

(b)

$$\csc \theta = \frac{1}{\frac{y}{r}} \quad \sec \theta = \frac{1}{\frac{x}{r}} \quad \cot \theta = \frac{1}{\frac{y}{x}}$$

$$\csc \theta = \frac{r}{y}, y \neq 0 \quad \sec \theta = \frac{r}{x}, x \neq 0 \quad \cot \theta = \frac{x}{y}, y \neq 0$$

(c)

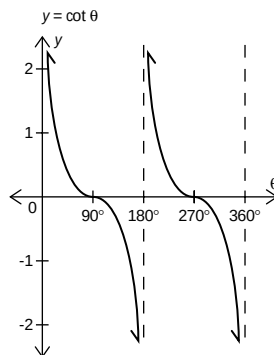
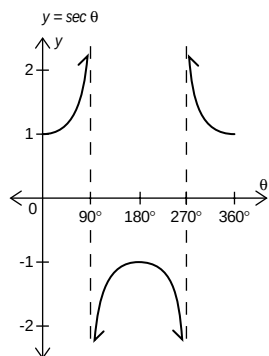
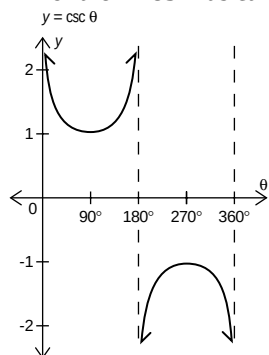
| $\theta$      | $0^\circ$   | $30^\circ$  | $60^\circ$  | $90^\circ$  | $120^\circ$ | $150^\circ$ |
|---------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\csc \theta$ | —           | 2           | 1.2         | 1           | 1.2         | 2           |
| $\theta$      | $180^\circ$ | $210^\circ$ | $240^\circ$ | $270^\circ$ | $300^\circ$ | $330^\circ$ |
| $\csc \theta$ | —           | -2          | -1.2        | -1          | -1.2        | -2          |
| $\theta$      | $360^\circ$ |             |             |             |             |             |
| $\csc \theta$ | —           |             |             |             |             |             |

| $\theta$      | $0^\circ$   | $30^\circ$  | $60^\circ$  | $90^\circ$  | $120^\circ$ | $150^\circ$ |
|---------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\sec \theta$ | 1           | 1.2         | 2           | —           | -2          | -1.2        |
| $\theta$      | $180^\circ$ | $210^\circ$ | $240^\circ$ | $270^\circ$ | $300^\circ$ | $330^\circ$ |
| $\sec \theta$ | -1          | -1.2        | -2          | —           | 2           | 1.2         |
| $\theta$      | $360^\circ$ |             |             |             |             |             |
| $\sec \theta$ | 1           |             |             |             |             |             |

| $\theta$      | $0^\circ$   | $30^\circ$  | $60^\circ$  | $90^\circ$  | $120^\circ$ | $150^\circ$ |
|---------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\cot \theta$ | —           | 1.7         | 0.6         | 0           | -0.6        | -1.7        |
| $\theta$      | $180^\circ$ | $210^\circ$ | $240^\circ$ | $270^\circ$ | $300^\circ$ | $330^\circ$ |
| $\cot \theta$ | —           | 1.7         | 0.6         | 0           | -0.6        | -1.7        |
| $\theta$      | $360^\circ$ |             |             |             |             |             |
| $\cot \theta$ | —           |             |             |             |             |             |

Be sure that the calculator is set to degree mode. To calculate the value of a secondary trigonometric function, determine the value of the primary trigonometric function and then

apply the reciprocal key  $\frac{1}{x}$  key for a scientific calculator and  $x^{-1}$  for the TI-83 Plus calculator).



(d) The restrictions are shown as vertical asymptotes on the graphs.

