

$$D(t) = -6 \cos \frac{\pi}{6} t + 11.5$$

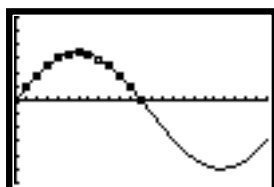
The difference is the $\frac{\pi}{2}$ phase shift.

For question 17:

L1	L2	L3	3
0	0		
.25	.22		
.5	.45		
.75	.61		
1	.75		
1.25	.82		
1.5	.85		
L3(1)=			

WINDOW
Xmin=0
Xmax=6
Xscl=.25
Ymin=-1.5
Ymax=1.5
Yscl=.25
Xres=1

Plot1 Plot2 Plot3
\Y1=1.0469230379
278*sin(.9236713
9391227X+.186010
79196801)+-.1937
4434364401
\Y2=
\Y3=



$$V(t) = 0.85 \sin \frac{\pi}{3} t$$

The equations are close but not the same. The calculator equation will be more accurate.

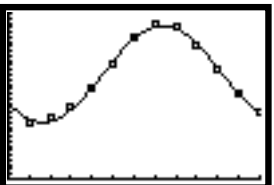
For question 18:

L1	L2	L3	1
1	12	13	
2	13	14	
3	15	16	
4	16	18	
5	19	21	
6	24	24	
7	30	24	
8	33	26	
L1={1,2,3,4,5,6...			

L2	L3	L4	4
12	13	-9	
13	14	-6	
14	15	0	
15	16	10	
16	18	19	
19	21	19	
24	24	21	
30	24	23	
33	26		
L4={-9,-6,0,10,...			

For Athens

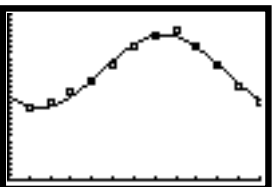
SinReg
y=a*sin(bx+c)+d
a=10.19726841
b=.552017908
c=-2.488461214
d=22.22142363



$$T(t) = -10.5 \cos \frac{\pi}{6} t + 22.5$$

For Lisbon

SinReg
y=a*sin(bx+c)+d
a=6.578138137
b=.5366067104
c=-2.393731518
d=19.72658359



$$T(t) = -7 \cos \frac{\pi}{6} t + 20$$

For Moscow

SinReg
y=a*sin(bx+c)+d
a=16.73321108
b=.4789684699
c=-1.773064691
d=6.986533286



$$T(t) = -16 \cos \frac{\pi}{6} t + 7$$

The equations generate the same graphs so have no real differences other than the phase angle and reflection.

3. The equation found by hand is

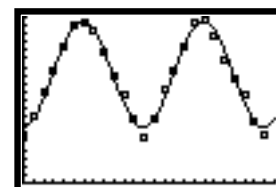
$$H(t) = 7 \sin \frac{\pi}{6} t(t-2) + 12.5$$

and is close to the graphing calculator equation. The calculator generated equation is more accurate.

L1	L2	L3	2
0	5.5		
1	8		
2	11		
3	13.5		
4	16.3		
5	19		
6	19.5		
L2={5.5,8,11,13...			

WINDOW
Xmin=0
Xmax=25
Xscl=1
Ymin=0
Ymax=20
Yscl=1
Xres=1

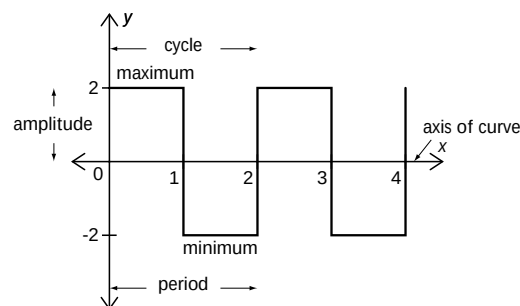
SinReg
y=a*sin(bx+c)+d
a=6.405276996
b=.5239257953
c=-1.504648375
d=13.09928365



Chapter 5, Review and Practice, page 485

1. If a function is self-replicating, then it is periodic.

2.



(a) cycle - the exact portion of the curve that repeats

(b) period - the length of one cycle

(c) amplitude - the distance from the axis of the curve to the maximum or minimum value. $\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$

(d) axis of the curve - the horizontal line that is halfway between the maximum and minimum values.

$$\text{Axis of the curve} = \frac{\text{max} + \text{min}}{2}$$

(e) maximum - the upper most point reached in one cycle

minimum - the lowest point reached in one cycle

3. (a) Ice cream consumption follows a regular pattern as it increases from a low in the cooler months, to a high in the warmer months and then decreases as the temperature cools. This cycle repeats each year.

(b) If production were constant, natural gas use would be constant. However, as temperatures cool, natural gas for heating purposes would increase and as temperatures

increase, heating needs would decrease. This cycle is repeated each year.

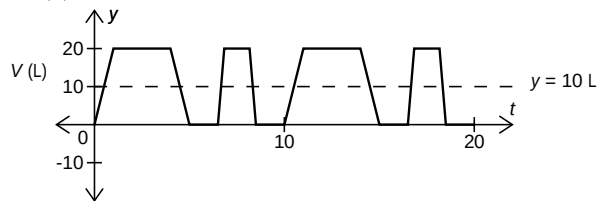
(c) The pendulum swings from a point to the right of the vertical, through the vertical position to a point on the left of the vertical. It then swings back to its starting point. A spring keeps the cycle going.

(d) When the string is plucked it moves away from rest, then back through rest to the other side. This vibration, over a short period of time could be considered as periodic, although it will eventually stop.

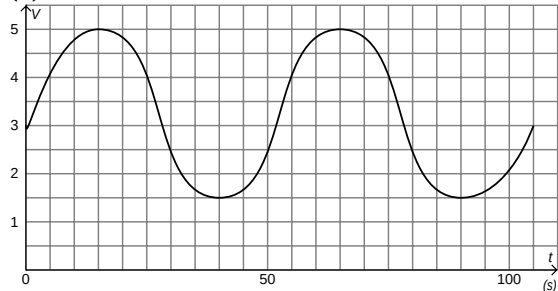
(e) Temperatures rise and fall with the seasons in a predictable manner from one year to the next.

