

# Practise, Apply, Solve 5.8, page 474

**21. (a)** The nail is most likely picked up where the tire contacts the ground, which means at the bottom of the tire. The graph resembles a reflected cosine curve. One cycle is the circumference of the tire.

$$C = 2\pi r \quad k\theta = 2\pi$$

$$C = 2\pi(25) \quad k = \frac{2\pi}{50\pi}$$

$$C = 50\pi \quad k = \frac{1}{25}$$

The axis of the curve is at  $H(d) = 25$ ,  $\therefore$  the vertical shift is  $d = 25$ .

The cosine curve begins at the minimum value  $\therefore$  there is a reflection and no phase shift.

$\therefore$  an equation for the height is  $H(d) = -25 \cos \frac{1}{25}d + 25$

**(b)** Half a kilometre = 500 m = 50 000 cm

$$H(50\ 000) = -25 \cos \frac{1}{25}(50\ 000) + 25$$

$$\doteq 34.19 \text{ cm}$$

$\therefore$  the nail will be 34.19 cm above the ground after the car has travelled 0.5 km.

**(c)** Draw the graph to show the 6<sup>th</sup> time that the nail reaches 10 cm. That means there are three complete pieces of the curve. Using the graphing calculator and the TRACE feature gives a more accurate answer.

The distance the car has travelled by the time the nail reaches 10 cm for the sixth time is about 450 cm.

**Alternative Method:**

$$10 = -25 \cos \frac{1}{25}d + 25$$

$$-15 = -25 \cos \frac{1}{25}d$$

$$0.6 = \cos \frac{1}{25}d$$

$$\cos^{-1}(0.6) = \frac{d}{25}$$

$$23.2 = d$$

$\therefore$  the car has travelled 23.2 m the first time that the nail reaches a height of 10 cm.

$\therefore$  the second time the nail reaches this height will occur at  $50\pi - 23.2 = 133.9$  m (Quadrant IV)

The sixth time that the nail will reach this height occurs two revolutions later ( $100\pi$ ).

$$133.9 + 100\pi = 448.056 \doteq 450 \text{ cm}$$

$\therefore$  the sixth time the nail reaches this height will occur at approximately 450 m.

**(d)** The function will give an accurate height provided the driver moves in a smooth continuous way. Therefore, no spinning of tires and no skidding to a stop.

**22.** Students should try to find an Internet site on their own. A back up site is

<http://www.mgw.dinet.de/physik/Drivers/sunclocks/sunclocks.html>

The data they find will most likely be different because, each day, the sun's angle of declination with respect to the equator changes. This has a change on the elevation of the sun from the horizon at the given coordinates that is also periodic.

$$1. \text{ (a) } \sin \theta = 1 \therefore \theta = \frac{\pi}{2}$$

$$\text{(b) } \sin \theta = -1 \therefore \theta = \frac{3\pi}{2}$$

$$\text{(c) } \sin \theta = 0.5 \therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\text{(d) } \sin \theta = -0.5 \therefore \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$2. \text{ (a) } \cos \theta = 1 \therefore \theta = 0, 2\pi$$

$$\text{(b) } \cos \theta = -1 \therefore \theta = \pi$$

$$\text{(c) } \cos \theta = 0.5 \therefore \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\text{(d) } \cos \theta = -0.5 \therefore \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

**3. (a)** There are two possible solutions.

**(b)** The solutions will lie in Quadrant I and Quadrant II.

$$\text{(c) } \sin x = \frac{\sqrt{3}}{2}$$

$$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$x = \frac{\pi}{3}$$

$\therefore$  the related acute angle is  $x = \frac{\pi}{3}$ .

**(d)** The two solutions are  $x = \frac{\pi}{3}$  and  $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ .

**4. (a)** There are two possible solutions.

**(b)** The solutions can be found in Quadrant II and Quadrant III.

$$\text{(c) } \cos x = -0.8667$$

$$x = \cos^{-1}(-0.8667)$$

$$x = 150^\circ$$

$\therefore$  the related acute angle is  $180^\circ - 150^\circ = 30^\circ$

**(d)** The two solutions are  $x = 150^\circ$  and  $x = 180^\circ + 30^\circ = 210^\circ$

**5. (a)** There are two possible solutions.

**(b)** The solutions will lie in Quadrant I and Quadrant III.

$$\text{(c) } \tan \theta = 2.7553$$

$$\theta = \tan^{-1}(2.7553)$$

$$\theta = 1.22$$

$\therefore$  the related acute angle is 1.22

**(d)** The two solutions are  $\theta = 1.22$  and  $\theta = \pi + 1.22 = 4.36$ .

$$6. \text{ (a) } \sin x = 0.65$$

$$x = \sin^{-1}(0.65)$$

$$x = 40.5^\circ$$

There are two solutions: Quadrant I and Quadrant II

$\therefore x = 40.5^\circ$  and  $x = 180^\circ - 40.5^\circ = 139.5^\circ$

$$\text{(b) } \cos x = 0.8$$

$$x = \cos^{-1}(0.8)$$

$$x = 36.9^\circ$$

There are two solutions: Quadrant I and Quadrant IV

$\therefore x = 36.9^\circ$  and  $x = 360^\circ - 36.9^\circ = 323.1^\circ$

$$\begin{aligned} \text{(c)} \quad \tan x &= 1.5 \\ x &= \tan^{-1}(1.5) \\ x &= 56.3^\circ \end{aligned}$$

There are two solutions: Quadrant I and Quadrant III  
 $\therefore x = 56.3^\circ$  and  $x = 180^\circ + 56.3^\circ = 236.3^\circ$

$$\begin{aligned} \text{(d)} \quad \sin x &= 0.15 \\ x &= \sin^{-1}(0.15) \\ x &= 8.6^\circ \end{aligned}$$

There are two solutions: Quadrant I and Quadrant II  
 $\therefore x = 8.6^\circ$  and  $x = 180^\circ - 8.6^\circ = 171.4^\circ$

$$\begin{aligned} \text{(e)} \quad \tan x &= 0.75 \\ x &= \tan^{-1}(0.75) \\ x &= 36.9^\circ \end{aligned}$$

There are two solutions: Quadrant I and Quadrant III  
 $\therefore x = 36.9^\circ$  and  $x = 180^\circ + 36.9^\circ = 216.9^\circ$

$$\begin{aligned} \text{(f)} \quad \cos x &= -0.655 \\ x &= \cos^{-1}(-0.655) \\ x &= 130.9^\circ \end{aligned}$$

There are two solutions: Quadrant II and Quadrant III  
 $\therefore$  related acute angle  $= 180^\circ - 130.9^\circ = 49.1^\circ$   
 $\therefore x = 130.9^\circ$  and  $x = 180^\circ + 49.1^\circ = 229.1^\circ$

$$\begin{aligned} 7. \text{ (a)} \quad \sin \theta &= 0.4255 \\ \theta &= \sin^{-1}(0.4255) \\ \theta &= 0.44 \end{aligned}$$

