

(c)

$$25 = 14.9 \sin \frac{\pi}{6}(t-3) + 13$$

$$14.9 \sin \frac{\pi}{6}(t-3) = 12$$

$$\sin \frac{\pi}{6}(t-3) = \frac{12}{14.9}$$

$$\frac{\pi}{6}(t-3) = \sin^{-1}\left(\frac{12}{14.9}\right)$$

$$\sin^{-1}\left(\frac{12}{14.9}\right) \doteq 0.72$$

$\therefore 0.72$ is the related acute angle. Then,

$$\sin^{-1}\left(\frac{12}{14.9}\right) \doteq 0.72 \quad \text{and}$$

$$\sin^{-1}\left(\frac{12}{14.9}\right) \doteq \pi - 0.72$$

$$\doteq 2.42$$

So:

$$\frac{\pi}{6}(t-3) \doteq 0.72$$

$$t-3 \doteq 1.38$$

$$t = 4.38, \text{ which is early to mid May}$$

and

$$\frac{\pi}{6}(t-3) \doteq 2.42$$

$$t-3 \doteq 4.62$$

$$t \doteq 7.62, \text{ which is late August}$$

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1. (a) exponential

(b) linear

(c) quadratic

(d) periodic: Period is 4 s, Maximum is 25 cm, Minimum is -25 cm

2. If the function models a sinusoidal function it will take on the form of: $y = a \sin k(\theta - b) + d$ or $y = a \cos k(\theta - b) + d$. To determine the amplitude, a , find the maximum and minimum

points of the graph. $a = \left| \frac{\text{max} - \text{min}}{2} \right|$. To find k , first

determine the period of the graph and then solve:

period = $\frac{2\pi}{k}$ for k . To determine the phase shift, b , find a

typical starting point of the graph and determine how far that point is from $\theta = 0^\circ$. If the starting point is right of 0, $b > 0$. If the starting point is left of 0, $b < 0$. To determine the axis of symmetry or the vertical shift, find the maximum and

minimum points of the graph and solve: $d = \frac{\text{max} + \text{min}}{2}$.

3. (a) $\theta = 47^\circ$ (Quadrant I); $\theta = 47^\circ + 360^\circ = 407^\circ$;
 $\theta = -360^\circ + 47^\circ = -313^\circ$

(b) $\theta = \frac{2\pi}{3}$ (Quadrant II), $\theta = -2\pi + \frac{2\pi}{3} = -\frac{4\pi}{3}$

(c) $\theta = 15^\circ$ (Quadrant I); $\theta = 15^\circ + 360^\circ = 375^\circ$;
 $\theta = 15^\circ + 720^\circ = 735^\circ$

4. To convert back and forth between degree and radian measure use the proportion:

$$\frac{\text{degree measure}}{\text{degree measure}} = \frac{\text{radian measure}}{\text{radian measure}}$$

with the fact that $180^\circ = \pi$ radians

(a)

$$\frac{260^\circ}{180^\circ} = \frac{x}{\pi}$$

$$x = \frac{260^\circ \pi}{180^\circ} = \frac{13\pi}{9}$$

(b)

$$\frac{-15^\circ}{180^\circ} = \frac{x}{\pi}$$

$$x = \frac{-15^\circ \pi}{180^\circ} = -\frac{\pi}{12}$$

5. To convert back and forth between radian and degree measure use the proportion:

$$\frac{\text{radian measure}}{\text{radian measure}} = \frac{\text{degree measure}}{\text{degree measure}}$$

with the fact that $\pi = 180^\circ$ radians

(a)

$$\frac{5\pi}{\pi} = \frac{x}{180^\circ}$$

$$x = \frac{5(180^\circ)}{2} = 450^\circ$$

(b)

$$\frac{-7\pi}{\pi} = \frac{x}{180^\circ}$$

$$x = \frac{-7(180^\circ)}{2} = -630^\circ$$

6.

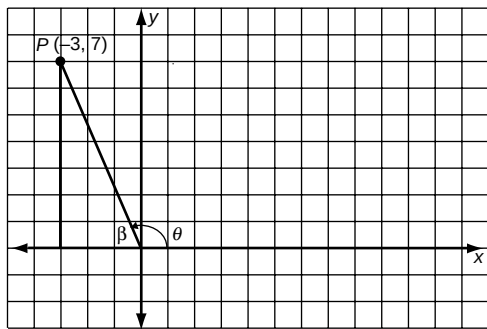
$$\frac{x}{\pi} = \frac{27^\circ}{180^\circ}$$

$$x = \frac{27^\circ \pi}{180^\circ}$$

$$x \doteq 0.47$$

Thus, 27° is approximately 0.47 radians.