(f) The sound is repeated over and over again for the same length of time. The interval between sounds is also constant.

4. (a)



(b)



5. (a) maximum is 11 m; minimum is 1 m

(b) One cycle is 60 s. Therefore the ride goes around in a circle once every 60 s.

(c)

$$\begin{aligned}\text{Amplitude} &= \frac{|\text{max} - \text{min}|}{2} \\ &= \frac{11 - 1}{2} \quad , \text{ radius of the Ferris wheel} \\ &= 5 \text{ m}\end{aligned}$$

(d) Axis of the curve:

$$\begin{aligned}h &= \frac{\text{max} + \text{min}}{2} \\ &= \frac{11 + 1}{2} \\ &= \frac{12}{2} \\ h &= 6 \text{ m}\end{aligned}$$

6. (a) angle in standard position: the vertex of the angle is located at the origin and the initial arm lies along the positive x-axis

(b) positive angle: the angle is formed by a counter clockwise rotation of the terminal arm

(c) negative angle: the angle is formed by a clockwise rotation of the terminal arm

(d) co-terminal angle: angles which share the same terminal arm

(e) principal angle: the angle, θ , which lies between 0° and 360°

(f) related acute angle: the angle formed by the terminal arm and the x-axis. The angle is always considered to be positive.

7. (a) Quadrant I: 90° , since $0^\circ \leq \theta \leq 90^\circ$

Quadrant II: 180° , since $90^\circ \leq \theta \leq 180^\circ$

Quadrant III: 270° , since $180^\circ \leq \theta \leq 270^\circ$

Quadrant IV: 360° , since $270^\circ \leq \theta \leq 360^\circ$

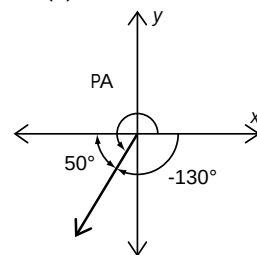
(b) Quadrant I: $\frac{\pi}{2}$, since $0 \leq \theta \leq \frac{\pi}{2}$

Quadrant II: π , since $\frac{\pi}{2} \leq \theta \leq \pi$

Quadrant III: $\frac{3\pi}{2}$, since $\pi \leq \theta \leq \frac{3\pi}{2}$

Quadrant IV: 2π , since $\frac{3\pi}{2} \leq \theta \leq 2\pi$

8. (a)

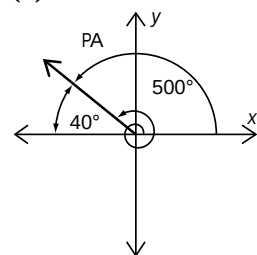


$\theta = -130^\circ$

principal angle: 230°

related acute angle: 50°

(b)

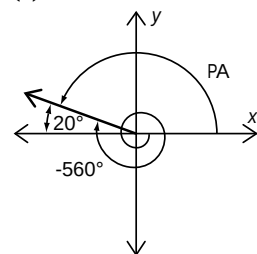


$\theta = 500^\circ$

principal angle: 140°

related acute angle: 40°

(c)

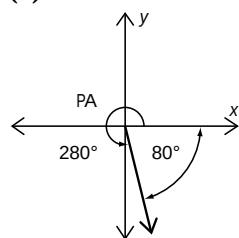


$\theta = -560^\circ$

principal angle: 160°

related acute angle: 20°

(d)



$$\theta = 280^\circ$$

principal angle: 280°

related acute angle: 80°

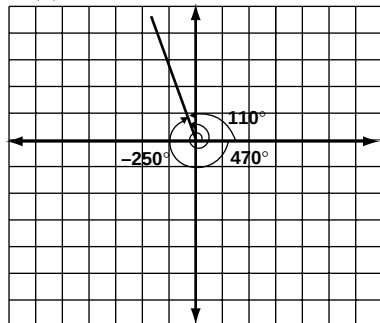
9. (a)

n	$\theta = 62^\circ + 360^\circ n$
-2	-658°
-1	-298°
0	62°
1	422°
2	782°
3	1142°

(b)

n	$\theta = -137^\circ + 360^\circ n$
-1	-497°
0	-137°
1	223°
2	583°

10. (a)

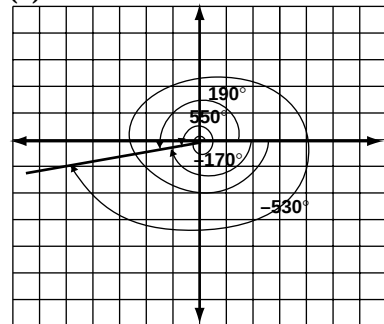


Positive angles: 110° , $110^\circ + 360^\circ = 470^\circ$

Negative angles: $-360^\circ + 110^\circ = -250^\circ$

$\therefore$ the co-terminal angles are: -250° , 110° , 470°

(b)



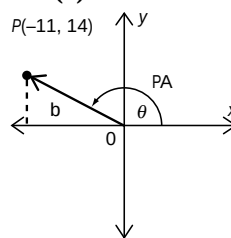
Positive angles: 190° , $190^\circ + 360^\circ = 550^\circ$

Negative angles: $-360^\circ + 190^\circ = -170^\circ$

and $-170^\circ - 360^\circ = -530^\circ$

$\therefore$ the co-terminal angles are: -530° , -170° , 190° , 550°

11. (a)



The angle terminates in Quadrant II.

