

Practise, Apply, Solve 5.6 page 455

1. (a) The graph is congruent to $y = \sin \theta$ and shifted up 2 units.
- (b) Horizontal compression of the graph of $y = \sin \theta$ by a factor of $\frac{1}{2}$. That means, two cycles of the graph where $y = \sin \theta$ has one cycle. (The period is 180° instead of 360° .)
- (c) Vertical stretch of the graph of $y = \sin \theta$ by a factor of 2. That means the graph has the same zeros as the graph of $y = \sin \theta$ but the amplitude is two.
- (d) Vertical stretch of the graph of $y = \sin \theta$ by a factor of -2 . That means the zeros are the same as $y = \sin \theta$ but the amplitude is two and the graph is reflected in the θ -axis.

2. (a) Vertical compression of $y = \cos \theta$ by a factor of $\frac{1}{2}$.
- (b) Horizontal expansion of $y = \cos \theta$ by a factor of 2. (The period 4π instead of 2π .)
- (c) Vertical shift upwards by $\frac{1}{2}$.
- (d) Vertical compression by a factor of $\frac{1}{2}$ and a reflection about the θ -axis.

3. (a) $3\theta = 360^\circ$
 $\theta = 120^\circ$
- (b) $\frac{\theta}{4} = 360^\circ$
 $\theta = 1440^\circ$

- (c) $6\theta = 360^\circ$
 $\theta = 60^\circ$
- (d) $5\theta = 360^\circ$
 $\theta = 72^\circ$
- (e) $\frac{3\theta}{2} = 360^\circ$
 $\theta = 240^\circ$
- (f) $\frac{\theta}{3} = 360^\circ$
 $\theta = 1080^\circ$
- (g) $2\theta = 180^\circ$
 $\theta = 90^\circ$
- (h) $\frac{\theta}{2} = 180^\circ$
 $\theta = 360^\circ$
- (i) $\frac{\theta}{3} = 180^\circ$
 $\theta = 540^\circ$

4. (a) $4\theta = 2\pi$

$$\theta = \frac{2\pi}{4} = \frac{\pi}{2}$$

- (b) $6\theta = 2\pi$

$$\theta = \frac{2\pi}{6} = \frac{\pi}{3}$$

- (c) $\frac{5}{3}\theta = 2\pi$

$$\theta = \frac{3}{5}(2\pi) = \frac{6\pi}{5}$$

- (d) $2\pi\theta = 2\pi$

$$\theta = \frac{2\pi}{2\pi} = 1$$

- (e) $\frac{2}{5}\theta = 2\pi$

$$\theta = \frac{5}{2}(2\pi) = 5\pi$$

- (f) $\frac{2}{3}\theta = 2\pi$

$$\theta = \frac{3}{2}(2\pi) = 3\pi$$

- (g) $3\theta = \pi$

$$\theta = \frac{\pi}{3}$$

- (h) $\frac{2}{3}\theta = \pi$

$$\theta = \frac{3\pi}{2}$$

5. (a) graph ii: $y = \sin\left(\theta + \frac{\pi}{4}\right)$ is congruent to the graph of $y = \sin \theta$ with a horizontal shift left $\frac{\pi}{4}$ radians.

- (b) graph iii: $y = \sin\left(\theta - \frac{\pi}{4}\right)$ is congruent to the graph of $y = \sin \theta$ with a horizontal shift right $\frac{\pi}{4}$ radians.

(c) graph i: $y = \cos\left(\theta + \frac{\pi}{4}\right)$ is congruent to the graph of $y = \cos \theta$ with a horizontal shift left $\frac{\pi}{4}$ radians.

6. (a) graph ii: Divide out 2 to put the equation in standard form.

$$y = \sin(2\theta - 90^\circ)$$

$$y = \sin 2(\theta - 45^\circ)$$

Horizontal shift 45° to the right. Horizontal compression by a factor of $\frac{1}{2}$.

