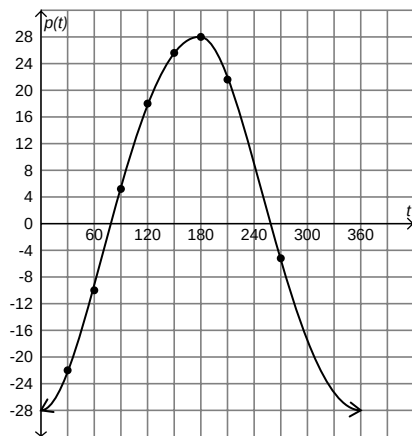


May 15: January 31
 February 28
 March 31
 April 30
 May 15 15

135 days

$$P(135) = 28 \sin \left(\frac{2\pi}{365} (135) - 1.4 \right) = 22.3^\circ$$

(b)



(c) The maximum angle north of due west is 28° . The minimum angle south of due west is -28° . To find these angles without a graph, use the axis of the curve and the amplitude. The axis of the curve is 0° and the amplitude is 28° .

$$\begin{aligned} \text{maximum} &= \text{axis of the curve} + \text{amplitude} \\ &= 0^\circ + 28^\circ \\ &= 28^\circ \end{aligned}$$

$$\begin{aligned} \text{minimum} &= \text{axis of the curve} - \text{amplitude} \\ &= 0^\circ - 28^\circ \\ &= -28^\circ \end{aligned}$$

(d) $\frac{2\pi}{365} t = 2\pi$

$$t = 2\pi \left(\frac{365}{2\pi} \right) = 365$$

The period is 365 days and there are 365 days in a year.

(e) Rewrite the function as

$$P(t) = 28 \sin \left[\frac{2\pi}{365} \left(t - \frac{365}{2\pi} (1.4) \right) \right]. \text{ Factor out the constant,}$$

$$k, \text{ and adjust the value of } 1.4 \text{ accordingly. } \frac{365}{2\pi} (1.4) = 81.$$

This is the horizontal shift to the right by 81 days. It represents the spring equinox at around June 22.

25. (a) a : vertical stretch by factor of a , b : horizontal phase shift by b units, shift right for $b < 0$, shift left for $b > 0$,

k : horizontal stretch by factor of $\frac{1}{k}$, d : vertical translation of

d units, shift up for $d > 0$, shift down for $d < 0$

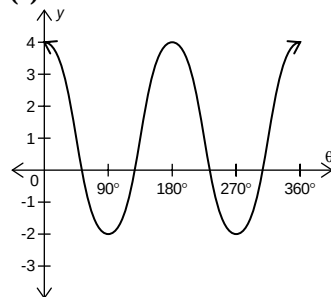
(b) $y = 3 \sin (2\theta + 90)^\circ + 1$

$$y = 3 \sin 2(\theta + 45)^\circ + 1$$

To graph the function, use the sketch of $y = \sin \theta$ as a starting graph. Horizontally compress by a factor of $\frac{1}{2}$, shift left 45° ,

vertically expand by factor of 3, and shift vertically up by 1.

(c)



26. Draw the graphs of both relations on the same axes. Show two full cycles of each.

For R, the period is $\frac{2\pi t}{24} = 2\pi$. Then, $t = \frac{24}{2\pi} (2\pi) = 24$.

For F, the period is $\frac{2\pi t}{24} = 2\pi$. Then, $t = 24$.

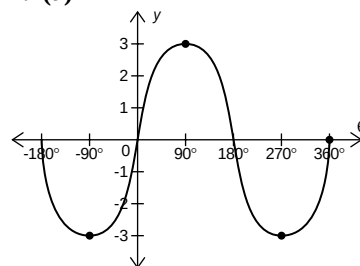
Use a graphing calculator to draw both graphs. Set the domain to show two cycles of both graphs ($2 \times 24 = 48$). Set the range to include the maximum and minimum values of both graphs: ($10\,000 + 5000 = 15\,000$, $10\,000 - 5000 = 5000$, and $1000 + 500 = 1500$, $1000 - 500 = 500$).

Enter both equations into the equation editor and graph both equations.

When the rabbit population is high, the fox population increases causing a decrease in the rabbit population. The decrease in rabbits causes a corresponding decrease in foxes. As the number of fox decrease, there is a corresponding increase in the number of rabbits. This starts the cycle over again.