(b)

$$\tan \beta = \frac{14}{-11}$$

$$\beta = \tan^{-1}\left(\frac{14}{-11}\right)$$

$$\beta \doteq -52^\circ$$

$\therefore$ the related acute angle is 52° .

$$(c) \theta = 180^\circ - 52^\circ = 128^\circ$$

12. (a)

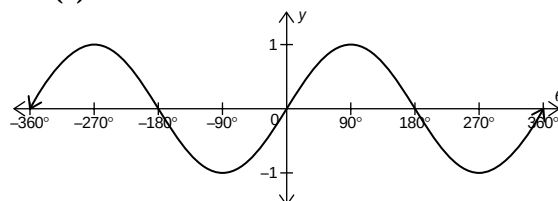
$$\sin \theta = \frac{y}{r}, \quad \cos \theta = \frac{x}{r}, \quad \tan \theta = \frac{y}{x}, \quad x \neq 0$$

$$\text{where } r = \sqrt{x^2 + y^2}.$$

(b) Solve $\sin \theta = \frac{y}{r}$ and $\cos \theta = \frac{x}{r}$ for x and y . Then,

$x = r \cos \theta$ and $y = r \sin \theta$. Replace x and y with their equivalent expressions. $\therefore P(x, y) = P(r \cos \theta, r \sin \theta)$.

13. (a)



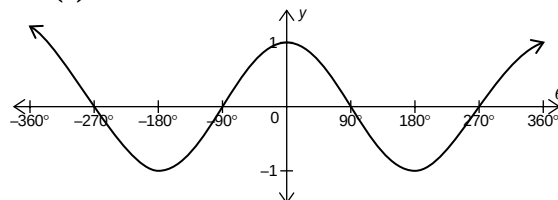
(b) period: 360° ; amplitude: 1; range: $\{-1 \leq y \leq 1, y \in \mathbf{R}\}$; axis of the curve: $y = 0$

(c) maximum values: $(-270^\circ, 1)$, $(90^\circ, 1)$

minimum values: $(-90^\circ, -1)$, $(270^\circ, -1)$

zeros: -360° , -180° , 0 , 180° , 360°

14. (a)



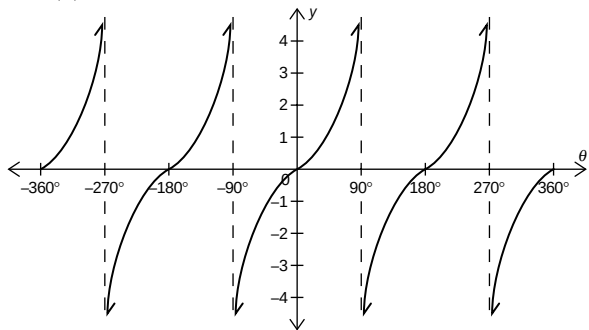
(b) period: 360° ; amplitude: 1; range: $\{-1 \leq y \leq 1, y \in \mathbf{R}\}$; axis of the curve: $y = 0$

(c) maximum values: $(-360^\circ, 1)$, $(0^\circ, 1)$, $(360^\circ, 1)$

minimum values: $(-180^\circ, -1)$, $(180^\circ, -1)$

zeros: -270° , -90° , 90° , 270°

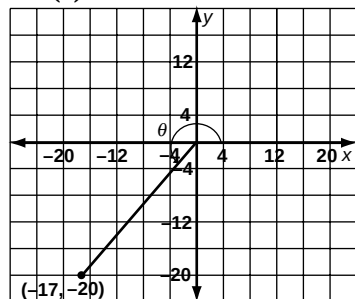
15. (a)



(b) The function $\tan \theta = \frac{y}{x}$ is undefined when $x = 0$.

(c) There is a vertical asymptote at the points where the graph is undefined.

16. (a)



$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-17)^2 + (-20)^2} \\ &= \sqrt{689} \end{aligned}$$

$$\sin \theta = \frac{y}{r} = \frac{-20}{\sqrt{689}} = -\frac{20}{\sqrt{689}}$$

$$\cos \theta = \frac{x}{r} = \frac{-17}{\sqrt{689}} = -\frac{17}{\sqrt{689}}$$

$$\tan \theta = \frac{y}{x} = \frac{-20}{-17} = \frac{20}{17}$$

(b) θ terminates in Quadrant III.

$$\tan \theta = \frac{20}{17} \doteq 49.6^\circ$$

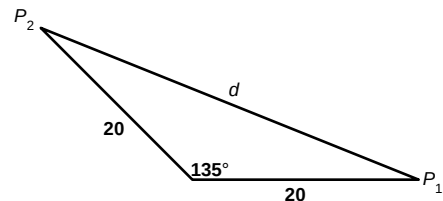
Then in quadrant III, $\theta = 180^\circ + 49.6^\circ \doteq 230^\circ$

17. $P(x, y) = (20 \cos (7.5)t^\circ, 20 \sin (7.5)t^\circ)$

After 18 minutes the angle of rotation $\theta = (7.5(18))^\circ = 135^\circ$

The radius is $r = 20$ m.

Then



The distance, d , between P_2 and P_1 can be found using the Law of Cosines.

Then,

$$d^2 = 20^2 + 20^2 - 2(20)(20) \cos 135^\circ$$

$$d \doteq 36.95$$

The shortest distance is about 37.0 m

Alternate Solution

When $t = 0$:

$$P_1(x_1, y_1) = (20 \cos 0^\circ, 20 \sin 0^\circ) = (20, 0)$$

When $t = 18$:

$$P_2(x_2, y_2) = (20 \cos 135^\circ, 20 \sin 135^\circ) \doteq (-14.142, 14.142)$$

Find the length of the line between the two points $P_1(x_1, y_1)$

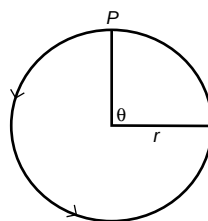
and $P_2(x_2, y_2)$:

$$\begin{aligned} d &= \sqrt{(x_1 - y_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(20 - (-14.142))^2 + (0 - 14.142)^2} \\ &\doteq 36.95 \end{aligned}$$

The shortest distance is about 37.0 m.

18. Both degrees and radians are used to determine the measure of an angle. Degree measure is based on 360° in one rotation of a point about a circle and does not depend on the radius of the circle. Radian measure is also based on rotation about a circle but is dependent on the radius of the circle. For the unit circle, $\pi = 180^\circ$.

19.



Let θ be the angle formed as the point P rotates about the circumference of a circle with radius r . For one complete rotation about the circle, $\theta = 2\pi r$. When $r = 1$, $\theta = 2\pi(1) = 2\pi$.