There are two solutions: Quadrant I and Quadrant II  
 $\therefore \theta = 0.44$  and  $\theta = \pi - 0.44 = 2.70$

$$\begin{aligned} \text{(b)} \quad \cos \theta &= 0.1576 \\ \theta &= \cos^{-1}(0.1576) \\ \theta &= 1.41 \end{aligned}$$

There are two solutions: Quadrant I and Quadrant IV  
 $\therefore \theta = 1.41$  and  $\theta = 2\pi - 1.41 = 4.87$

$$\begin{aligned} \text{(c)} \quad \tan \theta &= 2.35 \\ \theta &= \tan^{-1}(2.35) \\ \theta &= 1.17 \end{aligned}$$

There are two solutions: Quadrant I and Quadrant III  
 $\therefore \theta = 1.17$  and  $\theta = \pi + 1.17 = 4.31$

$$\begin{aligned} \text{(d)} \quad \sin \theta &= -0.5005 \\ \theta &= \sin^{-1}(-0.5005) \\ \theta &= -0.52 \end{aligned}$$

$\therefore$  the related acute angle is 0.52

There are two solutions: Quadrant III and Quadrant IV  
 $\therefore \theta = \pi + 0.52 = 3.67$  and  $\theta = 2\pi - 0.52 = 5.76$

$$\begin{aligned} \text{(e)} \quad \tan \theta &= -0.6341 \\ \theta &= \tan^{-1}(-0.6341) \\ \theta &= -0.57 \end{aligned}$$

$\therefore$  the related acute angle is 0.57

There are two solutions: Quadrant II and Quadrant IV  
 $\therefore \theta = \pi - 0.57 = 2.58$  and  $\theta = 2\pi - 0.57 = 5.72$

$$\begin{aligned} \text{(f)} \quad \cos \theta &= -0.7583 \\ \theta &= \cos^{-1}(-0.7583) \\ \theta &= 2.43 \end{aligned}$$

$\therefore$  the related acute angle is  $\pi - 2.43 = 0.71$

There are two solutions: Quadrant II and Quadrant III  
 $\therefore \theta = 2.43$  and  $\theta = \pi + 0.71 = 3.85$

$$\begin{aligned} 8. \text{ (a)} \quad \tan \theta &= 1 \\ \theta &= \tan^{-1}(1) \\ \theta &= 45^\circ \end{aligned}$$

Two solutions: Quadrant I and Quadrant III  
 $\therefore \theta = 45^\circ$  and  $\theta = 180^\circ + 45^\circ = 225^\circ$

$$\begin{aligned} \text{(b)} \quad \sin \theta &= \frac{1}{\sqrt{2}} \end{aligned}$$

$$\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\theta = 45^\circ$$

There are two solutions: Quadrant I and Quadrant II  
 $\therefore \theta = 45^\circ$  and  $\theta = 180^\circ - 45^\circ = 135^\circ$

$$\begin{aligned} \text{(c)} \quad \cos \theta &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta = 30^\circ$$

There are two solutions: Quadrant I and Quadrant IV  
 $\therefore \theta = 30^\circ$  and  $\theta = 360^\circ - 30^\circ = 330^\circ$

$$\begin{aligned} \text{(d)} \quad \sin \theta &= -\frac{\sqrt{3}}{2} \\ \theta &= \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) \end{aligned}$$

$$\theta = -60^\circ$$

$\therefore$  the related acute angle is  $60^\circ$

There are two solutions: Quadrant III and Quadrant IV  
 $\therefore \theta = 180^\circ + 60^\circ = 240^\circ$  and  $\theta = 360^\circ - 60^\circ = 300^\circ$

$$\begin{aligned} \text{(e)} \quad \cos \theta &= -\frac{1}{\sqrt{2}} \\ \theta &= \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) \end{aligned}$$

$$\theta = 135^\circ$$

$\therefore$  the related acute angle is  $180^\circ - 135^\circ = 45^\circ$

There are two solutions: Quadrant II and Quadrant III  
 $\therefore \theta = 135^\circ$  and  $\theta = 180^\circ + 45^\circ = 225^\circ$

$$\begin{aligned} \text{(f)} \quad \tan \theta &= \sqrt{3} \\ \theta &= \tan^{-1}(\sqrt{3}) \\ \theta &= 60^\circ \end{aligned}$$

There are two solutions: Quadrant I and Quadrant III  
 $\therefore \theta = 60^\circ$  and  $\theta = 180^\circ + 60^\circ = 240^\circ$

$$\begin{aligned} 9. \text{ (a)} \quad 2 \sin \theta &= -1 \\ \sin \theta &= -\frac{1}{2} \end{aligned}$$

$$\theta = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = -30.0^\circ$$

$\therefore$  the related acute angle is  $30^\circ$

There are two solutions: Quadrant III and Quadrant IV  
 $\therefore \theta = 180^\circ + 30^\circ = 210.0^\circ$  and  $\theta = 360^\circ - 30^\circ = 330.0^\circ$

**(b)**  
 $2 \cos \theta = -3$

$$\cos \theta = -\frac{3}{2}$$

$$\theta = \cos^{-1}\left(-\frac{3}{2}\right)$$

There are no solutions to this equation because  $-1 \leq \cos \theta \leq 1$

**(c)**  
 $2 \tan \theta = 3$

$$\tan \theta = \frac{3}{2}$$

$$\theta = \tan^{-1}\left(\frac{3}{2}\right)$$

$$\theta \doteq 56.3^\circ$$

There are two solutions: Quadrant I and Quadrant III

$$\therefore \theta = 56.3^\circ \text{ and } \theta = 180^\circ + 56.3^\circ = 236.3^\circ$$

**(d)**  
 $-3 \sin \theta - 1 = 1$

$$-3 \sin \theta = 2$$

$$\sin \theta = -\frac{2}{3}$$

$$\theta = \sin^{-1}\left(-\frac{2}{3}\right)$$

$$\theta \doteq -41.8^\circ$$

$\therefore$  the related acute angle is  $41.8^\circ$

There are two solutions: Quadrant III and Quadrant IV

$$\therefore \theta = 180^\circ + 41.8^\circ = 221.8^\circ \text{ and } \theta = 360^\circ - 41.8^\circ = 318.2^\circ$$

**(e)**  
 $-5 \cos \theta + 3 = 2$

$$-5 \cos \theta = -1$$

$$\cos \theta = \frac{1}{5}$$

$$\theta = \cos^{-1}\left(\frac{1}{5}\right)$$

$$\theta \doteq 78.5^\circ$$

There are two solutions: Quadrant I and Quadrant IV

$$\therefore \theta = 78.5^\circ \text{ and } \theta = 360^\circ - 78.5^\circ = 281.5^\circ$$