(e) For  $y = \csc \theta$ :

$$D = \{\theta \mid \theta = m^\circ, m \in \mathbf{R}, \theta \neq 180^\circ n, n \in \mathbf{I}\}$$

$$R = \{y \mid y \leq -1 \text{ or } y \geq 1, y \in \mathbf{R}\}$$

For  $y = \sec \theta$ :

$$D = \{\theta \mid \theta = m^\circ, m \in \mathbf{R}, \theta \neq -90^\circ + 180^\circ n, n \in \mathbf{I}\}$$

$$R = \{y \mid y \leq -1 \text{ or } y \geq 1, y \in \mathbf{R}\}$$

For  $y = \cot \theta$ :

$$D = \{\theta \mid \theta = m^\circ, m \in \mathbf{R}, \theta \neq 180^\circ n, n \in \mathbf{I}\}$$

$$R = \{y \mid y \in \mathbf{R}\}$$

## Practise, Apply, Solve 5.4 page 442

1. The circumference of a circle of radius 1 is  $C = 2\pi r$ .

$$C = 2\pi$$

A complete rotation about a circle is  $360^\circ$ .

A counterclockwise rotation is positive.

A clockwise rotation is negative.

(a) positive rotation  $\frac{1}{2}$  circle,  $\frac{1}{2}(2\pi) = \pi$ ,  $\frac{1}{2}(360^\circ) = 180^\circ$

(b) positive rotation  $\frac{1}{4}$  circle,  $\frac{1}{4}(2\pi) = \frac{\pi}{2}$ ,  $\frac{1}{4}(360^\circ) = 90^\circ$

(c) negative rotation  $\frac{1}{2}$  circle,  $-\frac{1}{2}(2\pi) = -\pi$ ,  
 $-\frac{1}{2}(360^\circ) = -180^\circ$

(d) negative rotation  $\frac{3}{4}$  circle,  $-\frac{3}{4}(2\pi) = -\frac{3\pi}{2}$ ,  
 $-\frac{3}{4}(360^\circ) = -270^\circ$

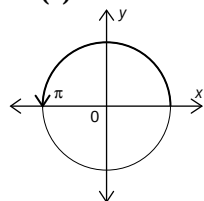
(e) negative rotation full circle,  $-(2\pi) = -2\pi$ ,  $-(360^\circ) = -360^\circ$

(f) positive rotation  $\frac{3}{4}$  circle,  $\frac{3}{4}(2\pi) = \frac{3\pi}{2}$ ,  $\frac{3}{4}(360^\circ) = 270^\circ$

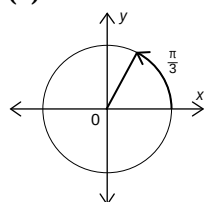
(g) negative rotation  $\frac{4}{6}$  circle,  $-\frac{4}{6}(2\pi) = -\frac{4\pi}{3}$ ,  
 $-\frac{4}{6}(360^\circ) = -240^\circ$

(h) positive rotation  $\frac{3}{8}$  circle,  $\frac{3}{8}(2\pi) = \frac{3\pi}{4}$ ,  $\frac{3}{8}(360^\circ) = 135^\circ$

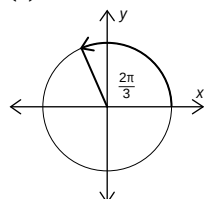
2. (a)



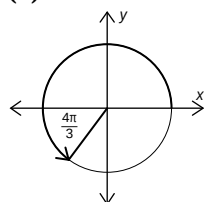
(b)



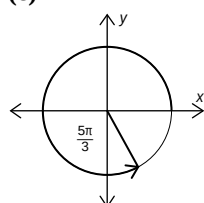
(c)



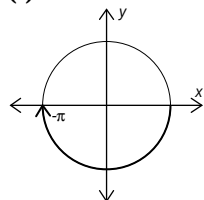
(d)



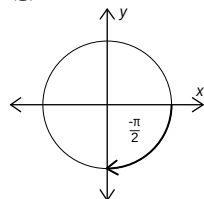
(e)



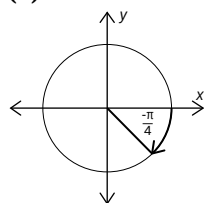
(f)



(g)