20. To convert back and forth between degree and radian measure use the proportion:

$$\frac{\text{degree measure}}{\text{degree measure}} = \frac{\text{radian measure}}{\text{radian measure}}$$

with the fact that $180^\circ = \pi$ radians

(a)

$$\frac{20^\circ}{180^\circ} = \frac{x}{\pi}$$

$$x = \frac{\pi}{9} \doteq 0.3$$

(b)

$$\frac{-50^\circ}{180^\circ} = \frac{x}{\pi}$$

$$x = \frac{-5\pi}{18} \doteq -0.9$$

(c)

$$\frac{160^\circ}{180^\circ} = \frac{x}{\pi}$$

$$x = \frac{8\pi}{9} \doteq 2.8$$

(d)

$$\frac{420^\circ}{180^\circ} = \frac{x}{\pi}$$

$$x = \frac{7\pi}{3} \doteq 7.3$$

(e)

$$\frac{-280^\circ}{180^\circ} = \frac{x}{\pi}$$

$$x = \frac{-14\pi}{9} \doteq -4.9$$

21. To convert back and forth between radian and degree measure use the proportion:

$$\frac{\text{radian measure}}{\text{radian measure}} = \frac{\text{degree measure}}{\text{degree measure}}$$

with the fact that $\pi = 180^\circ$ radians

(a)

$$\frac{\pi}{4} = \frac{x}{180^\circ}$$

$$x = 180^\circ \left(\frac{1}{4} \right) = 45^\circ$$

(b)

$$\frac{-5\pi}{4} = \frac{x}{180^\circ}$$

$$x = 180^\circ \left(\frac{-5}{4} \right) = -225^\circ$$

(c)

$$\frac{8\pi}{3} = \frac{x}{180^\circ}$$

$$x = 180^\circ \left(\frac{8}{3} \right) = 480^\circ$$

(d)

$$\frac{-2\pi}{3} = \frac{x}{180^\circ}$$

$$x = 180^\circ \left(\frac{-2}{3} \right) = -120^\circ$$

(e)

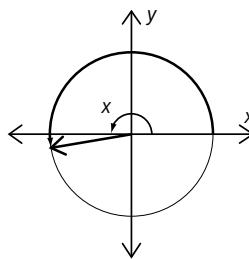
$$\frac{11\pi}{6} = \frac{x}{180^\circ}$$

$$x = 180^\circ \left(\frac{11}{6} \right) = 330^\circ$$

22. (a)

$$\frac{3.2}{\pi} = \frac{x}{180^\circ}$$

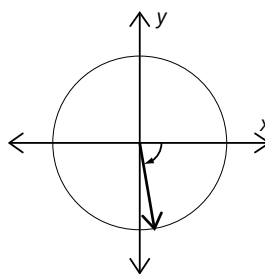
$$x = \frac{180^\circ(3.2)}{\pi} \doteq 183^\circ$$



(b)

$$\frac{-1.4}{\pi} = \frac{x}{180^\circ}$$

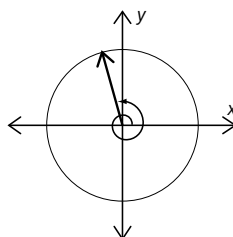
$$x = \frac{180^\circ(-1.4)}{\pi} \doteq -80^\circ$$



(c)

$$\frac{8.3}{\pi} = \frac{x}{180^\circ}$$

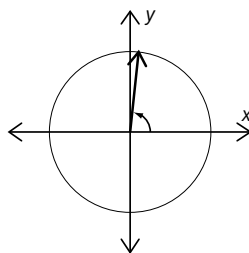
$$x = \frac{180^\circ(8.3)}{\pi} \doteq 476^\circ$$



(d)

$$\frac{1.5}{\pi} = \frac{x}{180^\circ}$$

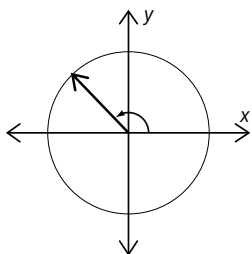
$$x = \frac{180^\circ(1.5)}{\pi} \doteq 86^\circ$$



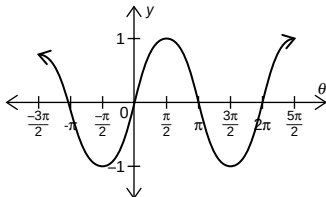
(e)

$$\frac{2.2}{\pi} = \frac{x}{180^\circ}$$

$$x = \frac{180^\circ(2.2)}{\pi} \doteq 126^\circ$$



23.



24. (a)

$$\sin \theta = 0.75$$

$$\theta = \sin^{-1}(0.75)$$

$$\theta \doteq 0.8$$

There are four values :

$$\theta_1 \doteq 0.8, \quad \theta_2 \doteq \pi - 0.8 \doteq 2.3$$

$$\theta_3 \doteq -\pi - 0.8 \doteq -3.9, \quad \theta_4 \doteq -2\pi + 0.8 \doteq -5.4$$

(b)

$$\cos \theta = -0.35$$

$$\theta = \cos^{-1}(-0.35)$$

$$\theta \doteq 1.9 \quad \text{and} \quad \theta \doteq 4.4$$

There are four values :

$$\theta_1 \doteq 1.9, \quad \theta_2 \doteq 4.4$$

$$\theta_3 \doteq 1.9 - 2\pi \doteq -4.4, \quad \theta_4 \doteq 4.4 - 2\pi \doteq -1.9$$

(c)

$$\tan \theta = 6.33$$

$$\theta = \tan^{-1}(6.33)$$

$$\theta \doteq 1.4$$

There are four values :

$$\theta_1 \doteq 1.4, \quad \theta_2 \doteq \pi + 1.4 \doteq 4.6$$

$$\theta_3 \doteq -\pi + 1.4 \doteq -1.7, \quad \theta_4 \doteq -2\pi + 1.4 \doteq -4.9$$

25. (a)

$$h(22) = \sin\left(\frac{\pi}{5}(22)\right) \doteq 0.95 \text{ m}$$

The vertical position of the ship is approximately 0.95 m.

