

If $y = \cos \theta$ then the period is 2π . Then for $h(t) = \cos\left(\frac{\pi}{5}t\right)$,

$$\frac{\pi}{5}t = 2\pi \text{ or } t = 10.$$

(c) When $t = 35$ s, $h(t) = \cos\left(\frac{\pi}{5}t\right)$ becomes

$$h(35) = \cos\left(\frac{\pi}{5}(35)\right) = \cos 7\pi = -1$$

The displacement is -1 m.

(d) $h(t) = -0.8$

$$-0.8 = \cos\left(\frac{\pi}{5}t\right)$$

$$\frac{\pi}{5}t = \cos^{-1}(-0.8)$$

$$\frac{\pi}{5}t = 2.498\ 091\ 545$$

$$\pi t = 12.490\ 457\ 72$$

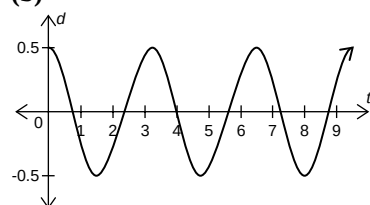
$$t = 3.975\ 836\ 177$$

Thus, the displacement first reaches -0.8 m at 4 s.

17. (a) $d = 0.5 \cos 2t$

t	0	0.5	1.0	1.5	2.0	2.5	3.0
d	0.5	0.27	-0.2	-0.49	-0.33	0.14	0.48
t	3.5	4.0	4.5	5.0	5.5	6.0	6.5
d	0.38	-0.07	-0.46	-0.42	0	0.42	0.45
t	7.0	7.5	8.0	8.5	9.0		
d	0.07	-0.38	-0.48	-0.14	0.33		

(b)



(c) The function models periodic behaviour because it repeats itself every 3.1 s.

(d) The amplitude of the function is 0.5 cm. The displacement from rest is also 0.5 cm. The rest position is the axis of the curve. This means that the amplitude and displacement from rest are the same.

18. Periodic phenomena repeat themselves over a specified interval. That interval can be the number of degrees of rotation of a point around a circle or it can be measured in terms of the actual circumference of a circle which is a real number.

19. (a) The larger gear turns clockwise when the smaller gear turns counterclockwise.

(b) The smaller gear makes one revolution in 2 s. A point on the circumference travels 2π metres in 2 s.

$$C = 2\pi r = 2\pi(1) = 2\pi$$

The circumference of the larger gear is $C = 2\pi(3)$ or 6π metres. At the same speed it will take 3 times as long for one revolution. It has a period of 6 s.

(c) Select convenient intervals of time to draw each graph.

Select times when a point on the circumference will be at

Part 3: Trigonometric Functions

zero or the maximum or minimum displacements for each gear. Use 0.5 s intervals for the smaller gear and 1.5 s intervals for the larger gear.

smaller gear

t	0	0.5	1	1.5	2	2.5	3	3.5	4
d	0	1	0	-1	0	1	0	-1	0
t	4.5	5	5.5	6	6.5	7	7.5	8	8.5
d	1	0	-1	0	1	0	-1	0	1
t	9	9.5	10	10.5	11	11.5	12		
d	0	-1	0	1	0	-1	0		

larger gear

t	0	1.5	3	4.5	6	7.5	9	10.5	12
d	0	3	0	-3	0	3	0	-3	0

(d) Interpolate from the graphs. When $d = -0.5$ for the smaller gear, the displacement of the larger gear is approximately 2.8 m.

(e) Interpolate from the graphs. When $d = 2$ for the larger gear, the displacement of the smaller gear is approximately 0.8 m.

(f) $5 \text{ min} = 5 \times 60 = 300 \text{ s}$

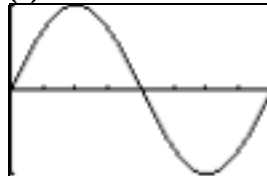
Each cycle is 6 s in length. Therefore, $300 \div 6 = 50$ full cycles completed.

$d = 0$ m at $t = 300$ s.

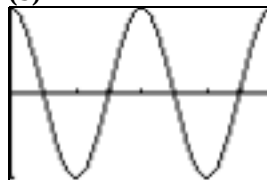
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1. For parts a to d set the mode to degrees and remember to change the window to reflect the given domain.