(h)



3. (a)

$$\frac{2\pi}{3} = \frac{x}{180^\circ}$$

$$\frac{2}{3} = \frac{x}{180^\circ}$$

$$x = 120^\circ$$

(b)

$$-\frac{5\pi}{3} = \frac{x}{180^\circ}$$

$$-\frac{5}{3} = \frac{x}{180^\circ}$$

$$x = -300^\circ$$

(c)

$$\frac{\pi}{4} = \frac{x}{180^\circ}$$

$$\frac{1}{4} = \frac{x}{180^\circ}$$

$$x = 45^\circ$$

(d)

$$-\frac{3\pi}{4} = \frac{x}{180^\circ}$$

$$-\frac{3}{4} = \frac{x}{180^\circ}$$

$$x = -135^\circ$$

(e)

$$\frac{7\pi}{6} = \frac{x}{180^\circ}$$

$$\frac{7}{6} = \frac{x}{180^\circ}$$

$$x = 210^\circ$$

(f)

$$-\frac{3\pi}{2} = \frac{x}{180^\circ}$$

$$-\frac{3}{2} = \frac{x}{180^\circ}$$

$$x = -270^\circ$$

(g)

$$\frac{11\pi}{6} = \frac{x}{180^\circ}$$

$$\frac{11}{6} = \frac{x}{180^\circ}$$

$$x = 330^\circ$$

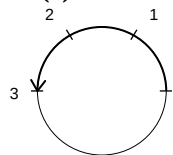
(h)

$$-\frac{9\pi}{2} = \frac{x}{180^\circ}$$

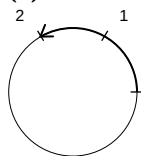
$$-\frac{9}{2} = \frac{x}{180^\circ}$$

$$x = -810^\circ$$

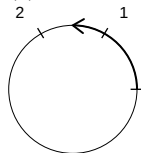
4. (a)



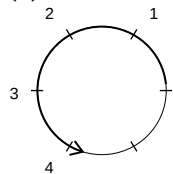
(b)



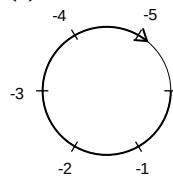
(c)



(d)



(e)



5. To convert from degree measure to radian measure use the proportion  $\frac{\text{Degree Measure}}{\text{Degree Measure}} = \frac{\text{Radian Measure}}{\text{Radian Measure}}$ .

(a)

$$\frac{x}{\pi} = \frac{90^\circ}{180^\circ}$$

$$x = \frac{\pi}{2}$$

(b)

$$\frac{x}{\pi} = \frac{270^\circ}{180^\circ}$$

$$x = \frac{3\pi}{2}$$

(c)

$$\frac{x}{\pi} = \frac{-180^\circ}{180^\circ}$$

$$x = -\pi$$

(d)

$$\frac{x}{\pi} = \frac{45^\circ}{180^\circ}$$

$$x = \frac{\pi}{4}$$

(e)

$$\frac{x}{\pi} = \frac{-135^\circ}{180^\circ}$$

$$x = -\frac{3\pi}{4}$$

(f)

$$\frac{x}{\pi} = \frac{60^\circ}{180^\circ}$$

$$x = \frac{\pi}{3}$$

(g)

$$\frac{x}{\pi} = \frac{240^\circ}{180^\circ}$$

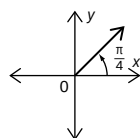
$$x = \frac{4\pi}{3}$$

(h)

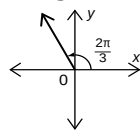
$$\frac{x}{\pi} = -\frac{120^\circ}{180^\circ}$$

$$x = -\frac{2\pi}{3}$$

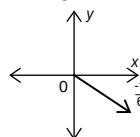
6. (a)  $\frac{\pi}{4}$



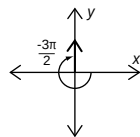
(b)  $\frac{\pi}{3}$



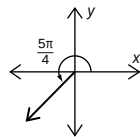
(c)  $\frac{\pi}{6}$

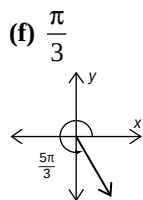


(d) There is no related acute angle.