(b)

$$\frac{\pi}{5}t = 2\pi$$

$$\text{Then, the period} = \frac{2\pi}{\frac{\pi}{5}} = 10$$

The period is 10 s. This means that the ship rises and falls with the waves and takes 10 s for one cycle.

(c)

$$h(t) = -0.9$$

$$-0.9 = \sin\left(\frac{\pi}{5}t\right)$$

$$\sin^{-1}(-0.9) = \frac{\pi}{5}t$$

$$\sin^{-1}(-0.9) \doteq -1.119$$

$$\theta = \pi + 1.119 \doteq 4.260$$

$$\theta = 2\pi - 1.119 \doteq 5.164$$

Then :

$$\frac{\pi}{5}t = 4.260 \quad \text{and} \quad \frac{\pi}{5}t = 5.164$$

$$t \doteq 6.78$$

$$t \doteq 8.22$$

The curve has a period of 10 s, then for $0 \leq t \leq 60$, $t = 6.8$ s, 8.2 s, 16.8 s, 18.2 s, 26.8 s, 28.2 s, 36.8 s, 38.2 s, 46.8 s, 48.2 s, 56.8 s, 58.2 s.

26.

- Use the curve $y = \sin \theta$ as the beginning graph.
- k determines how much of the curve is completed. between 0° and 360° . For instance, if $k = 2$, 2 full curves are completed. If $k = \frac{1}{2}$, only half the curve is completed.
- The amplitude is a . All y values are multiplied by a .
- The curve is shifted horizontally a distance of b units to the right if $b < 0$ and b units to the left if $b > 0$.
- The curve is shifted vertically a distance of d units upwards if $d > 0$ and d units downward if $d < 0$.

27. (a) The period is found using:

$$\frac{\pi}{6}t = 2\pi$$

$$\text{Then } t = 2\pi\left(\frac{6}{\pi}\right) = 12$$

The temperature is on a 12 month cycle. There are 12 months in a year.

(b) $y = \cos \theta$ has a maximum of $+1$ and a minimum of -1 .

Substituting $+1$ and -1 for $\cos \frac{\pi}{6}t$ will determine the

maximum and minimum temperatures of $-24(-1) + 9 = 33$ and $-24(+1) + 9 = -15$.

28. (a) y -coordinate is changed by a factor of -3

$$\left(\frac{\pi}{2}, -3\right), (\pi, 0)$$

(b) y -coordinate is shifted up 1 unit

$$\left(\frac{\pi}{2}, 2\right), (\pi, 1)$$

(c) x -coordinate is shifted left $\frac{\pi}{4}$ units

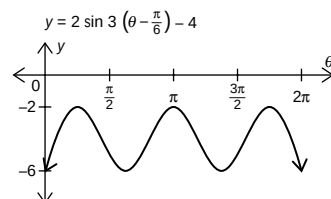
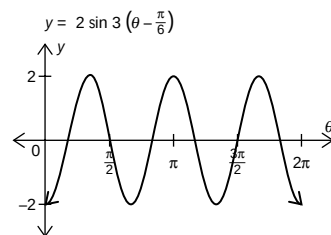
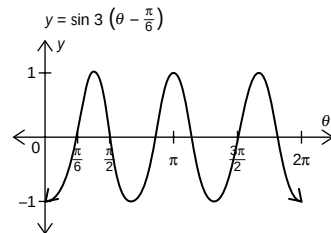
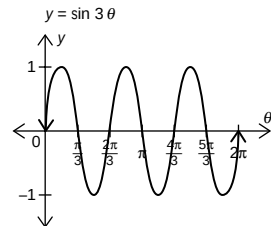
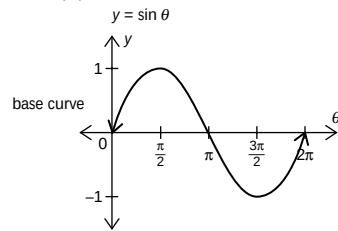
$$\left(\frac{\pi}{2} - \frac{\pi}{4}, 1\right) = \left(\frac{\pi}{4}, 1\right)$$

$$\left(\pi - \frac{\pi}{4}, 0\right) = \left(\frac{3\pi}{4}, 0\right)$$

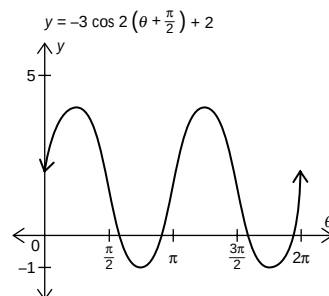
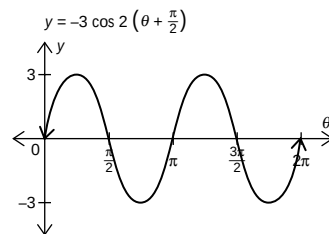
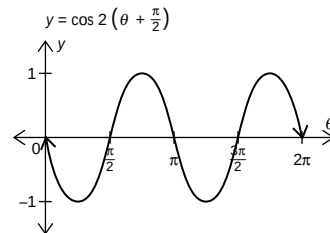
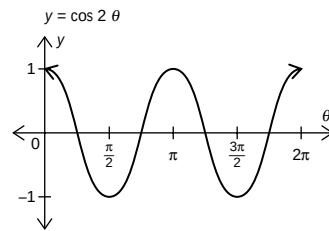
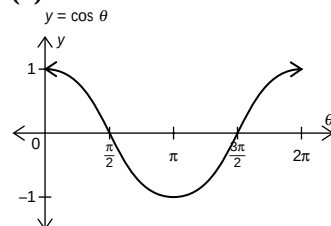
(d) x -coordinate is changed by a factor of $\frac{1}{2}$

$$\left(\frac{\pi}{4}, 1\right), \left(\frac{\pi}{2}, 0\right)$$

29. (a)



(b)



30. (a)

Amplitude : 2

Period :

$$\frac{\theta}{2} = 360^\circ$$

$$\theta = 720^\circ$$

$$\text{Phase Shift : } y = 2 \cos \frac{1}{2}(\theta - 120^\circ) + 1$$

120° to the right

Vertical Shift : up 1 unit

(b)

Amplitude : 3

Period :

$$2\theta = 360^\circ$$

$$\theta = 180^\circ$$

$$\text{Phase Shift : } y = -3 \sin 2(\theta + 45^\circ) - 1$$

45° to the left

Vertical Shift : down 1 unit

(c)

Amplitude : 2

Period :

$$3\theta = 2\pi$$

$$\theta = \frac{2\pi}{3}$$

$$\text{Phase Shift : } y = 2 \cos 3\left(\theta - \frac{\pi}{6}\right) + 1$$

$\frac{\pi}{6}$ to the right

Vertical Shift : up 1 unit

31. (a) Graphing the data gives the general shape of the trigonometric model and whether or not it is sinusoidal. It shows the maximum and minimum value and a starting point for the typical sinusoidal curve. These are necessary when completing $y = a \sin k(\theta + b) + d$ or $y = a \cos k(\theta + b) + d$.