**(f)**  
 $8 - \tan \theta = 10$

$$\tan \theta = -2$$

$$\theta = \tan^{-1}(-2)$$

$$\theta \doteq -63.4^\circ$$

$\therefore$  the related acute angle is  $63.4^\circ$

There are two solutions: Quadrant II and Quadrant IV

$$\therefore \theta = 180^\circ - 63.4^\circ = 116.6^\circ \text{ and } \theta = 360^\circ - 63.4^\circ = 296.6^\circ$$

**10. (a)** There are two solutions for  $0^\circ \leq x \leq 360^\circ$  unless the solution is at the minimum value.

**(b)**  
 $5 \cos x + 3 = 6$

$$5 \cos x = 3$$

$$\cos x = \frac{3}{5}$$

$$x = \cos^{-1}\left(\frac{3}{5}\right)$$

$$x \doteq 53.1^\circ$$

There are two solutions: Quadrant I and Quadrant IV

$$\therefore \theta = 53.1^\circ \text{ and } \theta = 360^\circ - 53.1^\circ = 307^\circ$$

**11. (a)**  
 $3 \sin x = \sin x + 1$

$$2 \sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = \sin^{-1}\left(\frac{1}{2}\right)$$

$$x \doteq 0.52$$

There are two solutions: Quadrant I and Quadrant II

$$\therefore x = 0.52 \text{ and } x = \pi - 0.523 = 2.62$$

**(b)**  
 $5 \cos x - \sqrt{3} = 3 \cos x$

$$2 \cos x = \sqrt{3}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$x \doteq 0.52$$

There are two solutions: Quadrant I and Quadrant IV

$$\therefore x = 0.52 \text{ and } x = 2\pi - 0.52 = 5.76$$

**(c)**  
 $\cos x - 1 = -\cos x$

$$2 \cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$x = \cos^{-1}\left(\frac{1}{2}\right)$$

$$x \doteq 1.05$$

There are two solutions: Quadrant I and Quadrant IV

$$\therefore x = 1.05 \text{ and } x = 2\pi - 1.05 = 5.24$$

**(d)**  
 $5 \sin x + 1 = 3 \sin x$

$$2 \sin x = -1$$

$$\sin x = \left(-\frac{1}{2}\right)$$

$$x = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$x \doteq -0.52$$

$\therefore$  the related acute angle is  $0.52$ .

There are two solutions: Quadrant III and Quadrant IV

$$\therefore x = \pi + 0.52 = 3.67 \text{ and } x = 2\pi - 0.52 = 5.76$$

12. (a)

$$\sin 2x = \frac{1}{\sqrt{2}}$$

$$2x = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$2x \doteq 0.785 \text{ (Quadrant I)}$$

$$\text{and } 2x \doteq \pi - 0.785$$

$$\doteq 2.356 \text{ (Quadrant II)}$$

Then,

$$2x \doteq 0.785 \quad \text{and} \quad 2x \doteq 2.356$$

$$x \doteq 0.393 \quad \quad \quad x \doteq 1.178$$

$\sin 2x$  has a period of  $\pi$ . Four solutions exist for  $0 \leq x \leq 2\pi$ .

$$0.393 + \pi = 3.5 \text{ and } 1.178 + \pi = 4.3$$

$\therefore$  the values are 0.4, 1.2, 3.5, and 4.3.

(b)

$$\sin 4x = \frac{1}{2}$$

$$4x = \sin^{-1}\left(\frac{1}{2}\right)$$

$$4x \doteq 0.5236 \text{ (Quadrant I)}$$

$$\text{and } 4x = \pi - 0.5236$$

$$4x \doteq 2.618 \text{ (Quadrant II)}$$

Then,

$$4x \doteq 0.5236 \quad \text{and} \quad 4x \doteq 2.618$$

$$x \doteq 0.131 \quad \quad \quad x \doteq 0.654$$

$\sin 4x$  has a period of  $\frac{\pi}{2}$ . Eight solutions exist for  $0 \leq x \leq 2\pi$ .

$$0.13 + \frac{\pi}{2} = 1.70, 0.13 + 2\left(\frac{\pi}{2}\right) = 3.27, 0.13 + 3\left(\frac{\pi}{2}\right) = 4.84,$$

$$0.65 + \frac{\pi}{2} = 2.23, 0.65 + 2\left(\frac{\pi}{2}\right) = 3.80, 0.65 + 3\left(\frac{\pi}{2}\right) = 5.37$$

$\therefore$  the eight values are 0.13, 0.65, 1.70, 2.23, 3.27, 3.80, 4.84, and 5.37.

(c)

$$\sin 3x = -\frac{\sqrt{3}}{2}$$

$$3x = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$3x \doteq -1.047$$

$$\text{In Quadrant III, } \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \pi + 1.047 = 4.19$$

$$\text{In Quadrant IV, } \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = 2\pi - 1.047 = 5.24$$

$$\text{Then } 3x = 4.19 \quad \text{and} \quad 3x = 5.24$$

$$x = 1.40 \quad \quad \quad x = 1.75$$

The period of  $\sin 3x$  is  $\frac{2\pi}{3}$ . Six solutions exist for  $0 \leq x \leq 2\pi$ .

$$x_1 = 1.40, x_2 = 1.75, x_3 = 1.40 + \frac{2\pi}{3} = 3.49,$$

$$x_4 = 1.75 + \frac{2\pi}{3} = 3.84, x_5 = 1.40 + 2\left(\frac{2\pi}{3}\right) = 5.59,$$

$$x_6 = 1.75 + 2\left(\frac{2\pi}{3}\right) = 5.93$$

(d)

$$\cos 4x = -\frac{1}{\sqrt{2}}$$

$$4x = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

$$4x \doteq 2.356$$

$$\text{In Quadrant II, } \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = 2.356$$

$$\text{In Quadrant III, } \pi - 2.356 = 0.785, \therefore \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = 3.927$$

$$\text{Then, } 4x = 2.356 \quad \text{and} \quad 4x = 3.927$$

$$x = 0.589 \quad \quad \quad x = 0.982$$

$\sin 4x$  has a period of  $\frac{\pi}{2}$ . Eight solutions exist for  $0 \leq x \leq 2\pi$ .

$$x_1 = 0.59, x_2 = 0.98, x_3 = 0.59 + \frac{\pi}{2} = 2.16,$$

$$x_4 = 0.98 + \frac{\pi}{2} = 2.55, x_5 = 0.59 + 2\left(\frac{\pi}{2}\right) = 3.73,$$

$$x_6 = 0.98 + 2\left(\frac{\pi}{2}\right) = 4.12, x_7 = 0.59 + 3\left(\frac{\pi}{2}\right) = 5.30,$$

$$x_8 = 0.98 + 3\left(\frac{\pi}{2}\right) = 5.69$$

(e)

$$\cos 2x = -\frac{1}{2}$$

$$2x = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$2x \doteq 2.094$$

$$\text{In Quadrant II, } \cos^{-1}\left(-\frac{1}{2}\right) = 2.094$$

$$\text{In Quadrant III, } \cos^{-1}\left(-\frac{1}{2}\right) = \pi + (\pi - 2.094) = 4.189$$

$$\text{Then, } 2x = 2.094 \quad \text{and} \quad 2x = 4.189$$

$$x = 1.05 \quad \quad \quad x = 2.09$$

$\cos 2x$  has a period of  $\pi$ . Four solutions exist for  $0 \leq x \leq 2\pi$ .