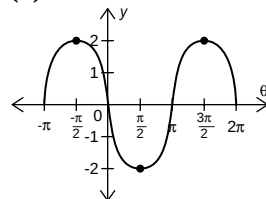
Practise, Apply, Solve 5.7, page 463

1. (a)



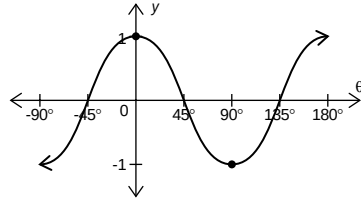
period is 360° , amplitude is 3, $\therefore y = 3 \sin \theta$

(b)



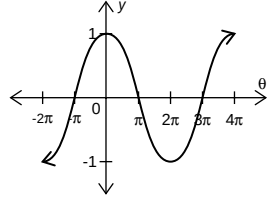
period is 2π , amplitude is 2, reflection about θ -axis, $\therefore$ the equation of the graph is $y = -2 \sin \theta$

2. (a)



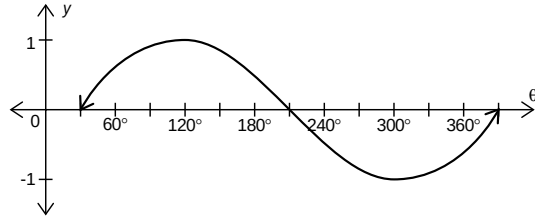
period is 180° so $k = 2$, amplitude is 1, $\therefore y = \cos 2\theta$

(b)



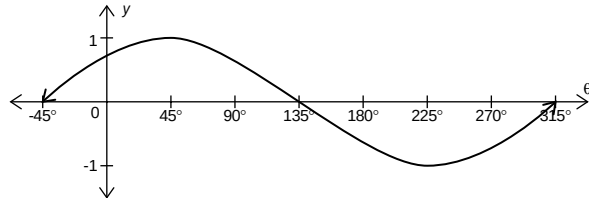
period is 4π so $k = \frac{1}{2}$, amplitude is 1, $\therefore y = \cos \frac{\theta}{2}$

3. (a)



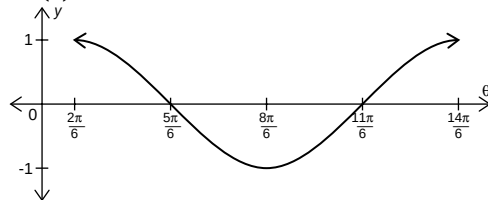
period is $360^\circ - 30^\circ = 360^\circ$, amplitude is 1, sine curve starts at 30° $\therefore$ shift 30° right; $\therefore y = \sin(\theta - 30^\circ)$

(b)



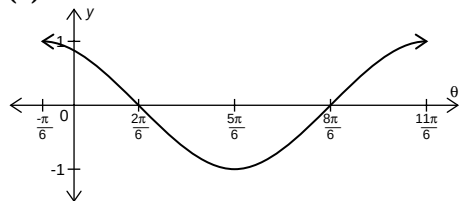
period is $315^\circ - (-45^\circ) = 360^\circ$, amplitude is 1, sine curve starts at -45° $\therefore$ shift 45° left; $\therefore y = \sin(\theta + 45^\circ)$

4. (a)



period is $\frac{14\pi}{6} - \frac{2\pi}{6} = \frac{12\pi}{6}$ or 2π , amplitude is 1, cosine curve starts at $\frac{2\pi}{6}$ or $\frac{\pi}{3}$ $\therefore$ shift $\frac{\pi}{3}$ right, $\therefore y = \cos\left(\theta - \frac{\pi}{3}\right)$

(b)



period is $\frac{11\pi}{6} - \left(-\frac{\pi}{6}\right) = \frac{11\pi}{6} + \frac{\pi}{6} = \frac{12\pi}{6}$ or 2π , amplitude is 1, cosine curve starts at $-\frac{\pi}{6}$ $\therefore$ shift $\frac{\pi}{6}$ left $\therefore$ equation of the graph is $y = \cos\left(\theta + \frac{\pi}{6}\right)$

5. Draw the graph using the ordered pairs. The graph has a period of $150^\circ - (-30^\circ) = 180^\circ$.

$$\text{Period} = \frac{360^\circ}{k}$$

$$180^\circ = \frac{360^\circ}{k}$$

$$k = 2$$

The amplitude is 1. Sine curve starts at -30° $\therefore$ shift the graph 30° left $\therefore$ the equation of the graph is $y = \sin 2(\theta + 30^\circ)$

6. Draw the graph using the ordered pairs. Sine curve starts at 0° $\therefore$ no horizontal shift, period is 360° , amplitude is 1, axis of curve is $y = 5$ $\therefore$ the vertical shift is up 5 $\therefore d = 5$

7. Draw the graph using the ordered pairs. Cosine curve starts at 0° $\therefore$ no horizontal shift, period is 2π , amplitude is 1, axis of curve is $y = -9$ $\therefore$ vertical shift down 9 $\therefore d = -9$

8. (a) period is 180° $\therefore k = 2$, amplitude is 2, the axis of the sine curve is $y = 0$ $\therefore$ no vertical shift, sine curve starts at -45° $\therefore$ shift 45° left $\therefore$ the equation of the graph is $y = 2 \sin 2(\theta + 45^\circ)$.