(b) Key information: period, phase shift, amplitude, vertical shift and axis of the curve.

Period: observe one full cycle and determine the length from the beginning to the end of the cycle. Period = $\frac{2\pi}{k}$

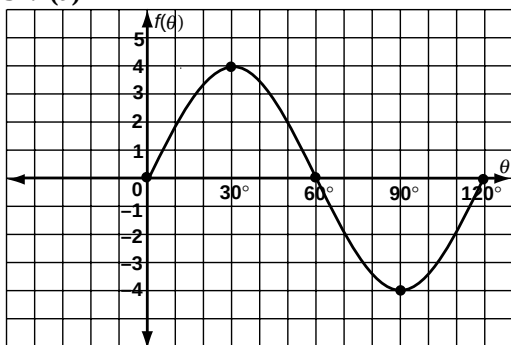
$$\text{Amplitude : } \left| \frac{\text{max} - \text{min}}{2} \right|$$

Also check if the curve is a reflection in the horizontal axis.

$$\text{Axis of the curve : } \frac{\text{max} + \text{min}}{2}$$

The axis of the curve is the vertical shift. The starting point of the typical curve determines the horizontal or phase shift, b . If the starting point is right of 0, $b < 0$. If the starting point is left of 0, $b > 0$.

32. (a)



Resembles the sine function.

The period is 120° . Then $120^\circ k = 360^\circ \therefore k = 3$

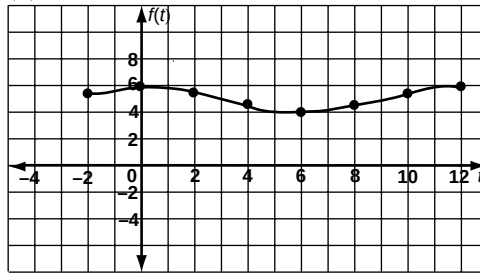
No phase shift. No vertical shift.

The amplitude is 4.

$$\therefore f(\theta) = 4 \sin 3\theta$$

$$\text{or } f(\theta) = 4 \cos 3(\theta - 30^\circ)$$

(b)



Resembles the cosine curve with no phase shift.

The period is 12. Then $12k = 2\pi \therefore k = \frac{\pi}{6}$

Max = 6 and Min = 4

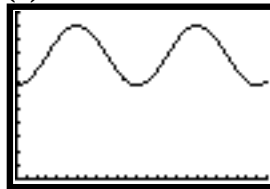
$$\text{Axis of the curve is } f(\theta) = \frac{6+4}{2} \text{ or } f(\theta) = 5$$

The vertical shift is up 5 units.

$$f(t) = \cos \frac{\pi}{6}t + 5$$

33. (a) Ice cream production rises to a maximum with warmer weather and then falls to a minimum with cooler weather. Thus, it is cyclical.

(b)



Resembles a sine curve.

The period is 12. Then $12k = 2\pi \therefore k = \frac{\pi}{6}$

The vertical shift is up 222 units.

$$\text{Axis of the curve is } P(t) = \frac{276+168}{2} = 222.$$

The vertical shift is up 222 units.

The phase shift is 3 units to the right. Then $b = -3$.

$$\text{Amplitude : } a = \frac{276-168}{2} = 54$$

$$\text{Then, } P(t) = 54 \sin \frac{\pi}{6}(t-3) + 222$$

$$\text{or } P(t) = -54 \cos \frac{\pi}{6}t + 222$$

(c) Draw a scatter plot using the graphing calculator and plot

$$P(t) = 54 \sin \frac{\pi}{6}(t-3) + 222.$$
 The scatter plot and graph follow the same pattern with the graph touching over 70% of the data points.

```

Plot1 Plot2 Plot3
Y1=54sin(π/6(X-
3))+222
Y2=
Y3=
Y4=
Y5=
Y6=

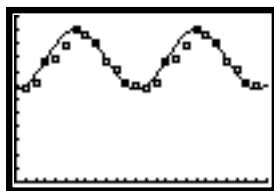
```

```

WINDOW
Xmin=0
Xmax=25
Xscl=1
Ymin=0
Ymax=300
Yscl=25
Xres=1

```

L1	L2	L3	3
1	168		
2	181		
3	194		
4	207		
5	220		
6	233		
7	246		
8	259		
9	272		
10	285		
L3(1)=			



34. Solutions can be found by:

- Sketching the graph and interpolating
- Determining the points of intersection between two corresponding functions that make up the original equation
- Determine the zeros of the single corresponding function

35. (a)

$$\text{L.S} = \cos \theta$$

$$= \cos \left(\frac{5\pi}{3} \right)$$

$$= 0.5$$

$$= \text{R.S}$$

$$\therefore \theta = \frac{5\pi}{3} \text{ is a solution}$$

(b)

$$\text{L.S} = 5 \sin \theta + 6$$

$$= 5 \sin 25^\circ + 6$$

$$\doteq 8.11$$

$$\text{R.S} = 10$$

$$\text{L.S} \neq \text{R.S}$$

$$\therefore \theta = 25^\circ \text{ is not a solution}$$

36. (a)

$$x = \sin^{-1} \left(\frac{2}{3} \right)$$

$$x \doteq 41.8^\circ \text{ (Quadrant I)}$$

$$\text{and } x \doteq 180^\circ - 41.8^\circ = 138.2^\circ \text{ (Quadrant II)}$$