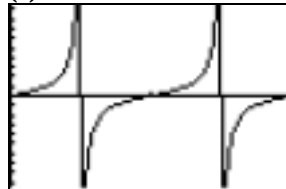
(a)



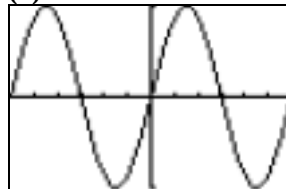
(b)

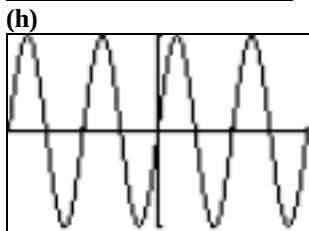
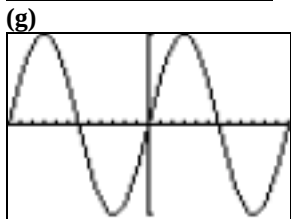
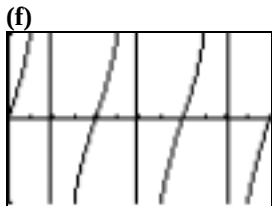
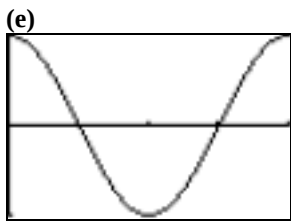


(c)



(d)





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1. (a) The graph is congruent to $y = \sin \theta$ and shifted up 2 units.
- (b) Horizontal compression of the graph of $y = \sin \theta$ by a factor of $\frac{1}{2}$. That means, two cycles of the graph where $y = \sin \theta$ has one cycle. (The period is 180° instead of 360° .)
- (c) Vertical stretch of the graph of $y = \sin \theta$ by a factor of 2. That means the graph has the same zeros as the graph of $y = \sin \theta$ but the amplitude is two.
- (d) Vertical stretch of the graph of $y = \sin \theta$ by a factor of -2 . That means the zeros are the same as $y = \sin \theta$ but the amplitude is two and the graph is reflected in the θ -axis.

2. (a) Vertical compression of $y = \cos \theta$ by a factor of $\frac{1}{2}$.
- (b) Horizontal expansion of $y = \cos \theta$ by a factor of 2. (The period 4π instead of 2π .)
- (c) Vertical shift upwards by $\frac{1}{2}$.
- (d) Vertical compression by a factor of $\frac{1}{2}$ and a reflection about the θ -axis.

3. (a) $3\theta = 360^\circ$
 $\theta = 120^\circ$
- (b) $\frac{\theta}{4} = 360^\circ$
 $\theta = 1440^\circ$

- (c) $6\theta = 360^\circ$
 $\theta = 60^\circ$
- (d) $5\theta = 360^\circ$
 $\theta = 72^\circ$
- (e) $\frac{3\theta}{2} = 360^\circ$
 $\theta = 240^\circ$
- (f) $\frac{\theta}{3} = 360^\circ$
 $\theta = 1080^\circ$
- (g) $2\theta = 180^\circ$
 $\theta = 90^\circ$
- (h) $\frac{\theta}{2} = 180^\circ$
 $\theta = 360^\circ$
- (i) $\frac{\theta}{3} = 180^\circ$
 $\theta = 540^\circ$

4. (a) $4\theta = 2\pi$

$$\theta = \frac{2\pi}{4} = \frac{\pi}{2}$$

- (b) $6\theta = 2\pi$

$$\theta = \frac{2\pi}{6} = \frac{\pi}{3}$$

- (c) $\frac{5}{3}\theta = 2\pi$

$$\theta = \frac{3}{5}(2\pi) = \frac{6\pi}{5}$$

- (d) $2\pi\theta = 2\pi$

$$\theta = \frac{2\pi}{2\pi} = 1$$

- (e) $\frac{2}{5}\theta = 2\pi$

$$\theta = \frac{5}{2}(2\pi) = 5\pi$$

- (f) $\frac{2}{3}\theta = 2\pi$

$$\theta = \frac{3}{2}(2\pi) = 3\pi$$

- (g) $3\theta = \pi$

$$\theta = \frac{\pi}{3}$$

- (h) $\frac{2}{3}\theta = \pi$

$$\theta = \frac{3\pi}{2}$$

5. (a) graph ii: $y = \sin\left(\theta + \frac{\pi}{4}\right)$ is congruent to the graph of $y = \sin \theta$ with a horizontal shift left $\frac{\pi}{4}$ radians.

- (b) graph iii: $y = \sin\left(\theta - \frac{\pi}{4}\right)$ is congruent to the graph of $y = \sin \theta$ with a horizontal shift right $\frac{\pi}{4}$ radians.