(b) period is 180° $\therefore k = 2$, amplitude is 3, the axis of the sine curve is $y = 2$ $\therefore$ vertical shift up 2, sine curve starts at $\theta = 0^\circ$, $\therefore$ the equation of the graph is $y = 3 \sin 2\theta + 2$

(c) period is $900^\circ - 180^\circ = 720^\circ$ $\therefore k = \frac{1}{2}$, amplitude is 2.5, the axis of the curve is $y = 2.5$ $\therefore$ vertical shift up 2.5, sine curve starts at $\theta = 180^\circ$ $\therefore$ shift 180° right, $\therefore$ the equation of the graph is $y = 2.5 \sin \frac{1}{2}(\theta - 180^\circ) + 2.5$

(d) period is 180° $\therefore k = 2$, amplitude is 2, the axis of the curve is $y = 4$ $\therefore$ vertical shift up 4, sine curve starts at $\theta = -90^\circ$ $\therefore$ shift 90° left $\therefore$ the equation of the graph is $y = 2 \sin 2(\theta + 90^\circ) + 4$

Alternatively, one could also note that the sine curve also starts at $\theta = 90^\circ$ $\therefore$ the equation of the graph could also be $y = 2 \sin 2(\theta - 90^\circ) + 4$ (the difference between the two starting points is 180° , which is the period of the graph).

(e) period is 90° $\therefore k = 4$, amplitude is 3, the axis of the curve is $y = -2$ $\therefore$ vertical shift down 2, sine curve starts at $\theta = 22.5^\circ$ $\therefore$ shift 22.5° right $\therefore$ the equation of the graph is $y = 3 \sin 4(\theta - 22.5^\circ) - 2$

9. The difference between question 8 and this question lies in the phase shift.

(a) cosine curve starts at $\theta = 0^\circ$ $\therefore$ the equation of the graph is $y = 2 \cos 2\theta$

(b) cosine curve starts at $\theta = 45^\circ$ $\therefore$ shift 45° right, $\therefore$ the equation of the graph is $y = 3 \cos 2(\theta - 45^\circ) + 2$

(c) cosine curve starts at $\theta = 0^\circ$ provided that there is a reflection in the axis of the curve $\therefore$ the equation of the graph

$$\text{is } y = -2.5 \cos \frac{1}{2} \theta + 2.5.$$

(d) cosine curve starts at $\theta = -45^\circ \therefore$ shift 45° left $\therefore$ the equation of the graph is $y = 2 \cos 2(\theta + 45^\circ) + 4$

(e) cosine curve starts at $\theta = 0^\circ$ provided there is a reflection in the axis of the curve $\therefore$ the equation of the graph is $y = -3 \cos 4\theta - 2$

10. (a) Solve for k .

$$k\theta = 2\pi$$

$$k(2\pi) = 2\pi$$

$$k = 1$$

$$\therefore \text{the equation of the graph is } y = 3 \sin \left(\theta - \frac{\pi}{4} \right) - 1$$

(b) Solve for k .

$$k\theta = 2\pi$$

$$k(\pi) = 2\pi$$

$$k = 2$$

$$\therefore \text{the equation of the graph is } y = \frac{1}{2} \sin 2 \left(\theta + \frac{\pi}{3} \right) + 2$$

(c) Solve for k .

$$k\theta = 2\pi$$

$$k \left(\frac{\pi}{2} \right) = 2\pi$$

$$k = 4$$

$$\therefore \text{the equation of the graph is } y = 2 \sin 4 \left(\theta - \frac{\pi}{6} \right) + 3$$

(d) Solve for k .

$$k\theta = 2\pi$$

$$k(4\pi) = 2\pi$$

$$k = \frac{1}{2}$$

$$\therefore \text{the equation of the graph is } y = -2 \sin \frac{1}{2} \left(\theta - \frac{\pi}{8} \right) - 3$$

(e) Solve for k .

$$k\theta = 2\pi$$

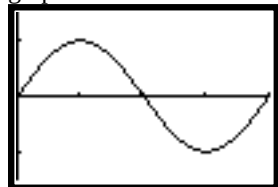
$$k(3\pi) = 2\pi$$

$$k = \frac{2}{3}$$

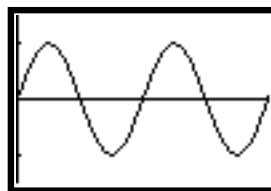
$$\therefore \text{the equation of the graph is } y = \frac{-3}{4} \sin \frac{2}{3} \left(\theta - \frac{\pi}{2} \right) - 2$$