7.



(a)

$$r = \sqrt{(-3)^2 + 7^2} = \sqrt{58}$$

$$\sin \theta = \frac{7}{\sqrt{58}}, \cos \theta = \frac{-3}{\sqrt{58}} = -\frac{3}{\sqrt{58}}, \tan \theta = \frac{7}{-3} = -\frac{7}{3}$$

(b)

$$\tan \theta = -\frac{7}{3}$$

$$\theta = \tan^{-1}\left(-\frac{7}{3}\right) \doteq -66.8^\circ, \text{ (Quadrant IV)}$$

But $P(-3, 7)$ terminates in Quadrant II.

$$\therefore \theta = 180^\circ - 66.8^\circ \doteq 113^\circ$$

8. (a)

$$3\theta = 360^\circ$$

$$\theta = \frac{360^\circ}{3} = 120^\circ$$

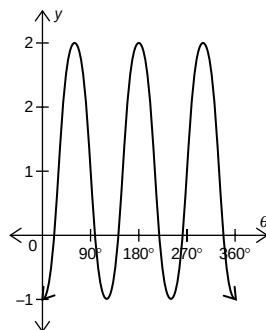
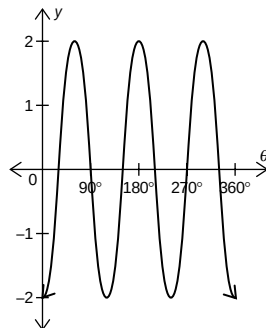
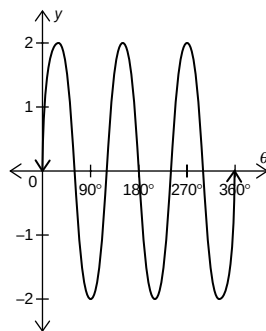
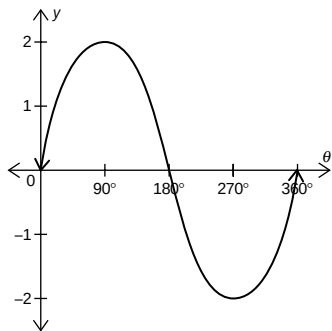
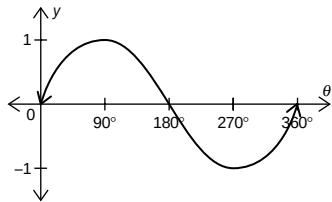
The period is 120° .

Phase shift is 30° to the right.

Amplitude is 2.

Vertical shift is up 1.

(b)



9. (a)

$$5 \cos \theta = -0.42$$

$$\cos \theta = \frac{-0.42}{5}$$

$$\theta = \cos^{-1}\left(\frac{-0.42}{5}\right)$$

$$\theta \doteq 94.8^\circ \doteq 95^\circ$$

and

$$\theta \doteq 360^\circ + 94.8^\circ \doteq 274.8^\circ \doteq 275^\circ$$

(b)

$$\sin 3\theta = -0.6$$

$$3\theta = \sin^{-1}(-0.6)$$

$$\sin^{-1}(-0.6) = -36.86^\circ$$

This angle gives:

$$180^\circ + 36.86^\circ = 216.86^\circ$$

$$\text{and } 360^\circ - 36.86^\circ = 323.14^\circ$$

$$\text{Then } 3\theta = 216.86^\circ \text{ and } 3\theta = 323.14^\circ$$

$$\therefore \theta \doteq 72^\circ \text{ and } \theta \doteq 108^\circ$$

The period of the curve is 120°

$$\therefore \text{the angles are: } 72^\circ, 108^\circ, 72^\circ + 120^\circ = 192^\circ, 108^\circ + 120^\circ = 228^\circ$$