(b) graph iii: Divide out 3 to put the equation in standard form.

$$y = \sin(3\theta + 90^\circ)$$

$$y = \sin 3(\theta + 30^\circ)$$

Horizontal shift 30° to the left. Horizontal compression by a factor of $\frac{1}{3}$.

(c) graph i: Divide out $\frac{1}{2}$ to put the equation in standard form.

$$y = \sin\left(\frac{\theta}{2} + 30^\circ\right)$$

$$y = \sin \frac{1}{2}(\theta + 60^\circ)$$

Horizontal shift 60° to the left. Horizontal expansion by a factor of 2.

7. Rewrite each equation in the form $y = a \sin k(\theta + b) + d$ or $y = a \cos k(\theta + b) + d$ and read the values of a , k , b , and d .

(a) $y = 3 \sin 2(\theta - 30^\circ) + 1$

(b) $y = 5 \cos 3(\theta + 15^\circ) - 2$

(c) $y = -2 \sin \frac{1}{3}(\theta + 45^\circ) + 2$

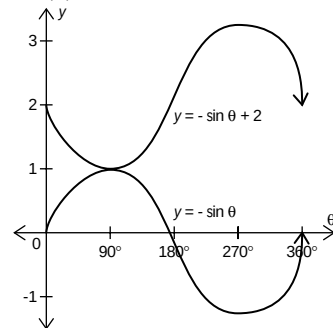
(d) $y = \frac{1}{2} \cos \frac{1}{2}(\theta - 15^\circ) - 3$

(e) $y = -4 \cos 6\left(\theta + \frac{\pi}{18}\right) + 1$

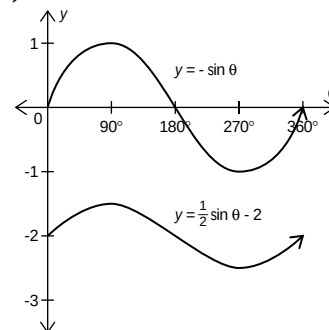
(f) $y = 3 \sin 4\left(\theta - \frac{\pi}{2}\right) + 2$

Part	Amplitude	Period	Phase Shift	Vertical Shift
(a)	3	180°	30° right	1 unit up
(b)	5	120°	15° left	2 units down
(c)	-2	1080°	45° left	2 units up
(d)	$\frac{1}{2}$	720°	15° right	3 units down
(e)	-4	$\frac{\pi}{3}$	$\frac{\pi}{18}$ left	1 unit up
(f)	3	$\frac{\pi}{2}$	$\frac{\pi}{2}$ right	2 units up

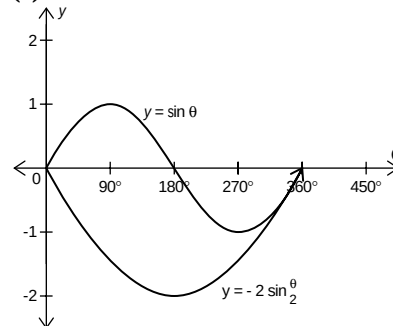
8. (a)



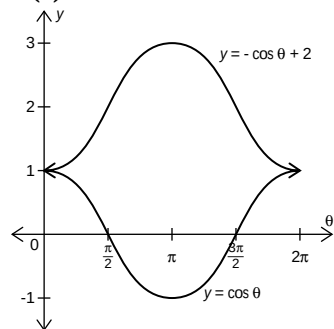
(b)



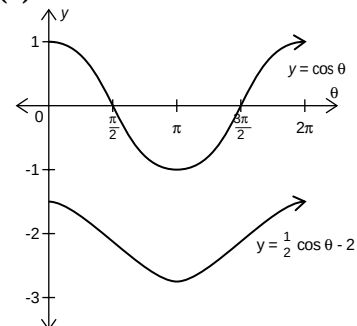
(c)