11. (a) both $y = \sin \theta$ and $y = \cos \left(\theta - \frac{\pi}{2} \right)$ are the same

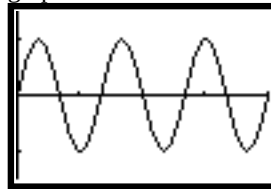
graph



(b) both $y = \sin 2\theta$ and $y = \cos 2 \left(\theta - \frac{\pi}{4} \right)$ have the same graph



(c) both $y = \sin 3\theta$ and $y = \cos 3 \left(\theta - \frac{\pi}{6} \right)$ have the same graph



Generalization: if $y = \sin k\theta$, then $y = \cos k \left(\theta - \frac{\pi}{2k} \right)$.

12. If $y = \sin k\theta$, then $y = \cos k \left(\theta - \frac{\pi}{2k} \right)$.

(a) if $y = 3 \sin \left(\theta - \frac{\pi}{4} \right) - 1$, then $k = 1$

$$\therefore y = 3 \cos \left(\theta - \frac{\pi}{4} - \frac{\pi}{2} \right) - 1 = 3 \cos \left(\theta - \frac{3\pi}{4} \right) - 1$$

(b) if $y = \frac{1}{2} \sin 2 \left(\theta + \frac{\pi}{3} \right) + 2$, then $k = 2$

$$\therefore y = \frac{1}{2} \cos 2 \left(\theta + \frac{\pi}{3} - \frac{\pi}{4} \right) + 2 = \frac{1}{2} \cos 2 \left(\theta + \frac{\pi}{12} \right) + 2$$

(c) if $y = 2 \sin 4 \left(\theta - \frac{\pi}{6} \right) + 3$, then $k = 4$

$$\therefore y = 2 \cos 4 \left(\theta - \frac{\pi}{6} - \frac{\pi}{8} \right) + 3 = 2 \cos 4 \left(\theta - \frac{7\pi}{24} \right) + 3$$

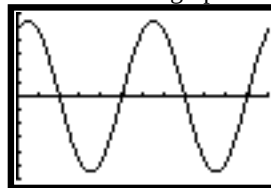
(d) if $y = -2 \sin \frac{1}{2} \left(\theta - \frac{\pi}{8} \right) - 3$, then $k = \frac{1}{2}$

$$\therefore y = -2 \cos \frac{1}{2} \left(\theta - \frac{\pi}{8} - \pi \right) - 3 = -2 \cos \frac{1}{2} \left(\theta - \frac{9\pi}{8} \right) - 3$$

(e) if $y = \frac{-3}{4} \sin \frac{2}{3} \left(\theta - \frac{\pi}{2} \right) - 2$, then $k = \frac{2}{3}$

$$\therefore y = \frac{-3}{4} \cos \frac{2}{3} \left(\theta - \frac{\pi}{2} - \frac{3\pi}{4} \right) - 2 = \frac{-3}{4} \cos \frac{2}{3} \left(\theta - \frac{5\pi}{4} \right) - 2$$

13. Sketch the graph of the function.



Halfway between 5 s and 35 s is $\frac{35-5}{2} = 15$. $\therefore$ the graph

returns to the vertical at $5 + 15 = 20$ s.

The graph could be looked upon as either a sine function or a cosine function. In either case, the amplitude, axis of the curve, and period are the same. The only difference is the phase (or horizontal) shift.

Amplitude is 5 s.

Axis of the curve is $m(t) = 0 \therefore$ no vertical shift

Length of the period is 60 s $\therefore k = \frac{2\pi}{60} = \frac{\pi}{30}$

The equations are determined by the phase shift.

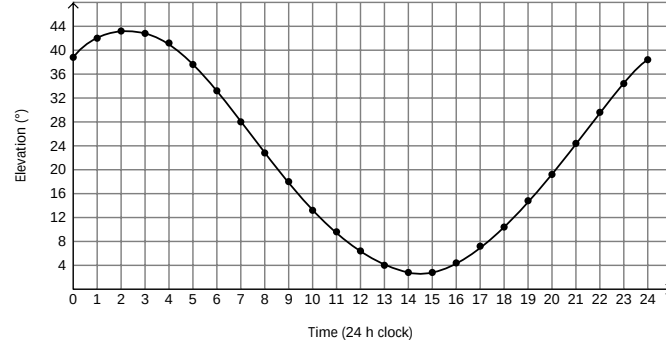
If creating a sine function, then the graph has shifted 10 s left:

$$y = 55 \sin \frac{\pi}{30}(t + 10)$$

If creating a cosine function, then the graph has shifted 5 s

$$\text{right: } y = 55 \sin \frac{\pi}{30}(t - 5)$$

14. (a)



(b) A sinusoidal model could describe the graph.