(e)  $\frac{\pi}{4}$



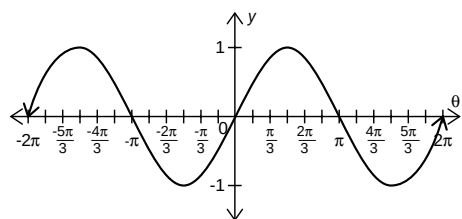


7. (a)

| $\theta$      | $-2\pi$ | $-\frac{11\pi}{6}$ | $-\frac{10\pi}{6}$ | $-\frac{9\pi}{6}$ | $-\frac{8\pi}{6}$ | $-\frac{7\pi}{6}$ | $-\pi$ |
|---------------|---------|--------------------|--------------------|-------------------|-------------------|-------------------|--------|
| $\sin \theta$ | 0       | 0.5                | 0.9                | 1                 | 0.9               | 0.5               | 0      |

| $\frac{-5\pi}{6}$ | $\frac{-4\pi}{6}$ | $\frac{-3\pi}{6}$ | $\frac{-2\pi}{6}$ | $\frac{-\pi}{6}$ | 0 | $\frac{\pi}{6}$ | $\frac{2\pi}{6}$ | $\frac{3\pi}{6}$ |
|-------------------|-------------------|-------------------|-------------------|------------------|---|-----------------|------------------|------------------|
| -0.5              | -0.9              | -1                | -0.9              | -0.5             | 0 | 0.5             | 0.9              | 1                |

| $\frac{4\pi}{6}$ | $\frac{5\pi}{6}$ | $\pi$ | $\frac{7\pi}{6}$ | $\frac{8\pi}{6}$ | $\frac{9\pi}{6}$ | $\frac{10\pi}{6}$ | $\frac{11\pi}{6}$ | $2\pi$ |
|------------------|------------------|-------|------------------|------------------|------------------|-------------------|-------------------|--------|
| 0.9              | 0.5              | 0     | -0.5             | -0.9             | -1               | -0.9              | -0.5              | 0      |



(b) maximum points:  $\left(-\frac{3\pi}{2}, 1\right)$  and  $\left(\frac{\pi}{2}, 1\right)$

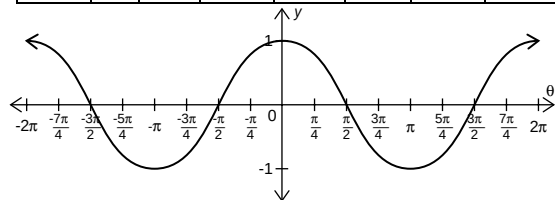
minimum points:  $\left(\frac{3\pi}{2}, -1\right)$  and  $\left(-\frac{\pi}{2}, -1\right)$

(c) zeros:  $(-2\pi, 0)$ ,  $(-\pi, 0)$ ,  $(0, 0)$ ,  $(\pi, 0)$ , and  $(2\pi, 0)$

8. (a)

| $\theta$      | $-2\pi$ | $-\frac{7\pi}{4}$ | $-\frac{6\pi}{4}$ | $-\frac{5\pi}{4}$ | $-\pi$ | $-\frac{3\pi}{4}$ | $-\frac{2\pi}{4}$ |
|---------------|---------|-------------------|-------------------|-------------------|--------|-------------------|-------------------|
| $\cos \theta$ | 1       | 0.7               | 0                 | -0.7              | -1     | -0.7              | 0                 |

| $-\frac{\pi}{4}$ | 0 | $\frac{\pi}{4}$ | $\frac{2\pi}{4}$ | $\frac{3\pi}{4}$ | $\pi$ | $\frac{5\pi}{4}$ | $\frac{6\pi}{4}$ | $\frac{7\pi}{4}$ | $2\pi$ |
|------------------|---|-----------------|------------------|------------------|-------|------------------|------------------|------------------|--------|
| 0.7              | 1 | 0.7             | 0                | -0.7             | -1    | -0.7             | 0                | 0.7              | 1      |