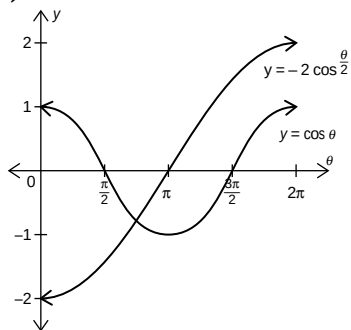
9. (a)



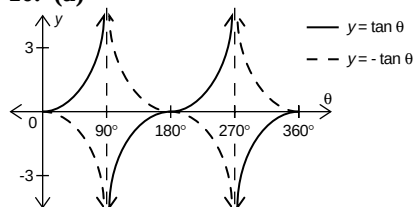
(b)



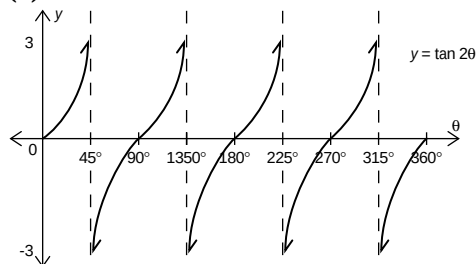
(c)



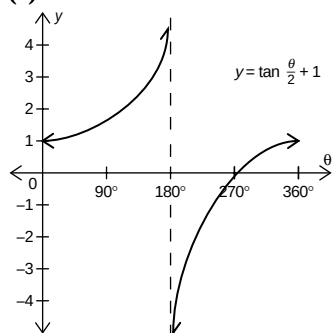
10. (a)



(b)



(c)



11. $f(\theta) = 3 \sin(2\theta - 90^\circ) - 1$

(a) $f(30^\circ) = 3 \sin(60^\circ - 90^\circ) - 1$
 $= 3 \sin(-30^\circ) - 1$
 $= -2.5$

(b) $f(45^\circ) = 3 \sin(90^\circ - 90^\circ) - 1$
 $= 3 \sin(0^\circ) - 1$
 $= -1.0$

(c) $f(125^\circ) = 3 \sin(250^\circ - 90^\circ) - 1$
 $= 3 \sin(160^\circ) - 1$
 $= 0.026$
 $= 0.0$

(d) $f(-225^\circ) = 3 \sin(-450^\circ - 90^\circ) - 1$
 $= 3 \sin(-540^\circ) - 1$
 $= -1.0$

12. $f(\theta) = -2 \cos\left(3\theta + \frac{\pi}{2}\right) - 1$

(a) $f\left(\frac{\pi}{3}\right) = -2 \cos\left(3\left(\frac{\pi}{3}\right) + \frac{\pi}{2}\right) - 1$
 $= -2 \cos\left(\frac{3\pi}{2}\right) - 1$
 $= -1.0$

(b) $f\left(\frac{2\pi}{3}\right) = -2 \cos\left(3\left(\frac{2\pi}{3}\right) + \frac{\pi}{2}\right) - 1$
 $= -2 \cos\left(2\pi + \frac{\pi}{2}\right) - 1$
 $= -2 \cos\left(\frac{5\pi}{2}\right) - 1$
 $= -1.0$

(c) $f\left(-\frac{\pi}{6}\right) = -2 \cos\left(3\left(-\frac{\pi}{6}\right) + \frac{\pi}{2}\right) - 1$
 $= -2 \cos\left(-\frac{\pi}{2} + \frac{\pi}{2}\right) - 1$
 $= -2 \cos 0 - 1$
 $= -3.0$

(d) $f\left(-\frac{3\pi}{4}\right) = -2 \cos\left(3\left(-\frac{3\pi}{4}\right) + \frac{\pi}{2}\right) - 1$
 $= -2 \cos\left(\frac{-9\pi + 2\pi}{4}\right) - 1$
 $= -2 \cos\left(-\frac{7\pi}{4}\right) - 1$
 $= -2.414$
 $= -2.4$

13. $f(\theta) = 2 \tan(2\theta - 45^\circ) - 1$

(a) $f(15^\circ) = 2 \tan(30^\circ - 45^\circ) - 1$
 $= 2 \tan(-15^\circ) - 1$
 $= -1.536$
 $= -1.5$