$$x_1 = 1.05, x_2 = 2.09, x_3 = 1.05 + \pi = 4.19, x_4 = 2.09 + \pi = 5.23$$

(f)

$$\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\frac{x}{2} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{x}{2} \doteq 0.523$$

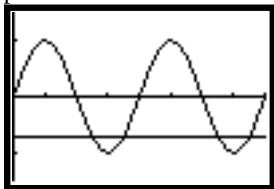
$$x \doteq 1.05$$

$\cos\left(\frac{x}{2}\right)$  has a period of  $4\pi$ . One solution exists for

$0 \leq x \leq 2\pi$ .  $\therefore x = 1.05$  is the only solution.

**13.** Use a graphing calculator to graph both  $y = \sin 2\theta$  and  $y = -\frac{1}{\sqrt{2}}$  and determine the intersection points of the two relations. Use the CALC-intersection feature to determine the intersection points.

The period of  $y = \sin 2\theta$  is  $\pi$ . Set Xscl at  $\frac{\pi}{4}$  to show strategic points of the curve. Set mode to radian.



There are 4 points of intersection. Therefore, there are 4 values of  $\theta$  that are solutions. Use 2<sup>nd</sup> CALC to activate the required screen. Cursor to the first intersection point and press ENTER 3 times.

Then  $\theta \doteq 1.96$ .

Reactivate the CALC-interest screen. Cursor to the second intersection point.

Then  $\theta \doteq 2.75$ .

Similarly,  $\theta \doteq 5.11$  and  $\theta \doteq 5.89$ .

**14.** Answers will vary. A search of sunrise calculators gives several alternative calculators. A possible Internet source is <http://aa.usno.navy.mil/AA/docs/RS-OneYear.html>

**15. (a)** At the start of the ride,  $t = 0$  s.

$$h(t) = 8 + 7 \sin(15^\circ t)$$

$$h(0) = 8 + 7 \sin[15^\circ(0)]$$

$$h(0) = 8 + 7 \sin 0^\circ$$

$$h(0) = 8$$

$\therefore$  the red seat is at a height of 8 m at the start of the ride.

**(b)** The maximum height is occurs when  $\sin 15^\circ t = 1$ .

$$\therefore \text{maximum} = 8 + 7(1) = 15 \text{ m}$$

$\therefore$  the maximum height is 15 m.

**(c)** When  $\sin 15^\circ t = 1$ , the red seat is at its maximum height.

Therefore,  $\sin 15^\circ t = 1$

$$15^\circ t = \sin^{-1}(1)$$

$$15^\circ t = 90^\circ$$

$$t = \frac{90^\circ}{15^\circ}$$

$$t = 6 \text{ s}$$

$\therefore$  during the first rotation, the red seat reaches its maximum height at 6 s.

**(d)** The minimum height occurs when  $\sin 15^\circ t = -1$ .

$$\text{Therefore, } h(t) = 8 + 7 \sin 15^\circ t$$

$$h(t) = 8 + 7(-1)$$

$$h(t) = 1$$

$\therefore$  the minimum height is 1 m.

**(e)** When  $\sin 15^\circ t = -1$ , the seat is at a minimum.

Therefore,  $\sin 15^\circ t = -1$

$$15^\circ t = \sin^{-1}(-1)$$

$$15^\circ t = 270^\circ$$

$$t = \frac{270^\circ}{15^\circ}$$

$$t = 18 \text{ s}$$

$\therefore$  during the first rotation, the red seat reaches its minimum height at 18 s.

**(f)** Each quarter rotation takes 6 s. Therefore, one rotation takes  $6(4) = 24$  s. Two rotations is 48 s.

This can also be determined using the period of the curve.

$$15^\circ t = 360^\circ$$

$$t = \frac{360^\circ}{15^\circ}$$

$$t = 24 \text{ s}$$

$\therefore$  one cycle is 24 s, so two cycles is  $2(24) = 48$  s.

**16. (a)** At the start of the ride,  $\theta = 0^\circ$ . Then,  $h(\theta) = 25 \cos 5\theta$

becomes  $h(0^\circ) = 25 \cos 0^\circ$

$$h(0^\circ) = 25$$

$\therefore$  the height of one horse at the start of the ride is 25 cm.

**(b)** Solve for  $\theta$  when  $h(t) = 15$  cm.

$$15 = 25 \cos 5\theta$$

$$\cos 5\theta = \frac{15}{25}$$

$$5\theta = \cos^{-1}\left(\frac{15}{25}\right)$$

$$5\theta = 53^\circ \quad \text{and} \quad 5\theta = 307^\circ$$

$$\theta = 10.6^\circ \quad \theta = 61.4^\circ$$

The period of the function is  $5\theta = 360^\circ$

$$\theta = 72^\circ$$

There are 10 angles in 1 revolution where the height is 15 cm.

The horse will be displaced 15 cm at the following rotation

angles:  $\theta_1 = 10.6^\circ$ ,  $\theta_2 = 61.4^\circ$ ,  $\theta_3 = 10.6^\circ + 72^\circ = 82.6^\circ$ ,

$\theta_4 = 61.4^\circ + 72^\circ = 133.4^\circ$ ,  $\theta_5 = 82.6^\circ + 72^\circ = 154.6^\circ$ ,

$\theta_6 = 133.4^\circ + 72^\circ = 205.4^\circ$ ,  $\theta_7 = 154.6^\circ + 72^\circ = 226.6^\circ$ ,

$\theta_8 = 205.4^\circ + 72^\circ = 277.4^\circ$ ,  $\theta_9 = 226.6^\circ + 72^\circ = 298.6^\circ$ ,

$\theta_{10} = 277.4^\circ + 72^\circ = 349.4^\circ$

**(c)** Solve for  $\theta$  when  $h = -20$ .

$$-20 = 25 \cos 5\theta$$

$$\cos 5\theta = -\frac{20}{25}$$

$$5\theta = \cos^{-1}\left(-\frac{20}{25}\right)$$

$$5\theta = 143^\circ$$

$$\theta \doteq 28.6^\circ$$

The period of the function is  $5\theta = 360^\circ$

$$\theta = 72^\circ$$

There are 10 angles when  $h = -20$ .