(b) maximum points:  $(-2\pi, 1)$ ,  $(0, 1)$ , and  $(2\pi, 1)$

minimum points:  $(-\pi, -1)$  and  $(\pi, -1)$

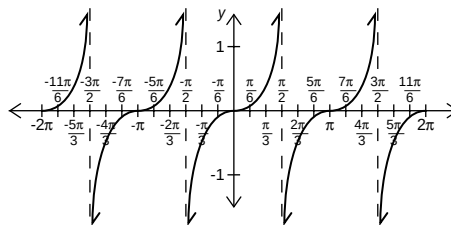
(c)  $\left(-\frac{3\pi}{2}, 0\right)$ ,  $\left(-\frac{\pi}{2}, 0\right)$ ,  $\left(\frac{\pi}{2}, 0\right)$ , and  $\left(\frac{3\pi}{2}, 0\right)$

9. (a)

| $\theta$      | $-2\pi$ | $-\frac{11\pi}{6}$ | $-\frac{10\pi}{6}$ | $-\frac{9\pi}{6}$ | $-\frac{8\pi}{6}$ | $-\frac{7\pi}{6}$ | $-\pi$ |
|---------------|---------|--------------------|--------------------|-------------------|-------------------|-------------------|--------|
| $\sin \theta$ | 0       | 0.6                | 1.7                | -                 | -1.7              | -0.6              | 0      |

| $\frac{-5\pi}{6}$ | $\frac{-4\pi}{6}$ | $\frac{-3\pi}{6}$ | $\frac{-2\pi}{6}$ | $\frac{-\pi}{6}$ | 0 | $\frac{\pi}{6}$ | $\frac{2\pi}{6}$ | $\frac{3\pi}{6}$ |
|-------------------|-------------------|-------------------|-------------------|------------------|---|-----------------|------------------|------------------|
| 0.6               | 1.7               | -                 | -1.7              | -0.6             | 0 | 0.6             | 1.7              | -                |

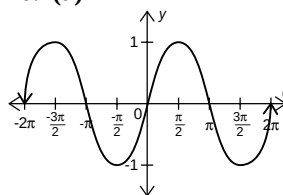
| $\frac{4\pi}{6}$ | $\frac{5\pi}{6}$ | $\pi$ | $\frac{7\pi}{6}$ | $\frac{8\pi}{6}$ | $\frac{9\pi}{6}$ | $\frac{10\pi}{6}$ | $\frac{11\pi}{6}$ | $2\pi$ |
|------------------|------------------|-------|------------------|------------------|------------------|-------------------|-------------------|--------|
| -1.7             | -0.6             | 0     | 0.6              | 1.7              | -                | -1.7              | -0.6              | 0      |



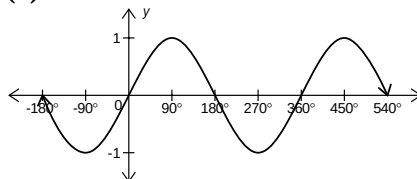
(b)  $\frac{\pi}{2} + \pi n$ ,  $-2 \leq n \leq 1$ ,  $n \in \mathbb{I}$ .

(c)  $(-2\pi, 0)$ ,  $(-\pi, 0)$ ,  $(0, 0)$ ,  $(\pi, 0)$ ,  $(2\pi, 0)$

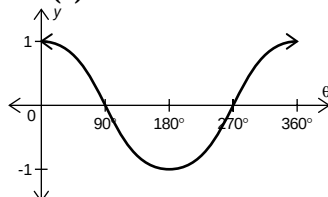
10. (a)



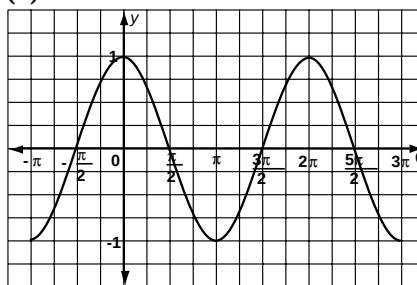
(b)