(c) The period is 24h, the length of one day $\therefore k = \frac{2\pi}{24} = \frac{\pi}{12}$

The axis of the curve is approximately $\frac{42.95 + 3.13}{2} \doteq 23.04$

The amplitude is $\left| \frac{42.95 - 3.13}{2} \right| \doteq 19.91$, which helps identify

the maximum and minimum angles of elevation.

The cosine curve is easier to identify for the phase shift. The shift is 2 hours right. Let $e(t)$ represent the elevation at a certain time, t .

$$e(t) = 19.91 \cos \frac{\pi}{12}(t - 2) + 23.04, 0 \leq t \leq 24$$

There must be a restriction because the equation does not take into consideration the movement of the sun north and south of the equator as the earth revolves around the sun.

(d) The equation could not be used to calculate the elevation of the sun at 02:00 for the given location because its domain is $0 \leq t \leq 24$. For the equation to predict on a yearly basis, the sun's movement about the equator must be considered.

(e)

$$e(t) = 19.91 \cos \frac{\pi}{12}(t - 2) + 23.04$$

$$30^\circ = 19.91 \cos \frac{\pi}{12}(t - 2) + 23.04$$

$$6.96 = 19.91 \cos \frac{\pi}{12}(t - 2)$$

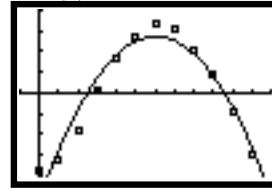
$$0.349\,573\,08 = \cos \frac{\pi}{12}(t - 2)$$

$$\cos^{-1}(0.349\,573\,08) = \frac{\pi}{12}(t - 2)$$

$$t \doteq 6.6 \text{ h}$$

$\therefore$ the elevation of the sun is 30° above the horizon between 6 a.m. and 7 a.m. and then again between 10 p.m. and 11 p.m.

15. (a)



(b) A sinusoidal model describes the graph.

(c) axis of the curve = $\frac{17.0 + (-18.6)}{2} = -0.8$

This is the vertical shift of the graph.

Amplitude is $\left| \frac{17.0 - (-18.6)}{2} \right| = 17.8$

The amplitude is the increase or decrease in temperature from the axis of the curve to the maximum and minimum temperatures.

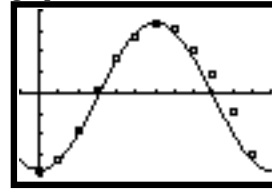
Period is 12 months $\therefore k = \frac{2\pi}{12} = \frac{\pi}{6}$

This represents the length of one year.

Using the cosine function and a reflection about the axis of the curve has the curve begin at $t = 0$. $\therefore$ no phase shift

$\therefore$ an equation is $T(t) = -17.8 \cos \frac{\pi}{6}t - 0.8$

(d) The graphing calculator and the hand-drawn scatter plot graph are almost identical.



(e) The 39th month occurs when $t = 37$ because the first month (January) occurs at $t = 0$.

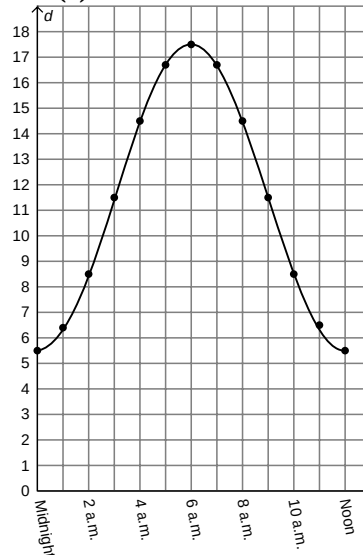
$$T(37) = -17.8 \cos \left(\frac{\pi}{6}(37) \right) - 0.8$$

$$= -17.8(0.866\,025\,403\,8) - 0.8$$

$$= -16.22$$

$\therefore$ the average monthly temperature for the 38th month is -16.22°C .