(b) $f(90^\circ) = 2 \tan(180^\circ - 45^\circ) - 1$
 $= 2 \tan(135^\circ) - 1$
 $= -3.0$

(c) $f(60^\circ) = 2 \tan(120^\circ - 45^\circ) - 1$
 $= 2 \tan(75^\circ) - 1$
 $= 6.464$
 $= 6.5$

(d) $f(-25^\circ) = 2 \tan(-50^\circ - 45^\circ) - 1$
 $= 2 \tan(-95^\circ) - 1$
 $= 21.860$
 $= 21.9$

14. (a) The graphs of $y = \sin \theta$ and $y = \cos \theta$ are both sinusoidal. They both have a period of 360° or 2π , amplitude of 1, maximum value of 1, and minimum value of -1 . The axis of each curve is $y = 0$.

To see the difference between the curves draw at least two cycles of each curve on the same axes.

The graphs of $y = \sin \theta$ and $y = \cos \theta$ are congruent to each other but appear to start at different places when both are compared to the graph of $y = \sin \theta$ or both to the graph of $y = \cos \theta$.

(b) The graph of $y = \cos \theta$ starts $\frac{\pi}{2}$ earlier than the graph of $y = \sin \theta$. Then, $y = \sin \theta$ can be written as a horizontal shift to the left $\frac{\pi}{2}$ or 90° of the graph of $y = \cos \theta$.

Then, $y = \sin \theta$ can be written as $y = \cos \left(\theta - \frac{\pi}{2} \right)$ or

$$y = \cos (\theta - 90^\circ).$$

Then, $\sin \theta = \cos (\theta - 90^\circ)$.

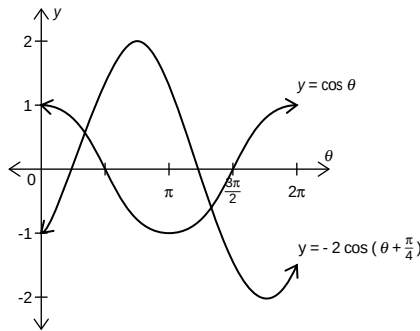
(c) The graph of $y = \sin \theta$ starts $\frac{\pi}{2}$ later than the graph of $y = \cos \theta$. Then, $y = \cos \theta$ can be written as a horizontal shift to the right $\frac{\pi}{2}$ or 90° of the graph of $y = \sin \theta$.

Then, $y = \cos \theta$ can be written as $y = \sin \left(\theta + \frac{\pi}{2} \right)$ or

$$y = \sin (\theta + 90^\circ).$$

Then, $\cos \theta = \sin (\theta + 90^\circ)$.

15. Horizontal shift $\frac{\pi}{4}$ units to the left. Vertical stretch by a factor of 2 and reflection about the θ -axis.



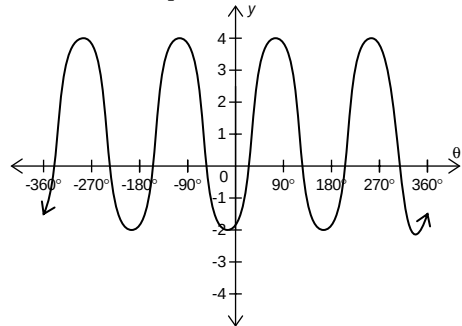
16. $y = 2 \sin 3 \left(\theta - \frac{\pi}{2} \right) - 2$

17. $y = -3 \cos \frac{1}{2} \left(\theta + \frac{2\pi}{3} \right) + 1$

18. (a) Write the equation in standard form.