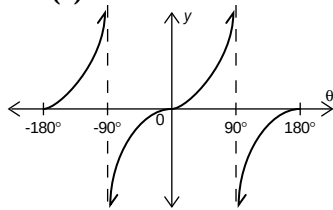
11. (a)



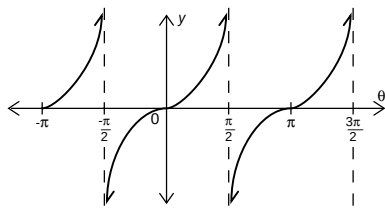
(b)



12. (a)



(b)



13. (a)  $\sin \theta = -\frac{1}{2}$

$\theta = \sin^{-1}\left(-\frac{1}{2}\right) = -30^\circ$

$\theta = -150^\circ, -30^\circ, 210^\circ, 330^\circ$

(b)  $\cos \theta = 0.825$

$\theta = \cos^{-1}(0.825) = 34^\circ$

$\theta = -326^\circ, -34^\circ, 34^\circ, 326^\circ$

(c)  $\tan \theta = 13.623$

$\theta = \tan^{-1}(13.623) = 86^\circ$

$\theta = -274^\circ, -94^\circ, 86^\circ, 266^\circ$

14. (a)  $\theta$  is in radian measure since  $-\pi \leq \theta \leq 3\pi$ . Set the calculator to radian mode. Do not round to one decimal until the last step.

$\sin \theta = \frac{7}{8}$

$\theta = \sin^{-1}\left(\frac{7}{8}\right) = 1.065\,435\,817$  (Do not round off this value

until all calculations are complete.)

Sketch the graph of  $y = \sin \theta$  remembering to label the  $\theta$ -axis in radians and not degrees. Every  $\pi$  represents approximately 3.14 on the graph.

Then  $\theta_1 = 1.1$ ,  $\theta_2 = \pi - 1.065\,435\,817$

$= 2.076\,156\,837$

$= 2.1$

$\theta_3 = \theta_1 + 2\pi$

$\theta_4 = \theta_2 + 2\pi$

$= 7.348\,621\,124$

$= 8.359\,342\,144$

$= 7.3$

$= 8.4$

(b)  $\cos \theta = -0.5$

$\theta = \cos^{-1}(-0.5) = 2.094\,395\,102$

There are four values of  $\theta$  when  $y = -0.5$ .

Then  $\theta_1 = 2.094\,395\,102$ ,  $\theta_2 = \theta_1 + 2\pi$

$= 2.1$

$= 4.188\,790\,205$

$= 4.2$

$\theta_3 = \theta_1 + 2\pi$

$\theta_4 = -2.094\,395\,102$

$= 8.377\,580\,409$

$= -2.1$

$= 8.4$

(c)  $\theta = \tan^{-1}(-4.25) = -1.339\,705\,566$

There are 4 values of  $\theta$  when  $\theta = -4.25$ .

To determine them, recall that  $y = \tan \theta$  repeats itself every  $\pi$ .

$\theta_1 = -1.339\,705\,566$ ,

$= -1.3$

$\theta_2 = \theta_1 + \pi$

$= 1.801\,866\,994$

$= 1.8$

$\theta_3 = \theta_2 + \pi$

$= 4.943\,479\,648$

$= 4.9$

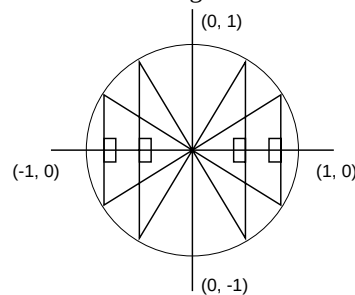
$\theta_4 = \theta_3 + \pi$

$= 8.085\,072\,302$

$= 8.1$

15. (a) The vertical displacement is calculated the same for each right triangle.  $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ ,  $\sin \theta = \frac{D}{1}$ , or

$D = \sin \theta$ .  $\theta$  is the angle at the origin.  $D$  is positive above the water line and negative below the water line.