16. (a)



This is a sinusoidal curve resembling the reflected cosine function.

$$\text{Axis of the curve} = \frac{17.5 + 5.5}{2} = 11.5$$

$\therefore$ a vertical shift up 11.5

$$\text{Amplitude} = \frac{17.5 - 5.5}{2} = 6$$

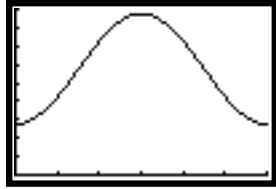
The amplitude is the increase or decrease in the water's depth from the axis of the curve to the maximum and minimum depth.

$$\text{Period is 12 hours, } \therefore k = \frac{2\pi}{12} = \frac{\pi}{6}$$

Using the cosine function and a reflection about the axis of the curve gives no phase shift.

$$\therefore \text{the equation is } D(t) = -6 \cos \frac{\pi}{6}t + 11.5$$

(b) The graphing calculator graph and the hand drawn scatter plot graph are almost identical.



(c) To evaluate the depth at 10:30, substitute $t = 10.5$.

$$D(10.5) = -6 \cos \frac{\pi}{6}(10.5) + 11.5 \doteq 7.3 \text{ m}$$

$\therefore$ at 10:30, the depth of the water is 7.3 m.

(d) Using the graphing calculator to determine t when $D(t) = 7$, draw the line $D(t) = 7$ and use the TRACE function until $y = 7$ (or very close to it). Then, $t = 1.4$ and $t = 10.6$. These values of t correspond to 1:24 a.m. ($0.4 \times 60 = 24$) and 10:36 a.m. ($0.6 \times 60 = 36$).

Alternative Method:

Solve the trigonometric equation for t when $D(t) = 7$.

$$7 = -6 \cos \frac{\pi}{6}t + 11.5$$

$$-4.5 = -6 \cos \frac{\pi}{6}t$$

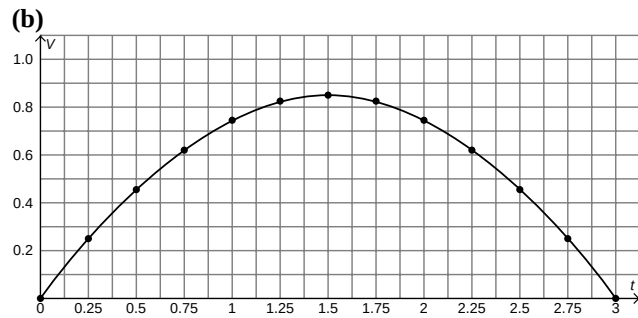
$$0.75 = \cos \frac{\pi}{6}t$$

$$0.7227 = \frac{\pi}{6}t$$

$$t = 1.4 \text{ and } t = 10.6$$

$\therefore$ the water is 7 m deep at approximately 1:24 and again at 10:36.

17. (a) The respiratory cycle is an example of a periodic function because the cycle is rhythmic. We inhale, rest, exhale, rest, inhale, and so on, in a very regular cyclical way.



The graph looks like the top half of a sine curve with a period of 6 s. $\therefore k\theta = 2\pi$ which gives $6k = 2\pi$ or $k = \frac{\pi}{3}$.

Amplitude is 0.85.

Axis of the curve is $v(t) = 0$.

No phase shift.

$$\therefore \text{an equation is } v(t) = 0.85 \sin \frac{\pi}{3}t$$

(c) The equation is almost an exact fit on the scatter plot.



(d) Substitute $t = 6$ into the equation for $v(t)$.

$$v(6) = 0.85 \sin \left(\frac{\pi}{3}(6) \right)$$

$$v(6) = 0$$

$\therefore$ the velocity is 0 L/s when $t = 6$ s.

Alternative Method:

Graph the equation which represents the velocity of Nicole's breathing. Determine $v(6)$ from the graph.

(e)

$$0.5 = 0.85 \sin \frac{\pi}{3}t$$

$$0.588 = \sin \frac{\pi}{3}t$$

$$\sin^{-1}(0.588) = \frac{\pi}{3}t$$

$$t \doteq 0.6$$

$$\text{Also, } 3 - 0.6 = 2.4 \text{ s}$$

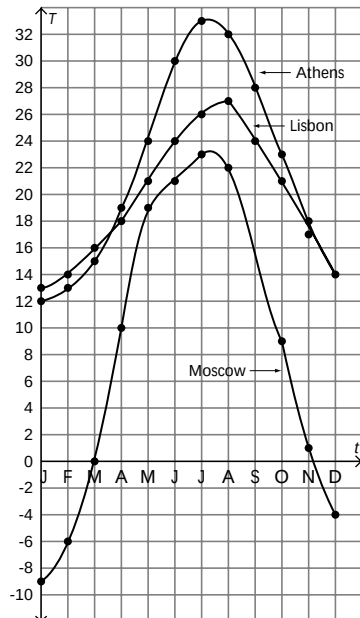
$\therefore$ after 0.6 s and after 2.4 s, the velocity is 0.5 L/s.