$\theta_1 = 28.6^\circ$ ,  $\theta_2 = 72^\circ - 28.6^\circ = 43.4^\circ$ ,

$\theta_3 = 28.6^\circ + 72^\circ = 100.6^\circ$ ,  $\theta_4 = 43.4^\circ + 72^\circ = 115.4^\circ$ ,

$\theta_5 = 100.6^\circ + 72^\circ = 172.6^\circ$ ,  $\theta_6 = 115.4^\circ + 72^\circ = 187.4^\circ$ ,

$\theta_7 = 172.6^\circ + 72^\circ = 244.6^\circ$ ,  $\theta_8 = 187.4^\circ + 72^\circ = 259.4^\circ$ ,

$\theta_9 = 244.6^\circ + 72^\circ = 316.6^\circ$ ,  $\theta_{10} = 259.4^\circ + 72^\circ = 331.4^\circ$

**(d)** One revolution =  $360^\circ$ .

$\therefore$  the time for one complete revolution is  $\frac{360^\circ}{24^\circ/\text{s}} = 15 \text{ s}$

**17.** Set the TI-83 calculator to degree mode.

**(a)** Choose an appropriate window to represent the domain  $0^\circ \leq x \leq 360^\circ$  and the range  $-7 \leq y \leq 1$ .

Enter  $y = 4 \sin x - 3$  and  $y = -4$ . Choose Graph. Use CALC-intersect to solve for the intersection points.

$x = 194.5^\circ$  and  $x = 345.5^\circ$

**(b)** Choose an appropriate window to represent the domain  $0^\circ \leq x \leq 360^\circ$  and the range  $0.5 \leq y \leq 0.5$ .

Enter  $y = 0.5 \cos x$  and  $y = 0.1$ . Choose Graph. Use CALC-intersect to solve for the intersection points.

$x = 78.5^\circ$  and  $x = 281.5^\circ$

**(c)** The base tangent function has a period of  $180^\circ$ . Set Xscl to  $45^\circ$ . The domain is  $0^\circ \leq x \leq 360^\circ$  and range is  $-\infty \leq x \leq \infty$ . Therefore, select sufficient  $y$  to include  $y = 5$ .

Enter  $y = 3 \tan x - 2$  and  $y = 5$ . Choose Graph. Use CALC-intersect to solve for the intersection points.

$x = 66.8^\circ$  and  $x = 246.8^\circ$

**(d)** The period of  $\sin 2x$  is  $180^\circ$ . Set Xscl to  $45^\circ$ . The domain is  $0^\circ \leq x \leq 360^\circ$  and the range is  $-2 \leq y \leq 2$ .

Enter  $y = 2 \sin 2x$  and  $y = -1$ . Choose Graph. Use CALC-intersect to solve for the intersection points. Since the period of  $\sin 2x = 180^\circ$ , there are four intersection points.

$x = 105.0^\circ, 165.0^\circ, 285.0^\circ$ , and  $345.0^\circ$

**(e)** The period of  $\cos 3x$  is  $120^\circ$ . Set Xscl to  $30^\circ$ . The domain is  $0^\circ \leq x \leq 360^\circ$  and the range is  $-2 \leq y \leq 2$ .

Enter  $y = -1 \cos 3x$  and  $y = 1$ . Choose Graph. Use CALC-intersect to solve for the intersection points. Since the period of  $\cos 3x$  is  $120^\circ$ , there are three intersection points.

$x = 60.0^\circ, 180.0^\circ$ , and  $300.0^\circ$

**(f)** The period of  $\tan\left(\frac{x}{2}\right)$  is  $360^\circ$ . Set Xscl to  $90^\circ$ . The domain is  $0^\circ \leq x \leq 360^\circ$  and the range is  $-\infty \leq x \leq \infty$ .

Therefore, select sufficient  $y$  to at least cover the amplitude.

Enter  $y = 5 \tan\left(\frac{x}{2}\right)$  and  $y = 1$ . Choose Graph. Use CALC-

intersect to solve for the intersection points. Since the period of  $\tan\left(\frac{x}{2}\right)$  is  $360^\circ$ , there is one intersection point.

$x = 22.6^\circ$

**(g)** The period of  $\sin(x + 90^\circ)$  is  $360^\circ$ . Since there is a phase shift of  $90^\circ$ , set Xscl to  $90^\circ$ . The domain is  $0^\circ \leq x \leq 360^\circ$  and the range is  $-2 \leq y \leq 2$ . Enter  $y = 2 \sin(x + 90^\circ)$  and  $y = 1$ .

Choose Graph. Use CALC-intersect to solve for the intersection points. There are two intersection points.

$x = 60.0^\circ$  and  $x = 300.0^\circ$

**(h)** The period of  $\cos(x - 60^\circ)$  is  $360^\circ$ . Since there is a phase shift of  $60^\circ$ , set Xscl to  $60^\circ$ . The domain is

$0^\circ \leq x \leq 360^\circ$  and the range is  $1 \leq y \leq 3$ . Set Ymin to 0 and Ymax to 4 to include the range. Enter  $y = \cos(x - 60^\circ) + 2$  and  $y = 3$ . Choose Graph. Use CALC-intersect to solve for the intersection points. There is one intersection point.

$x = 60.0^\circ$

**18.** The domain is  $0 \leq x \leq 2\pi$ . Set the TI-83 graphing calculator to radian mode.

**(a)** Choose an appropriate window to represent the domain  $0 \leq x \leq 2\pi$  and range  $3 \leq y \leq 9$ . Set the window to show the  $x$ -axis. Enter  $y = 3 \sin x + 6$  and  $y = 5$ . Choose Graph. Use CALC-intersect to solve for the intersection points.

$x = 3.48$  and  $x = 5.94$

**(b)** The period of  $\cos 0.5x$  is  $4\pi$ . Set Xscl to  $\pi$ . The domain is  $0 \leq x \leq 2\pi$  and the range is  $-2 \leq x \leq 2$ . Set  $y$  to include these points. Enter  $y = 2 \cos 0.5x$  and  $y = 2$ . Choose Graph. Use CALC-intersect to solve for the intersection points. Since the

period of  $\cos 0.5x$  is  $4\pi$ , there is one intersection point.

$x = 6.28$

**(c)** The period of  $\tan 2x$  is  $\frac{\pi}{2}$ . Set Xscl to  $\frac{\pi}{4}$ . The domain is

$0 \leq x \leq 2\pi$  and the range is  $-\infty \leq y \leq \infty$ . Set Ymin and Ymax to include  $\pm 2$ . Enter  $y = 2 \tan 2x$  and  $y = 1$ . Choose Graph.

Use CALC-intersect to solve for the intersection points. Since the period of  $\tan 2x$  is  $\frac{\pi}{2}$ , there are four intersection points.