(b)

$$x = \cos^{-1} \left(-\frac{5}{8} \right)$$

$$x \doteq 128.7^\circ \text{ (Quadrant II)}$$

$$\text{and } x \doteq 180^\circ + 51.3^\circ \doteq 231.3^\circ \text{ (Quadrant III)}$$

(c)

$$x = \tan^{-1}(2.8)$$

$$x \doteq 70.3^\circ \text{ (Quadrant I)}$$

$$\text{and } x \doteq 180^\circ + 70.3^\circ \doteq 250.3^\circ \text{ (Quadrant III)}$$

(d)

$$\sin 2x = \frac{-4}{3}$$

$$2x = \sin^{-1} \left(\frac{-4}{3} \right)$$

$$\therefore \text{no solution since } \sin 2x \text{ oscillates between } \pm 1.$$

(e)

$$5 \cos(0.5x) = 2$$

$$\cos(0.5x) = \frac{2}{5}$$

$$0.5x \doteq \cos^{-1} \left(\frac{2}{5} \right)$$

$$\cos^{-1} \left(\frac{2}{5} \right) \doteq 66.42^\circ$$

$$\text{Then } 0.5x \doteq 66.42^\circ$$

$$x \doteq 132.84^\circ \doteq 132.8^\circ$$

(f)

$$3 \tan \left(\frac{x}{2} \right) = 7$$

$$\tan \left(\frac{x}{2} \right) = \frac{7}{3}$$

$$\frac{x}{2} = \tan^{-1} \left(\frac{7}{3} \right)$$

$$\frac{x}{2} \doteq 66.8^\circ \text{ and } \frac{x}{2} \doteq 246.8^\circ$$

$$x \doteq 133.6^\circ \text{ and } x \doteq 493.6^\circ$$

$$\therefore \text{since } 0^\circ \leq x \leq 360^\circ, x = 133.6^\circ$$

37. (a)

$$x = \cos^{-1} \left(\frac{7}{8} \right)$$

$$x \doteq 0.505 \text{ (Quadrant I)}$$

$$x \doteq 0.51$$

$$\text{and } x \doteq 2\pi - 0.51 \doteq 5.78 \text{ (Quadrant IV)}$$

(b)

$$x = \sin^{-1} \left(\frac{-3}{11} \right)$$

$$x \doteq -0.276 \text{ (Quadrant IV)}$$

$$\therefore \text{does not satisfy the given domain}$$

$$\text{Then } x \doteq \pi + 0.276$$

$$x \doteq 3.42 \text{ (Quadrant III)}$$

$$\text{and } x \doteq 2\pi - 0.276 \doteq 6.01 \text{ (Quadrant IV)}$$

(c)

$$2x = \tan^{-1}(1)$$

$$2x \doteq 0.785$$

$$x \doteq 0.39 \text{ (Quadrant I)}$$

and

$$2x \doteq \pi + 0.785$$

$$2x \doteq 3.926$$

$$x \doteq 1.96 \text{ (Quadrant III)}$$

$$\text{Period is } \frac{\pi}{2}$$

$$\text{Then } x_1 \doteq 0.39, x_2 \doteq 0.39 + \frac{\pi}{2} \doteq 1.96, x_3 \doteq 1.96 + \frac{\pi}{2} \doteq 3.53$$

$$x_4 \doteq 3.53 + \frac{\pi}{2} \doteq 5.10$$

(d)

$$5 \sin x = -5$$

$$\sin x = -1$$

$$x = \sin^{-1}(-1)$$

$$x \doteq -1.57 \text{ (Quadrant IV)}$$

$\therefore$ does not satisfy the given domain

$$\text{So, } x \doteq 2\pi - 1.57$$

$$x \doteq 4.71 \text{ (Quadrant IV)}$$

(e)

$$5 \cos 3x = 2$$

$$\cos 3x = \frac{2}{5}$$

$$3x = \cos^{-1}\left(\frac{2}{5}\right)$$

$$3x \doteq 1.159$$

$$x \doteq 0.39 \text{ (Quadrant I)}$$

There are 6 solutions :

$$x_1 \doteq 0.39, \quad x_2 \doteq \frac{2\pi}{3} - 0.39 \doteq 1.71$$

Since it is $\frac{2\pi}{3}$ periodic then :

$$x_3 \doteq 0.39 + \frac{2\pi}{3} \doteq 2.48, \quad x_4 \doteq 1.71 + \frac{2\pi}{3} \doteq 3.80$$

$$x_5 \doteq 0.39 + \left(\frac{2\pi}{3}\right)(2) \doteq 4.58, \quad x_6 \doteq 1.71 + \left(\frac{2\pi}{3}\right)(2) \doteq 5.90$$

(f)

$$3 \tan\left(\frac{x}{2}\right) = 7$$

$$\tan\left(\frac{x}{2}\right) = \frac{7}{3}$$

$$\frac{x}{2} = \tan^{-1}\left(\frac{7}{3}\right)$$

$$\frac{x}{2} \doteq 1.165$$

$$x \doteq 2.33 \text{ (Quadrant II)}$$

There is only one solution for $0 \leq x \leq 2\pi$.

38. (a) $x = 53.1^\circ, 126.9^\circ$

(b) $x = 104.5^\circ, 255.5^\circ$

(c) $x = 71.6^\circ, 251.6^\circ$

(d) $x = 30.0^\circ, 150.0^\circ, 210.0^\circ, 330.0^\circ$

39. (a) $x = 0.46$ and $x = 3.61$

(b) $x = 1.05, 3.14, 5.24$

(c) $x = 0.52, 2.62, 4.71$

(d) $x = 1.57, 4.71$

40. (a)

The highest temperatures occur when:

$$\sin \frac{\pi}{6}(t-3) = 1$$

Then :

$$\frac{\pi}{6}(t-3) = \sin^{-1}(1)$$

$$\frac{\pi}{6}(t-3) = \frac{\pi}{2}, \quad \therefore \sin^{-1}(1) = \frac{\pi}{2}$$

$$t-3 = \frac{\pi}{2}\left(\frac{6}{\pi}\right)$$

$$t-3 = 3$$

$$t = 6$$

The highest temperature is in July.