Using the graphing calculator, graph both $v(t) = 0.5$ and

$$v(t) = 0.85 \sin \frac{\pi}{3}t \text{ and TRACE the intersection points}$$

$$\therefore t = 0.57 \text{ s and } t = 2.36 \text{ s}$$

18.



All three graphs are sinusoidal looking. The period is one

year, which is 12 months $\therefore k = \frac{2\pi}{12} = \frac{\pi}{6}$

Athens: axis of curve is $T(t) = \frac{33+12}{2} = 22.5$

$\therefore$ vertical shift up 22.5

Amplitude is $\frac{33-12}{2} = \frac{21}{2} = 10.5$

Graph starts at its minimum $\therefore$ no phase shift if reflected cosine function

$\therefore T_A(t) = -10.5 \cos \frac{\pi}{6}t + 22.5$

Lisbon: axis of curve is $T(t) = \frac{27+13}{2} = 20$

$\therefore$ vertical shift up 20

Amplitude is $\frac{27-13}{2} = 7$

Graph starts at its minimum $\therefore$ no phase shift if reflected cosine function

$\therefore T_L(t) = -7 \cos \frac{\pi}{6}t + 20$

Moscow: axis of curve $T(t) = \frac{23+(-9)}{2} = 7$

$\therefore$ vertical shift up 7

Amplitude is $\frac{23-(-9)}{2} = 16$

Graph starts at its minimum $\therefore$ no phase shift if reflected cosine function

$T_M(t) = -16 \cos \frac{\pi}{6}t + 7$

The closer a city is to the equator the greater the axis of the curve. Moscow, which is farthest north shows the greatest fluctuation between high and low temperatures (amplitude). Lisbon and Athens are both the same distance from the equator. However, Lisbon's temperature is moderated by its position on the ocean and shows slightly less variation than Athens.

19. (a) maximum height = 35 m, board at bottom of wheel is 2 m above ground (minimum height)

$\therefore$ wheel has a diameter of 33 m and a radius of 16.5 m

amplitude is 16.5

axis of curve is $h(t) = 16.5 + 2 = 18.5$

The curve begins at minimum value and proceeds to maximum value. This follows a reflected cosine curve with no phase shift.

One revolution is 2 min or 120 s.

$\therefore$ period is 120 s

Then $120k = 2\pi$ so $k = \frac{\pi}{60}$.

$\therefore$ an equation is $h(t) = -16.5 \cos \frac{\pi}{60}t + 18.5$

(b) To find the height after 45 s, substitute $t = 45$.

$h(45) = -16.5 \cos \frac{\pi}{60}(45) + 18.5$

$h(45) \doteq 30.2$ m

$\therefore$ the passenger is 30.2 m after 45 s.

(c) The maximum height is 35 m and the period is 2 min. If the ride lasts for 4 min, then the rider will reach this height twice.

$35 = -16.5 \cos \frac{\pi}{60}t + 18.5$

$16.5 = -16.5 \cos \frac{\pi}{60}t$

$-1 = \cos \frac{\pi}{60}t$

$\cos^{-1}(-1) = \frac{\pi}{60}t$

$\pi = \frac{\pi}{60}t$

$t = 60$ s

So, the second time the passenger reaches the maximum height would occur at $60 \text{ s} + 120 \text{ s} = 180 \text{ s}$.

$\therefore$ the passenger will be at the maximum height at 1 min and again at 3 min during a 4 min ride.

20. (a) Locate the maximum and minimum values from the graph. The amplitude, a , is found as

$$a = \left| \frac{\text{maximum} - \text{minimum}}{2} \right|$$

(b) Read the period of the sinusoidal function from the graph. The period of the base curve is 360° or 2π . Compare the given graph to the base graph to determine k .

$k\theta = 2\pi$ so that $k = \frac{2\pi}{\theta}$ or $k = \frac{360^\circ}{\theta}$, where θ is the period as

read from the graph.

(c) The horizontal or phase shift is b . Select a point on the given graph that can be taken as the starting point for the typical sine or cosine curve that is closest to the vertical axis. Determine the horizontal distance (θ - distance) this point is from the vertical axis. If to the right, $b < 0$. If to the left, $b > 0$.

(d) The value of d is the axis of the curve.

$$d = \frac{\text{maximum} + \text{minimum}}{2}$$

21. (a) The nail is most likely picked up where the tire contacts the ground, which means at the bottom of the tire. The graph resembles a reflected cosine curve. One cycle is the circumference of the tire.

$$C = 2\pi r \quad k\theta = 2\pi$$

$$C = 2\pi(25) \quad k = \frac{2\pi}{50\pi}$$

$$C = 50\pi \quad k = \frac{1}{25}$$

The axis of the curve is at $H(d) = 25$, $\therefore$ the vertical shift is $d = 25$.