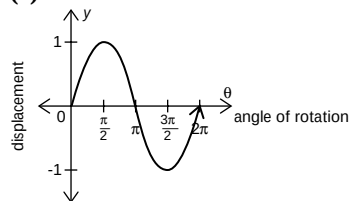


(b)

| $\theta$ | 0 | $\frac{\pi}{6}$ | $\frac{2\pi}{6}$ | $\frac{\pi}{2}$ | $\frac{4\pi}{6}$ | $\frac{5\pi}{6}$ | $\pi$ | $\frac{7\pi}{6}$ |
|----------|---|-----------------|------------------|-----------------|------------------|------------------|-------|------------------|
| $D$      | 0 | 0.5             | 0.87             | 1               | 0.87             | 0.5              | 0     | -0.5             |

| $\frac{8\pi}{6}$ | $\frac{3\pi}{2}$ | $\frac{10\pi}{6}$ | $\frac{11\pi}{6}$ | $2\pi$ |
|------------------|------------------|-------------------|-------------------|--------|
| -0.87            | -1               | -0.87             | -0.5              | 0      |

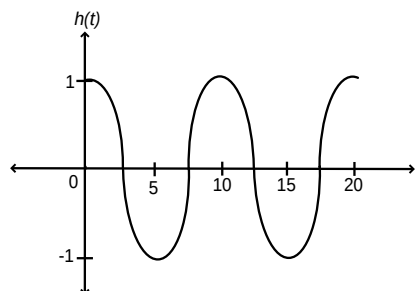
(c)



(d) The graph is periodic and sinusoidal. The equation is  $y = \sin \theta$ ,  $0 \leq \theta \leq 2\pi$ .

16. (a) Be sure to set the calculator to radian mode.

| $t(s)$        | 0 | 2.5 | 5  | 7.5 | 10 | 12.5 | 15 | 17.5 | 20 |
|---------------|---|-----|----|-----|----|------|----|------|----|
| $h(t)$<br>(m) | 1 | 0   | -1 | 0   | 1  | 0    | -1 | 0    | 1  |



b) The period is 10 s because the graph repeats itself every 10 s.

If  $y = \cos \theta$  then the period is  $2\pi$ . Then for  $h(t) = \cos\left(\frac{\pi}{5}t\right)$ ,

$$\frac{\pi}{5}t = 2\pi \text{ or } t = 10.$$

(c) When  $t = 35$  s,  $h(t) = \cos\left(\frac{\pi}{5}t\right)$  becomes

$$h(35) = \cos\left(\frac{\pi}{5}(35)\right) = \cos 7\pi = -1$$

The displacement is  $-1$  m.

(d)  $h(t) = -0.8$

$$-0.8 = \cos\left(\frac{\pi}{5}t\right)$$

$$\frac{\pi}{5}t = \cos^{-1}(-0.8)$$

$$\frac{\pi}{5}t = 2.498\ 091\ 545$$

$$\pi t = 12.490\ 457\ 72$$

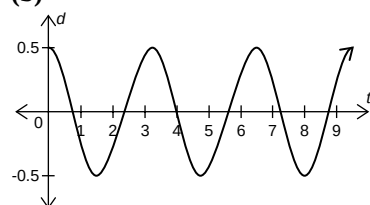
$$t = 3.975\ 836\ 177$$

Thus, the displacement first reaches  $-0.8$  m at 4 s.