$$y = 3 \sin 2 (\theta - 30^\circ) + 1$$

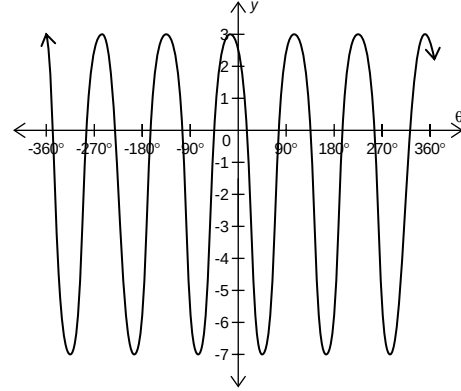
Period: 180° , horizontal compression factor: $\frac{1}{2}$, horizontal shift: right 30° , amplitude (vertical expansion factor): 3, vertical shift: up 1 unit



(b) Write the equation in standard form.

$$y = 5 \cos 3 (\theta + 15^\circ) - 2$$

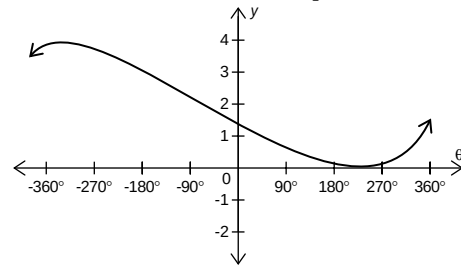
Period: 120° , horizontal compression factor: $\frac{1}{3}$, horizontal shift: left 15° , amplitude (vertical expansion factor): 5, vertical shift: down 2 units



(c) Write the equation in standard form.

$$y = -2 \sin \frac{1}{3} (\theta + 45^\circ) + 2$$

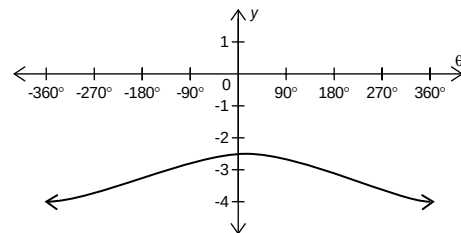
Period: 1080° , horizontal expansion factor: 3, horizontal shift: left 45° , amplitude (vertical expansion factor): 2, reflection about θ -axis, vertical shift: up 2 units



(d) Write the equation in standard form.

$$y = \frac{1}{2} \cos \frac{1}{2} (\theta - 15^\circ) - 3$$

Period: 720° , horizontal expansion factor: 2, horizontal shift: 15° right, amplitude (vertical expansion factor): $\frac{1}{2}$, vertical shift: down 3 units



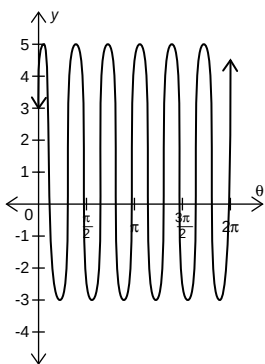
19. (a) Write the equation in standard form.

$$y = 1 - 4 \cos 6 \left(\theta + \frac{\pi}{18} \right)$$

Period: 60° , horizontal compression factor: $\frac{1}{6}$, horizontal

shift: left $\frac{\pi}{18}$, amplitude (vertical expansion factor): 4,

vertical shift: up 1 unit



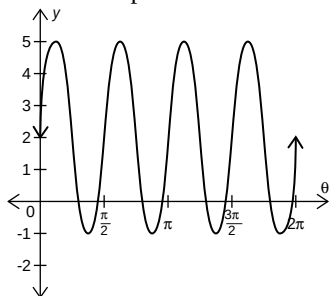
(b) Write the equation in standard form.

$$y = 2 + 3 \sin 4 \left(\theta - \frac{\pi}{2} \right)$$

Period: 90° , horizontal compression factor: $\frac{1}{4}$, horizontal

shift: right $\frac{\pi}{2}$, amplitude (vertical expansion factor): 3,

vertical shift: up 2 units



20. $y = -3 \cos \left(2\theta - \frac{\pi}{4} \right) + 1$ is built from the graph of

$y = \cos \theta$ through various transformations.