$x = 0.23, 1.80, 3.37$ , and  $4.94$

**(d)** The period of  $\sin 3x$  is  $\frac{2\pi}{3}$ . Set Xscl to  $\frac{\pi}{6}$ . The domain

is  $0 \leq x \leq 2\pi$  and the range is  $-4 \leq y \leq 4$ . Enter  $y = 4 \sin 3x$  and  $y = -2$ . Choose Graph. Use CALC-intersect to solve for

the intersection points. Since the period of  $\sin 3x$  is  $\frac{2\pi}{3}$ , there

are six intersection points.  $x = 1.22, 1.92, 3.32, 4.01, 5.41$ , and  $6.11$

**(e)** Choose an appropriate window to represent the domain

$0 \leq x \leq 2\pi$  and the range  $-2 \leq y \leq 2$ . Enter  $y = -2 \cos x$  and  $y = 1$ . Choose Graph. Use CALC-intersect to solve for the

intersection points. There are two points of intersection.

$x = 2.09$  and  $x = 4.19$ .

**(f)** Choose an appropriate window to represent the domain  $0 \leq x \leq 2\pi$  and the range  $-\infty \leq y \leq \infty$ . The  $y$  values must

include at least  $\pm 5$ . The period of the base tangent function is  $\pi$ . Enter  $y = -\tan x$  and  $y = 5$ . Choose Graph. Use CALC-

intersect to solve for the intersection points. There are two points of intersection.  $x = 1.77$  and  $x = 4.91$

**(g)** The period of  $\sin(x - \pi)$  is  $2\pi$ . Since there is a phase shift of  $\pi$ , set Xscl to  $\pi$ . The domain is  $0 \leq x \leq 2\pi$  and the

range is  $-1 \leq y \leq 1$ . Enter  $y = \sin(x - \pi)$  and  $y = 1$ . Choose Graph. Use CALC-intersect to solve for the intersection

points. There is one point of intersection.  $x = 4.71$

**(h)** The period of  $\cos(x + 1)$  is  $2\pi$ . Set Xscl to  $\frac{\pi}{2}$ . The

domain is  $0 \leq x \leq 2\pi$  and the range is  $-2 \leq y \leq 2$ . Enter

$y = 2 \cos(x + 1)$  and  $y = 0.5$ . Choose Graph. Use CALC-

intersect to solve for the intersection points. There are two

points of intersection.  $x = 0.32$  and  $x = 3.97$

**19. (a)** Substitute for  $d = 142$  m and  $\theta = 43^\circ$  and solve for  $v$ :

$$d = \frac{v^2}{9.8} \sin 2\theta + 0.5$$

$$142 = \frac{v^2}{9.8} \sin 2(43^\circ) + 0.5$$

$$142 = \frac{v^2}{9.8} \sin 86^\circ + 0.5$$

$$141.5 = \frac{v^2}{9.8} \sin 86^\circ$$

$$1386.7 = v^2 \sin 86^\circ$$

$$1386.7 = v^2 (0.99756405)$$

$$1390.1 = v^2$$

$$\pm 37.3 = v$$

Velocity is positive and  $37.3$  m/s at the moment of impact.

(b) Substitute  $d = 59$  m and  $v = 30$  m/s to get:

$$59 = \frac{(30)^2}{9.8} \sin 2\theta + 0.5$$

$$58.5 = \frac{(30)^2}{9.8} \sin 2\theta$$

$$573.3 = 900 \sin 2\theta$$

$$0.637 = \sin 2\theta$$

$$2\theta = \sin^{-1}(0.637)$$

$$2\theta \doteq 39.56^\circ$$

$$\theta \doteq 19.78^\circ$$

The period of  $\sin 2\theta$  is  $180^\circ$ . There are four possible solutions to the equation:

$$\theta_1 = 19.8^\circ, \theta_2 = 90^\circ - 19.8^\circ = 70.2^\circ,$$

$$\theta_3 = 19.8^\circ + 180^\circ = 199.8^\circ, \theta_4 = 70.2^\circ + 180^\circ = 250.2^\circ$$

(i) A line drive is the smallest angle:  $19.8^\circ$

(ii) A fly ball is the largest angle less than  $90^\circ$ :  $70.2^\circ$

The other angles ( $199.8^\circ$  and  $250.2^\circ$ ) are not possible in this context.

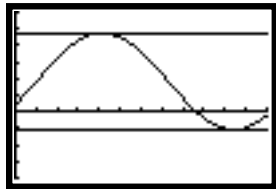
$$20. T(t) = 14.6 \sin 0.5(t-1) + 9.15$$

(a) The maximum occurs when  $\sin 0.5(t-1) = 1$ . Thus, the maximum is  $T(t) = 14.6(1) + 9.15 = 23.75$

The minimum occurs when  $\sin 0.5(t-1) = -1$ . Thus, the minimum is  $T(t) = 14.6(-1) + 9.15 = -5.45$

There are two ways to proceed: (1) solve graphically, or (2) solve algebraically.

(1) Solve by graphing  $T = 14.6 \sin 0.5(t-1) + 9.15$  and  $T = 23.75$  and  $T = -5.45$ .



Read the appropriate month from the graph at the maximum and minimum temperatures.  $t = 4.1$  and  $10.5$ , which correspond to the months of May and November/December.

(2) Solving algebraically:

$$\sin 0.5(t-1) = 1 \qquad \sin 0.5(t-1) = -1$$

$$0.5(t-1) = \sin^{-1}(1) \qquad 0.5(t-1) = \sin^{-1}(-1)$$

$$0.5(t-1) \doteq 1.57 \qquad 0.5(t-1) \doteq -1.57$$

$$t-1 = 3.14 \qquad t-1 = -3.14$$

$$t \doteq 4.1 \qquad t = -2.14$$

$$\text{Period is } 2\pi \text{ so } t = -2.14 + 2\pi = 10.5$$

$\therefore t = 4.1$  and  $t = 10.5$ , which correspond to the months of May and November/December.

(b) Substitute for  $T(t) = 0^\circ\text{C}$ :

$$0 = 14.6 \sin 0.5(t-1) + 9.15$$

$$14.6 \sin 0.5(t-1) = -9.15$$

$$\sin 0.5(t-1) = -\frac{9.15}{14.6}$$

$$0.5(t-1) = \sin^{-1}\left(-\frac{9.15}{14.6}\right)$$

$$\text{Then } \sin^{-1}\left(-\frac{9.15}{14.6}\right) = -0.677\,327$$

The sine function is negative in Quadrants III and IV.

$$t_1 = \pi + 0.677\,327 = 3.818 \text{ and } t_2 = 2\pi - 0.677\,327 = 5.606$$

Then,

$$0.5(t-1) = 3.818$$

$$0.5(t-1) = 5.606$$

$$t-1 = 7.636 \qquad \text{and} \qquad t-1 = 11.212$$

$$t \doteq 8.64$$

$$t \doteq 12.2$$

$\therefore$  the temperature is  $0^\circ\text{C}$  in late September and early December.