The lowest temperatures occur when:

$$\sin \frac{\pi}{6}(t-3) = -1$$

$$\frac{\pi}{6}(t-3) = \sin^{-1}(-1)$$

$$\frac{\pi}{6}(t-3) = \frac{3\pi}{2}, \quad \therefore \sin^{-1}(-1) = \frac{3\pi}{2}$$

$$t-3 = \frac{3\pi}{2}\left(\frac{6}{\pi}\right)$$

$$t-3 = 9$$

$$t = 12$$

The lowest temperatures occur in January.

(b)

$$0 = 14.9 \sin \frac{\pi}{6}(t-3) + 13$$

$$14.9 \sin \frac{\pi}{6}(t-3) = -13$$

$$\sin \frac{\pi}{6}(t-3) = \frac{-13}{14.9}$$

$$\frac{\pi}{6}(t-3) = \sin^{-1}\left(\frac{-13}{14.9}\right)$$

$$\sin^{-1}\left(\frac{-13}{14.9}\right) \doteq -1.06$$

$\therefore$ 1.06 is the related acute angle. Then,

$$\sin^{-1}\left(\frac{-13}{14.9}\right) \doteq \pi + 1.06 \doteq 4.20 \text{ and}$$

$$\sin^{-1}\left(\frac{-13}{14.9}\right) \doteq 2\pi - 1.06 \doteq 5.22$$

So:

$$\frac{\pi}{6}(t-3) \doteq 4.20$$

$$t-3 \doteq 8.02$$

$$t \doteq 11.02 \text{ which is December}$$

and

$$\frac{\pi}{6}(t-3) \doteq 5.22$$

$$t-3 \doteq 9.97$$

$$t \doteq 12.97 \text{ which is January}$$

(c)

$$25 = 14.9 \sin \frac{\pi}{6}(t-3) + 13$$

$$14.9 \sin \frac{\pi}{6}(t-3) = 12$$

$$\sin \frac{\pi}{6}(t-3) = \frac{12}{14.9}$$

$$\frac{\pi}{6}(t-3) = \sin^{-1}\left(\frac{12}{14.9}\right)$$

$$\sin^{-1}\left(\frac{12}{14.9}\right) \doteq 0.72$$

$\therefore 0.72$ is the related acute angle. Then,

$$\sin^{-1}\left(\frac{12}{14.9}\right) \doteq 0.72 \quad \text{and}$$

$$\sin^{-1}\left(\frac{12}{14.9}\right) \doteq \pi - 0.72$$

$$\doteq 2.42$$

So:

$$\frac{\pi}{6}(t-3) \doteq 0.72$$

$$t-3 \doteq 1.38$$

$$t = 4.38, \text{ which is early to mid May}$$

and

$$\frac{\pi}{6}(t-3) \doteq 2.42$$

$$t-3 \doteq 4.62$$

$$t \doteq 7.62, \text{ which is late August}$$

Chapter 5 Review Test, page 491

1. (a) exponential

(b) linear

(c) quadratic

(d) periodic: Period is 4 s, Maximum is 25 cm, Minimum is -25 cm

2. If the function models a sinusoidal function it will take on the form of: $y = a \sin k(\theta - b) + d$ or $y = a \cos k(\theta - b) + d$. To determine the amplitude, a , find the maximum and minimum

points of the graph. $a = \left| \frac{\text{max} - \text{min}}{2} \right|$. To find k , first

determine the period of the graph and then solve:

period = $\frac{2\pi}{k}$ for k . To determine the phase shift, b , find a

typical starting point of the graph and determine how far that point is from $\theta = 0^\circ$. If the starting point is right of 0, $b > 0$. If the starting point is left of 0, $b < 0$. To determine the axis of symmetry or the vertical shift, find the maximum and

minimum points of the graph and solve: $d = \frac{\text{max} + \text{min}}{2}$.

3. (a) $\theta = 47^\circ$ (Quadrant I); $\theta = 47^\circ + 360^\circ = 407^\circ$;
 $\theta = -360^\circ + 47^\circ = -313^\circ$

(b) $\theta = \frac{2\pi}{3}$ (Quadrant II), $\theta = -2\pi + \frac{2\pi}{3} = -\frac{4\pi}{3}$

(c) $\theta = 15^\circ$ (Quadrant I); $\theta = 15^\circ + 360^\circ = 375^\circ$;
 $\theta = 15^\circ + 720^\circ = 735^\circ$

4. To convert back and forth between degree and radian measure use the proportion:

$$\frac{\text{degree measure}}{\text{degree measure}} = \frac{\text{radian measure}}{\text{radian measure}}$$

with the fact that $180^\circ = \pi$ radians

(a)

$$\frac{260^\circ}{180^\circ} = \frac{x}{\pi}$$

$$x = \frac{260^\circ \pi}{180^\circ} = \frac{13\pi}{9}$$

(b)

$$\frac{-15^\circ}{180^\circ} = \frac{x}{\pi}$$

$$x = \frac{-15^\circ \pi}{180^\circ} = -\frac{\pi}{12}$$

5. To convert back and forth between radian and degree measure use the proportion:

$$\frac{\text{radian measure}}{\text{radian measure}} = \frac{\text{degree measure}}{\text{degree measure}}$$

with the fact that $\pi = 180^\circ$ radians

(a)

$$\frac{5\pi}{\pi} = \frac{x}{180^\circ}$$

$$x = \frac{5(180^\circ)}{2} = 450^\circ$$

(b)

$$\frac{-7\pi}{\pi} = \frac{x}{180^\circ}$$

$$x = \frac{-7(180^\circ)}{2} = -630^\circ$$

6.

$$\frac{x}{\pi} = \frac{27^\circ}{180^\circ}$$

$$x = \frac{27^\circ \pi}{180^\circ}$$

$$x \doteq 0.47$$

Thus, 27° is approximately 0.47 radians.