	Per.	Ampl.	Begins cycle at	Axis of Curve
$y = \cos \theta$	2π	1	$\theta = 0$	$y = 0$
$y = \sin \theta$	2π	1	$\theta = 0$	$y = 0$
$y = -3 \cos \left(2\theta - \frac{\pi}{4} \right) + 1$	π	3	$\theta = \frac{\pi}{8}$	$y = 1$
$= -3 \cos 2 \left(\theta - \frac{\pi}{8} \right) + 1$				

	y-coordinate at beginning of cycle	Max. Value	Min. Value
$y = \cos \theta$	1	1	-1
$y = \sin \theta$	0	1	-1
$y = -3 \cos 2 \left(\theta - \frac{\pi}{8} \right) + 1$	-2	4	-2

The graph of $y = -3 \cos 2 \left(\theta - \frac{\pi}{8} \right) + 1$ is different from

$y = \cos \theta$ in that it has been stretched vertically by a factor of 3, reflected in the θ -axis, shifted up one unit on the y-axis,

horizontally shifted to the right $\frac{\pi}{8}$ units, and the period has

been halved (180°).

The graph is different from $y = \sin \theta$ in that it has been stretched vertically by a factor of 3, reflected in the θ -axis, shifted up one unit on the y-axis, horizontally shifted right $\frac{\pi}{4}$ units, and the period has been halved (180°).

$$21. (a) \frac{5\pi t}{3} = 2\pi$$

$$t = \frac{3}{5\pi} (2\pi) = \frac{6}{5}$$

The period is $\frac{6}{5}$ s. The period represents the time between one beat of a person's heart and the next beat.

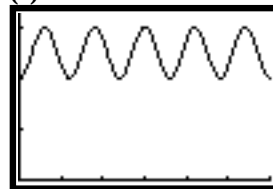
(b) There are 60 seconds in one minute.

$$\text{Then, } 60 \div \frac{6}{5} = 60 \times \frac{5}{6}$$

$$= 50$$

The person's heart beats 50 beats/min.

(c)



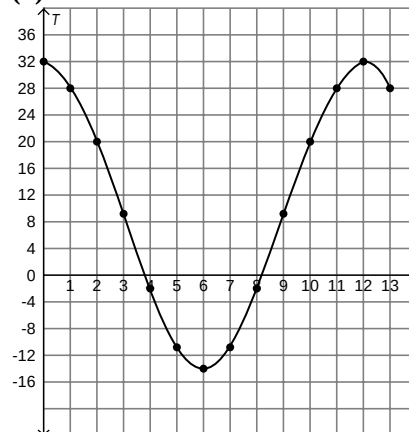
(d) The range is $80 \leq P(t) \leq 120$. This person's blood pressure, at rest, varies between 80 mm and 120 mm of mercury. The higher number, 120, is known as the systolic pressure and occurs when the heart contracts. The diastolic pressure, 80, is the lower number and occurs when the heart relaxes between beats.

22. (a)

t	0	1	2	3	4	5	6
T	32	28.9	20.5	9	-2.5	-10.9	-14

t	7	8	9	10	11	12	13
T	-10.9	-2.5	9	20.5	28.9	32	28.9

(b)



(c) The axis of the curve is $T = 9^\circ\text{C}$. The amplitude is the maximum distance away from the axis of the curve in both the positive and negative directions. The value of the amplitude is 23.

maximum = axis of the curve + amplitude

$$= 9^{\circ}\text{C} + 23^{\circ}\text{C}$$

$$= 32^{\circ}\text{C}$$

minimum = axis of the curve – amplitude

$$= 9^{\circ}\text{C} - 23^{\circ}\text{C}$$

$$= -14^{\circ}\text{C}$$

(d) The graph repeats itself every 12 months.

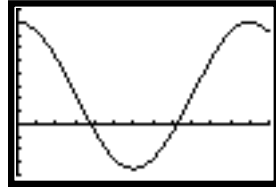
Algebraically,

$$\frac{\pi t}{6} = 2\pi$$

$$t = \frac{6}{\pi}(2\pi)$$

$$t = 12$$

(e)



(f) The amplitude of this graph is 23. Therefore, the graph of $y = \cos \theta$ is stretched vertically by a factor of 23. The axis of this graph is found at $T = 9$ so there is an upward vertical shift of 9 units. The period of the graph is 12 and the horizontal expansion is $\frac{6}{\pi}$. There is no horizontal phase shift.