(c) Substitute for  $T(t) = 20^\circ\text{C}$ :

$$20 = 14.6 \sin 0.5(t-1) + 9.15$$

$$14.6 \sin 0.5(t-1) = 10.85$$

$$\sin 0.5(t-1) = \frac{10.85}{14.6}$$

$$0.5(t-1) = \sin^{-1}\left(\frac{10.85}{14.6}\right)$$

$$\text{Then } \sin^{-1}\left(\frac{10.85}{14.6}\right) \doteq 0.8377$$

The sine function is positive in Quadrants I and II.

$$t_1 = 0.84 \text{ and } t_2 = \pi - 0.8377 = 2.30$$

Then,

$$0.5(t-1) = 0.84$$

$$0.5(t-1) = 2.30$$

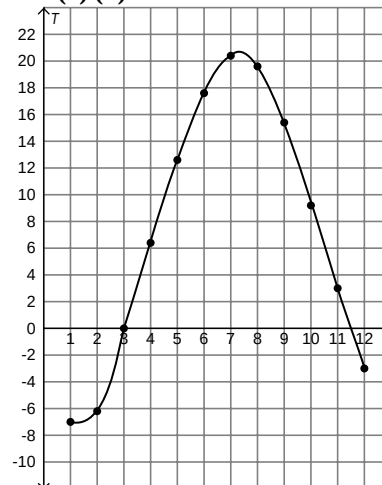
$$t-1 = 1.68 \qquad \text{and} \qquad t-1 = 4.6$$

$$t = 2.68$$

$$t = 5.6$$

$\therefore$  the temperature is  $20^\circ\text{C}$  in late March and late June.

21. (a) (b)



$$\begin{aligned} \text{(c) Axis of the curve} &= \frac{\text{maximum} + \text{minimum}}{2} \\ &= \frac{20.3 + (-6.7)}{2} \\ &= 6.8 \end{aligned}$$

$$\begin{aligned} \text{Amplitude} &= \frac{|\text{maximum} - \text{minimum}|}{2} \\ &= \frac{|20.3 - (-6.7)|}{2} \\ &= 13.5 \end{aligned}$$

The curve repeats itself every 12 months. Sinusoidal model

with  $12k = 2\pi$  so that  $k = \frac{\pi}{6}$ . The phase shift is about 4.5 to

the right if using a sine function. Then,

$$T(t) = 13.5 \sin \frac{\pi}{6}(t - 4.5) + 6.8$$

(d) Examining the table of values shows that 15°C occurs between May and June and again in September. Solving the equation gives:

$$15 = 13.5 \sin \frac{\pi}{6}(t - 4.5) + 6.8$$

$$8.2 = 13.5 \sin \frac{\pi}{6}(t - 4.5)$$

$$\frac{8.2}{13.5} = \sin \frac{\pi}{6}(t - 4.5)$$

$$\sin^{-1}\left(\frac{8.2}{13.5}\right) = \frac{\pi}{6}(t - 4.5)$$

$$\text{Then } \sin^{-1}\left(\frac{8.2}{13.5}\right) \doteq 0.6528$$

The sine function is positive in Quadrants I and II.

$$t_1 = 0.6528 \text{ and } t_2 = \pi - 0.6528 = 2.489$$

Then,

$$\frac{\pi}{6}(t - 4.5) = 0.6528$$

$$\frac{\pi}{6}(t - 4.5) = 2.489$$

$$t - 4.5 \doteq 1.247$$

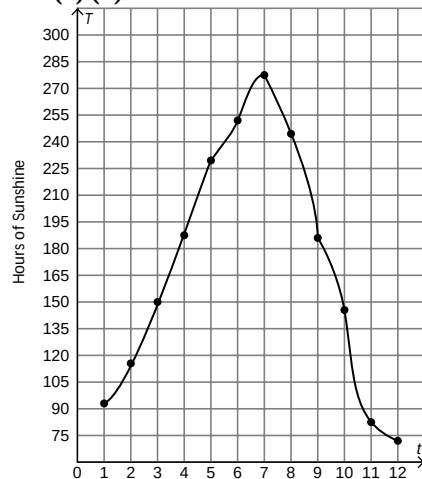
$$t - 4.5 \doteq 4.754$$

$$t \doteq 5.75$$

$$t \doteq 9.25$$

∴ the daily mean temperature will be 15°C in London in May and September. The model is built from real data and is very close to the original data. Some of the data do not lie directly on the curve.

22. (a) (b)



(c) The curve resembles a cosine function that is reflected.

$$\begin{aligned} \text{Axis of curve} &= \frac{\text{maximum} + \text{minimum}}{2} \\ &= \frac{278 + 72.6}{2} \\ &= 175.3 \end{aligned}$$

$$\begin{aligned} \text{Amplitude} &= \frac{|\text{maximum} - \text{minimum}|}{2} \\ &= \frac{|278 - 72.6|}{2} \\ &= 102.7 \end{aligned}$$

Horizontal shift right 0. Reflection about the  $t$ -axis. The period of the function is 12 months. Then  $12k = 2\pi$  so that

$$k = \frac{\pi}{6}. \text{ Then, } H(t) = -102.7 \cos \frac{\pi}{6}t + 175.3.$$

(d) The maximum occurs when  $-\cos \frac{\pi}{6}t = 1$ .

$$\cos \frac{\pi}{6}t = -1$$

$$\frac{\pi}{6}t = \cos^{-1}(-1)$$

$$t = \frac{6 \cos^{-1}(-1)}{\pi}$$

$$t = 6$$

∴ according to the function, the number of monthly hours of sunshine will be at a maximum in June.

The minimum occurs when  $-\cos \frac{\pi}{6}t = -1$ .

$$\cos \frac{\pi}{6}t = 1$$

$$\frac{\pi}{6}t = \cos^{-1}(1)$$

$$t = \frac{6 \cos^{-1}(1)}{\pi}$$

$$t = 0$$

∴ according to the function, the number of monthly hours of sunshine will be at a minimum in December.

(e) The model is a reasonable representation of the data, but it is not exact. Individual results vary up to 10%.

23. (a) The period of  $\cos \theta$  is  $2\pi$ . The cosine function is positive in Quadrants I and IV. Therefore, two solutions are possible for  $0 \leq \theta \leq 2\pi$ .

(b)

$$2 \cos \theta + 3 = 4$$

$$2 \cos \theta = 1$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

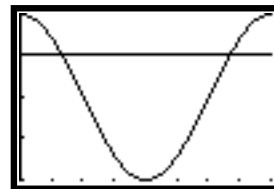
$$\theta \doteq 1.047 \text{ in Quadrant I}$$

$$\theta \doteq 2\pi - 1.047$$

$$\doteq 5.236 \text{ in Quadrant IV}$$

Then,  $\theta = 1.05$  and  $\theta = 5.24$

(c) Choose an appropriate window to represent the domain  $0 \leq \theta \leq 2\pi$  and the range  $1 \leq y \leq 5$ . Enter  $y = 2 \cos \theta + 3$  and  $y = 4$ . Choose Graph. Use CALC-intersect to solve for the intersection points. There are two points of intersection.  $x = 1.05$  and  $x = 5.24$ .



