circ$

4. (a) quadrant I, positive

(b) quadrant II, positive

(c) quadrant II, negative

(d) quadrant II, negative

(e) quadrant III, negative

(f) quadrant IV, negative

(g) quadrant II, negative

(h) quadrant IV, negative

5. (a)

$$r = \sqrt{(-3)^2 + (-4)^2} = 5$$

$$\sin \theta = \frac{y}{r} = \frac{-4}{5} = -\frac{4}{5}$$

$$\cos \theta = \frac{x}{r} = \frac{-3}{5} = -\frac{3}{5}$$

$$\tan \theta = \frac{y}{x} = \frac{-4}{-3} = \frac{4}{3}$$

(b)

$$r = \sqrt{(3)^2 + (4)^2} = 5$$

$$\sin \theta = \frac{y}{r} = \frac{4}{5}$$

$$\cos \theta = \frac{x}{r} = \frac{3}{5}$$

$$\tan \theta = \frac{y}{x} = \frac{4}{3}$$

(c)

$$r = \sqrt{(5)^2 + (12)^2} = 13$$

$$\sin \theta = \frac{y}{r} = \frac{12}{13}$$

$$\cos \theta = \frac{x}{r} = \frac{5}{13}$$

$$\tan \theta = \frac{y}{x} = \frac{12}{5}$$

(d)

$$r = \sqrt{(-12)^2 + (5)^2} = 13$$

$$\sin \theta = \frac{y}{r} = \frac{5}{13}$$

$$\cos \theta = \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13}$$

$$\tan \theta = \frac{y}{x} = \frac{5}{-12} = -\frac{5}{12}$$

(e)

$$r = \sqrt{(7)^2 + (-24)^2} = 25$$

$$\sin \theta = \frac{y}{r} = \frac{-24}{25} = -\frac{24}{25}$$

$$\cos \theta = \frac{x}{r} = \frac{7}{25}$$

$$\tan \theta = \frac{y}{x} = \frac{-24}{7} = -\frac{24}{7}$$

(f)

$$r = \sqrt{(-7)^2 + (24)^2} = 25$$

$$\sin \theta = \frac{y}{r} = \frac{24}{25}$$

$$\cos \theta = \frac{x}{r} = \frac{-7}{25} = -\frac{7}{25}$$

$$\tan \theta = \frac{y}{x} = \frac{24}{-7} = -\frac{24}{7}$$

(g)

$$r = \sqrt{(0)^2 + (-1)^2} = 1$$

$$\sin \theta = \frac{y}{r} = \frac{-1}{1} = -1$$

$$\cos \theta = \frac{x}{r} = \frac{0}{1} = 0$$

$$\tan \theta = \frac{y}{x} = \frac{-1}{0} = \text{undefined}$$

(h)

$$r = \sqrt{(-1)^2 + (0)^2} = 1$$

$$\sin \theta = \frac{y}{r} = \frac{0}{1} = 0$$

$$\cos \theta = \frac{x}{r} = \frac{-1}{1} = -1$$

$$\tan \theta = \frac{y}{x} = \frac{0}{-1} = 0$$

6. (a)

$$\tan \theta = \frac{y}{x} = \frac{11}{5}$$

$$\theta = \tan^{-1} \left(\frac{11}{5} \right) \doteq 66^\circ$$

This is in quadrant I

(b)

$$\tan \theta = \frac{y}{x} = \frac{-2}{9}$$

$$\theta = \tan^{-1} \left(\frac{-2}{9} \right) \doteq -12.5^\circ$$