23. (a) The number of hours of daylight increases to a maximum and decreases to a minimum in a regular cycle as the earth revolves around the sun.

(b) March 21: January 31
February 28
March 21 21
80 days

$$D(80) = 4 \sin \left(\frac{360}{365}(80 - 80) \right)^{\circ} + 12 = 12$$

September 21: January 31
February 28
March 31
April 30
May 31
June 30
July 31
August 31
September 1 21
264 days

$$D(264) = 4 \sin \left(\frac{360}{365}(264 - 80) \right)^{\circ} + 12 = 11.9$$

March 21 and September 21 are the spring and fall equinox. Between sunrise and sunset is 12 hours.

(c) The sine function is shifted to the right 80 days.

(d) June 21: January 31
February 28
March 31
April 30
May 31
June 21 21
172 days

$$D(172) = 4 \sin \left(\frac{360}{365}(172 - 80) \right)^{\circ} + 12$$

$$= 16 \text{ h}$$

There are 16 h of daylight on June 21.

June 21 is the beginning of summer and is the longest day of the year.

December 21: January 31
February 28
March 31
April 30
May 31
June 30
July 31
August 31
September 30
October 31
November 30
December 21 21
355 days

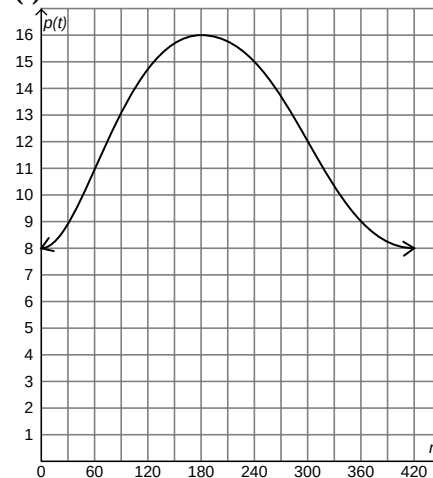
$$D(355) = 4 \sin \left(\frac{360}{365}(355 - 80) \right)^{\circ} + 12 = 8 \text{ h}$$

There are 8 h of daylight on December 21.

December 21 is the beginning of winter and is the shortest day of the year.

(e) The number 12 is the axis of the curve representing half the distance between the maximum and minimum hours of daylight and corresponds to the number of hours of daylight at the equator where the latitude is 0° .

(f)



(g) The maximum is approximately 16 h in late June. The minimum is approximately 8 h in late December or early January.

(h) $D(t) = 12$ at the intersection of the horizontal line and the sinusoidal curve. This occurs at the third and ninth months or March 21 and September 21.

24. Set the calculator to radian mode.

(a) February 28: January 31
February 28 28
59 days

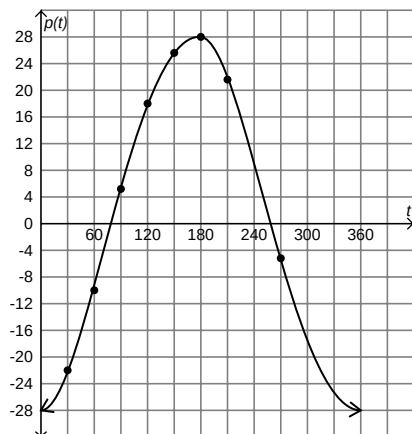
$$P(59) = 28 \sin \left(\frac{2\pi}{365}(59) - 1.4 \right) = -10.5^{\circ}$$

May 15: January 31
 February 28
 March 31
 April 30
 May 15 15

135 days

$$P(135) = 28 \sin \left(\frac{2\pi}{365} (135) - 1.4 \right) = 22.3^\circ$$

(b)