24.

$$\cos \theta - \sin \theta = 0$$

$$\cos \theta = \sin \theta$$

$$1 = \frac{\sin \theta}{\cos \theta}$$

$$1 = \tan \theta$$

$$\theta = \tan^{-1}(1)$$

$$\theta = \frac{\pi}{4} \text{ in Quadrant I}$$

$$\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4} \text{ in Quadrant III}$$

$\therefore$  the  $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$  are the solutions to the given equation.

25.

$$\sin(2\theta - 20^\circ) = \frac{1}{2}$$

$$(2\theta - 20^\circ) = \sin^{-1}\left(\frac{1}{2}\right)$$

$$(2\theta - 20^\circ) = 30^\circ$$

The sine function is positive in Quadrants I and II and  $\sin 2\theta$

has a period of  $180^\circ$ . Then,  $\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$  and

$$\sin^{-1}\left(\frac{1}{2}\right) = 180^\circ - 30^\circ = 150^\circ$$

Then,

$$(2\theta - 20^\circ) = 30^\circ$$

$$(2\theta - 20^\circ) = 150^\circ$$

$$2\theta = 50^\circ$$

$$2\theta = 170^\circ$$

$$\theta = 25^\circ$$

$$\theta = 85^\circ$$

$$\therefore \theta_1 = 25^\circ, \theta_2 = 85^\circ, \theta_3 = 25^\circ + 180^\circ = 205^\circ,$$

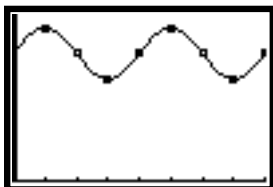
$$\theta_4 = 85^\circ + 180^\circ = 265^\circ$$

## Practice 5.9, page 480

1.  $y = 20 \sin(6.283x) + 100$

| L1   | L2  | L3 | 3 |
|------|-----|----|---|
| 0    | 100 |    |   |
| .25  | 120 |    |   |
| .5   | 100 |    |   |
| .75  | 80  |    |   |
| 1    | 100 |    |   |
| 1.25 | 120 |    |   |
| 1.5  | 100 |    |   |

| WINDOW   |  |  |  |
|----------|--|--|--|
| Xmin=0   |  |  |  |
| Xmax=2   |  |  |  |
| Xscl=.25 |  |  |  |
| Ymin=0   |  |  |  |
| Ymax=130 |  |  |  |
| Yscl=1   |  |  |  |
| Xres=1   |  |  |  |

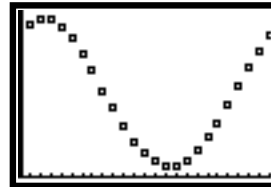


| 2nd F1 Plot2 Plot3 |  |  |  |
|--------------------|--|--|--|
| Y1=20*sin(6.283    |  |  |  |
| 1853071796X+3.63   |  |  |  |
| 6363636363E-14)    |  |  |  |
| +100               |  |  |  |
| Y2=                |  |  |  |
| Y3=                |  |  |  |
| Y4=                |  |  |  |

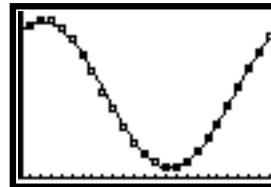
2. For question 14:

| L1 | L2    | L3 | 3 |
|----|-------|----|---|
| 0  | 38.59 |    |   |
| .1 | 41.49 |    |   |
| .2 | 42.85 |    |   |
| .3 | 42.75 |    |   |
| .4 | 40.93 |    |   |
| .5 | 37.73 |    |   |
| .6 | 33.51 |    |   |

| WINDOW  |  |  |  |
|---------|--|--|--|
| Xmin=0  |  |  |  |
| Xmax=24 |  |  |  |
| Xscl=1  |  |  |  |
| Ymin=0  |  |  |  |
| Ymax=45 |  |  |  |
| Yscl=.1 |  |  |  |
| Xres=1  |  |  |  |



| SinReg          |  |  |  |
|-----------------|--|--|--|
| y=a*sin(bx+c)+d |  |  |  |
| a=19.90990768   |  |  |  |
| b=.2566899513   |  |  |  |
| c=1.01432174    |  |  |  |
| d=22.59306804   |  |  |  |



The earlier equation was

$$E(t) = 19.91 \cos \frac{\pi}{12}(t-2) + 23.04$$

Notice that  $a$  and  $d$  are almost the same.

$$\frac{\pi}{12} \div 0.26 \div b$$

The difference between the values of  $c$  is about 1.53 or  $\frac{\pi}{2}$

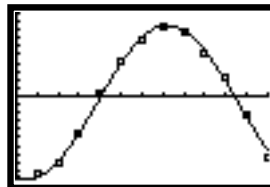
which is the phase shift difference between the sine function and the cosine function.

For question 15:

| L1 | L2    | L3 | 3 |
|----|-------|----|---|
| 1  | -18.6 |    |   |
| 2  | -16.3 |    |   |
| 3  | -9.1  |    |   |
| 4  | 4     |    |   |
| 5  | 8.5   |    |   |
| 6  | 13.8  |    |   |
| 7  | 17    |    |   |

| WINDOW   |  |  |  |
|----------|--|--|--|
| Xmin=0   |  |  |  |
| Xmax=12  |  |  |  |
| Xscl=1   |  |  |  |
| Ymin=-20 |  |  |  |
| Ymax=20  |  |  |  |
| Yscl=2   |  |  |  |
| Xres=1   |  |  |  |

| SinReg          |  |  |  |
|-----------------|--|--|--|
| y=a*sin(bx+c)+d |  |  |  |
| a=18.5443079    |  |  |  |
| b=.4697220476   |  |  |  |
| c=-1.812763069  |  |  |  |
| d=-1.437097362  |  |  |  |



For question 16:

| L1 | L2   | L3 | 3 |
|----|------|----|---|
| 0  | 5.5  |    |   |
| .1 | 8.2  |    |   |
| .2 | 8.5  |    |   |
| .3 | 11.5 |    |   |
| .4 | 14.5 |    |   |
| .5 | 16.7 |    |   |
| .6 | 17.5 |    |   |

| WINDOW  |  |  |  |
|---------|--|--|--|
| Xmin=0  |  |  |  |
| Xmax=12 |  |  |  |
| Xscl=1  |  |  |  |
| Ymin=0  |  |  |  |
| Ymax=20 |  |  |  |
| Yscl=1  |  |  |  |
| Xres=1  |  |  |  |

| 2nd F1 Plot2 Plot3 |  |  |  |
|--------------------|--|--|--|
| Y1=6.0017144209    |  |  |  |
| 984*sin(.5236688   |  |  |  |
| 5301325X+-1.5712   |  |  |  |
| 167912844)+11.50   |  |  |  |
| 08561869           |  |  |  |
| Y2=                |  |  |  |
| Y3=                |  |  |  |