The cosine curve begins at the minimum value $\therefore$ there is a reflection and no phase shift.

$\therefore$ an equation for the height is $H(d) = -25 \cos \frac{1}{25}d + 25$

(b) Half a kilometre = 500 m = 50 000 cm

$$H(50\ 000) = -25 \cos \frac{1}{25}(50\ 000) + 25$$

$$\doteq 34.19 \text{ cm}$$

$\therefore$ the nail will be 34.19 cm above the ground after the car has travelled 0.5 km.

(c) Draw the graph to show the 6th time that the nail reaches 10 cm. That means there are three complete pieces of the curve. Using the graphing calculator and the TRACE feature gives a more accurate answer.

The distance the car has travelled by the time the nail reaches 10 cm for the sixth time is about 450 cm.

Alternative Method:

$$10 = -25 \cos \frac{1}{25}d + 25$$

$$-15 = -25 \cos \frac{1}{25}d$$

$$0.6 = \cos \frac{1}{25}d$$

$$\cos^{-1}(0.6) = \frac{d}{25}$$

$$23.2 = d$$

$\therefore$ the car has travelled 23.2 m the first time that the nail reaches a height of 10 cm.

$\therefore$ the second time the nail reaches this height will occur at $50\pi - 23.2 = 133.9$ m (Quadrant IV)

The sixth time that the nail will reach this height occurs two revolutions later (100π).

$$133.9 + 100\pi = 448.056 \doteq 450 \text{ cm}$$

$\therefore$ the sixth time the nail reaches this height will occur at approximately 450 m.

(d) The function will give an accurate height provided the driver moves in a smooth continuous way. Therefore, no spinning of tires and no skidding to a stop.

22. Students should try to find an Internet site on their own. A back up site is

<http://www.mgw.dinet.de/physik/Drivers/sunclocks/sunclocks.html>

The data they find will most likely be different because, each day, the sun's angle of declination with respect to the equator changes. This has a change on the elevation of the sun from the horizon at the given coordinates that is also periodic.

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1. (a) $\sin \theta = 1 \therefore \theta = \frac{\pi}{2}$

(b) $\sin \theta = -1 \therefore \theta = \frac{3\pi}{2}$

(c) $\sin \theta = 0.5 \therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

(d) $\sin \theta = -0.5 \therefore \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

2. (a) $\cos \theta = 1 \therefore \theta = 0, 2\pi$

(b) $\cos \theta = -1 \therefore \theta = \pi$

(c) $\cos \theta = 0.5 \therefore \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

(d) $\cos \theta = -0.5 \therefore \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

3. (a) There are two possible solutions.

(b) The solutions will lie in Quadrant I and Quadrant II.

(c) $\sin x = \frac{\sqrt{3}}{2}$

$$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$x = \frac{\pi}{3}$$

$\therefore$ the related acute angle is $x = \frac{\pi}{3}$.

(d) The two solutions are $x = \frac{\pi}{3}$ and $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

4. (a) There are two possible solutions.

(b) The solutions can be found in Quadrant II and Quadrant III.

(c) $\cos x = -0.8667$

$$x = \cos^{-1}(-0.8667)$$

$$x = 150^\circ$$

$\therefore$ the related acute angle is $180^\circ - 150^\circ = 30^\circ$

(d) The two solutions are $x = 150^\circ$ and $x = 180^\circ + 30^\circ = 210^\circ$

5. (a) There are two possible solutions.

(b) The solutions will lie in Quadrant I and Quadrant III.

(c) $\tan \theta = 2.7553$

$$\theta = \tan^{-1}(2.7553)$$

$$\theta = 1.22$$

$\therefore$ the related acute angle is 1.22

(d) The two solutions are $\theta = 1.22$ and $\theta = \pi + 1.22 = 4.36$.

6. (a) $\sin x = 0.65$

$$x = \sin^{-1}(0.65)$$

$$x = 40.5^\circ$$

There are two solutions: Quadrant I and Quadrant II

$\therefore x = 40.5^\circ$ and $x = 180^\circ - 40.5^\circ = 139.5^\circ$

(b) $\cos x = 0.8$

$$x = \cos^{-1}(0.8)$$

$$x = 36.9^\circ$$

There are two solutions: Quadrant I and Quadrant IV

$\therefore x = 36.9^\circ$ and $x = 360^\circ - 36.9^\circ = 323.1^\circ$