17. (a)  $d = 0.5 \cos 2t$

|     |      |       |       |       |       |      |      |
|-----|------|-------|-------|-------|-------|------|------|
| $t$ | 0    | 0.5   | 1.0   | 1.5   | 2.0   | 2.5  | 3.0  |
| $d$ | 0.5  | 0.27  | -0.2  | -0.49 | -0.33 | 0.14 | 0.48 |
| $t$ | 3.5  | 4.0   | 4.5   | 5.0   | 5.5   | 6.0  | 6.5  |
| $d$ | 0.38 | -0.07 | -0.46 | -0.42 | 0     | 0.42 | 0.45 |
| $t$ | 7.0  | 7.5   | 8.0   | 8.5   | 9.0   |      |      |
| $d$ | 0.07 | -0.38 | -0.48 | -0.14 | 0.33  |      |      |

(b)



(c) The function models periodic behaviour because it repeats itself every 3.1 s.

(d) The amplitude of the function is 0.5 cm. The displacement from rest is also 0.5 cm. The rest position is the axis of the curve. This means that the amplitude and displacement from rest are the same.

18. Periodic phenomena repeat themselves over a specified interval. That interval can be the number of degrees of rotation of a point around a circle or it can be measured in terms of the actual circumference of a circle which is a real number.

19. (a) The larger gear turns clockwise when the smaller gear turns counterclockwise.

(b) The smaller gear makes one revolution in 2 s. A point on the circumference travels  $2\pi$  metres in 2 s.

$$C = 2\pi r = 2\pi(1) = 2\pi$$

The circumference of the larger gear is  $C = 2\pi(3)$  or  $6\pi$  metres. At the same speed it will take 3 times as long for one revolution. It has a period of 6 s.

(c) Select convenient intervals of time to draw each graph.

Select times when a point on the circumference will be at

Part 3: Trigonometric Functions

zero or the maximum or minimum displacements for each gear. Use 0.5 s intervals for the smaller gear and 1.5 s intervals for the larger gear.

smaller gear

|     |     |     |     |      |     |      |     |     |     |
|-----|-----|-----|-----|------|-----|------|-----|-----|-----|
| $t$ | 0   | 0.5 | 1   | 1.5  | 2   | 2.5  | 3   | 3.5 | 4   |
| $d$ | 0   | 1   | 0   | -1   | 0   | 1    | 0   | -1  | 0   |
| $t$ | 4.5 | 5   | 5.5 | 6    | 6.5 | 7    | 7.5 | 8   | 8.5 |
| $d$ | 1   | 0   | -1  | 0    | 1   | 0    | -1  | 0   | 1   |
| $t$ | 9   | 9.5 | 10  | 10.5 | 11  | 11.5 | 12  |     |     |
| $d$ | 0   | -1  | 0   | 1    | 0   | -1   | 0   |     |     |

larger gear

|     |   |     |   |     |   |     |   |      |    |
|-----|---|-----|---|-----|---|-----|---|------|----|
| $t$ | 0 | 1.5 | 3 | 4.5 | 6 | 7.5 | 9 | 10.5 | 12 |
| $d$ | 0 | 3   | 0 | -3  | 0 | 3   | 0 | -3   | 0  |

(d) Interpolate from the graphs. When  $d = -0.5$  for the smaller gear, the displacement of the larger gear is approximately 2.8 m.

(e) Interpolate from the graphs. When  $d = 2$  for the larger gear, the displacement of the smaller gear is approximately 0.8 m.

(f)  $5 \text{ min} = 5 \times 60 = 300 \text{ s}$

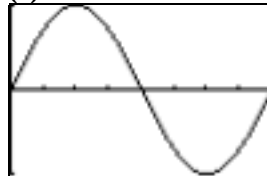
Each cycle is 6 s in length. Therefore,  $300 \div 6 = 50$  full cycles completed.

$d = 0$  m at  $t = 300$  s.

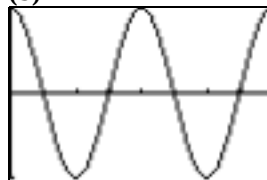
## Practice 5.5 page 446

1. For parts a to d set the mode to degrees and remember to change the window to reflect the given domain.

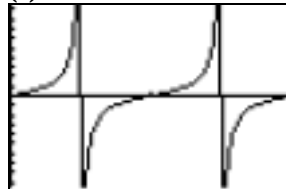
(a)



(b)



(c)



(d)