(c) The maximum angle north of due west is 28° . The minimum angle south of due west is -28° . To find these angles without a graph, use the axis of the curve and the amplitude. The axis of the curve is 0° and the amplitude is 28° .

$$\begin{aligned} \text{maximum} &= \text{axis of the curve} + \text{amplitude} \\ &= 0^\circ + 28^\circ \\ &= 28^\circ \end{aligned}$$

$$\begin{aligned} \text{minimum} &= \text{axis of the curve} - \text{amplitude} \\ &= 0^\circ - 28^\circ \\ &= -28^\circ \end{aligned}$$

(d) $\frac{2\pi}{365} t = 2\pi$

$$t = 2\pi \left(\frac{365}{2\pi} \right) = 365$$

The period is 365 days and there are 365 days in a year.

(e) Rewrite the function as

$$P(t) = 28 \sin \left[\frac{2\pi}{365} \left(t - \frac{365}{2\pi} (1.4) \right) \right]. \text{ Factor out the constant,}$$

$$k, \text{ and adjust the value of } 1.4 \text{ accordingly. } \frac{365}{2\pi} (1.4) = 81.$$

This is the horizontal shift to the right by 81 days. It represents the spring equinox at around June 22.

25. (a) a : vertical stretch by factor of a , b : horizontal phase shift by b units, shift right for $b < 0$, shift left for $b > 0$,

k : horizontal stretch by factor of $\frac{1}{k}$, d : vertical translation of

d units, shift up for $d > 0$, shift down for $d < 0$

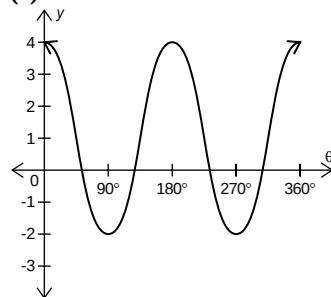
(b) $y = 3 \sin (2\theta + 90)^\circ + 1$

$$y = 3 \sin 2(\theta + 45)^\circ + 1$$

To graph the function, use the sketch of $y = \sin \theta$ as a starting graph. Horizontally compress by a factor of $\frac{1}{2}$, shift left 45° ,

vertically expand by factor of 3, and shift vertically up by 1.

(c)



26. Draw the graphs of both relations on the same axes. Show two full cycles of each.

For R, the period is $\frac{2\pi t}{24} = 2\pi$. Then, $t = \frac{24}{2\pi} (2\pi) = 24$.

For F, the period is $\frac{2\pi t}{24} = 2\pi$. Then, $t = 24$.

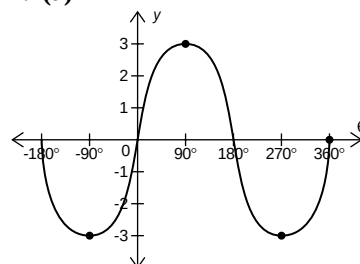
Use a graphing calculator to draw both graphs. Set the domain to show two cycles of both graphs ($2 \times 24 = 48$). Set the range to include the maximum and minimum values of both graphs: ($10\,000 + 5000 = 15\,000$, $10\,000 - 5000 = 5000$, and $1000 + 500 = 1500$, $1000 - 500 = 500$).

Enter both equations into the equation editor and graph both equations.

When the rabbit population is high, the fox population increases causing a decrease in the rabbit population. The decrease in rabbits causes a corresponding decrease in foxes. As the number of fox decrease, there is a corresponding increase in the number of rabbits. This starts the cycle over again.

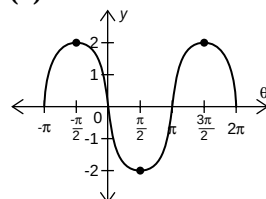
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1. (a)



period is 360° , amplitude is 3, $\therefore y = 3 \sin \theta$

(b)



period is 2π , amplitude is 2, reflection about θ -axis, $\therefore$ the equation of the graph is $y = -2 \sin \theta$