(c)

$$2 \tan\left(\theta - \frac{\pi}{2}\right) = 57.3$$

$$\tan\left(\theta - \frac{\pi}{2}\right) = \frac{57.3}{2}$$

$$\left(\theta - \frac{\pi}{2}\right) = \tan^{-1}\left(\frac{57.3}{2}\right)$$

$$\theta - \frac{\pi}{2} \doteq 1.5359$$

$$\theta \doteq 1.5359 + \frac{\pi}{2} \doteq 3.1066 \doteq 3.1$$

(d)

$$4 - 3 \sin\left(\theta - \frac{\pi}{2}\right) = 6.2$$

$$-3 \sin\left(\theta - \frac{\pi}{2}\right) = 6.2 - 4$$

$$\sin\left(\theta - \frac{\pi}{2}\right) = \frac{2.2}{-3}$$

$$\left(\theta - \frac{\pi}{2}\right) = \sin^{-1}\left(\frac{2.2}{-3}\right)$$

$$\sin^{-1}\left(\frac{2.2}{-3}\right) \doteq -0.8232 \text{ (Quadrant IV)}$$

$$\text{Then } \pi + 0.8232 \doteq 3.9647$$

$$\text{and } 2\pi - 0.8232 \doteq 5.4599$$

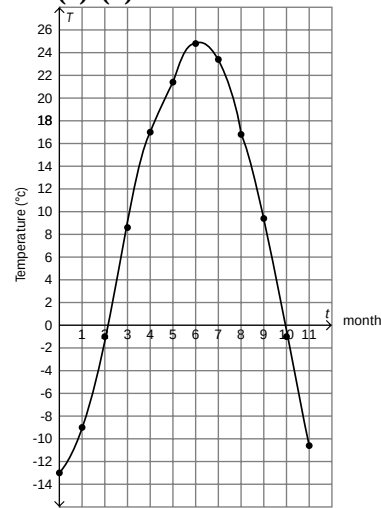
$$\therefore \theta - \frac{\pi}{2} \doteq 3.9647 \text{ and } \theta - \frac{\pi}{2} \doteq 5.4599$$

$$\theta \doteq 5.5 \text{ and } \theta \doteq 7.0256 \text{ (outside of the given domain)}$$

$$\text{So, } \theta \doteq 7.0256 - 2\pi \doteq 0.7425 \doteq 0.7$$

$$\text{Thus, } \theta \doteq 5.5 \text{ and } \theta \doteq 0.7$$

10. (a) (b)



This type of data could be expressed as a periodic function because on a yearly basis, the average temperature each month will be roughly the same.

(c) The maximum value is 24.7°C . The minimum value is -13.1° .

(d) The period is 12. The curve repeats itself after 12 months, representing one year.

(e) The period is 12 months.

$$\text{Therefore, } 12k = 2\pi \text{ or } k = \frac{\pi}{6}$$

The axis of the curve is

$$T(t) = \frac{\text{max} + \text{min}}{2} = \frac{24.7 + (-13.1)}{2} = 5.8$$

Then, the vertical shift is up 5.8 units

$$\text{The amplitude is: } a = \left| \frac{\text{max} - \text{min}}{2} \right| = \left| \frac{24.7 - (-13.1)}{2} \right| = 18.9$$

The sine function, cosine function or negative cosine function could be used. The negative cosine function would give the simplest equation because there would be no phase shift.

$$\text{Then: } T(t) = -18.9 \cos \frac{\pi}{6}t + 5.8$$

(f) If the cosine function acts as the base curve, the phase shift is 6 units to the right.

$$\text{(g) } T(t) = -18.9 \cos \frac{\pi}{6}t + 5.8$$

(h) The 38th month is $t = 37$ since January is $t = 0$.

$$T(37) = -18.9 \cos \frac{\pi}{6}(37) + 5.8 \doteq -10.6^\circ\text{C}$$

Confirm using the tables. January is $t = 0$ plus 12 month periodic. Then, $0 + 12 + 12 + 12$ returns to January after 36 months. Then, $t = 37$ is the next month which is February. $T(\text{February}) = -9.0^\circ\text{C}$. The equation and the table are pretty close.

11. (a)

$$\theta = 45^\circ, d(\theta) = 138 \text{ m}$$

$$\text{Substitute to get } 138 = \frac{v^2}{9.8} \sin 90^\circ + 0.5$$

Solve for v :

$$v^2 = \frac{(138 - 0.5)9.8}{\sin 90^\circ}$$

$$v = \sqrt{\frac{(138 - 0.5)9.8}{\sin 90^\circ}}$$

$$v \doteq 36.7 \text{ m/s}$$

Thus, the initial speed of the ball is 36.7 m/s

(b)

$$d(\theta) = 64 \text{ m}, v = 28 \text{ m/s}$$

Substitute and solve for θ :

$$64 = \frac{28^2}{9.8} \sin 2\theta + 0.5$$

$$\sin 2\theta = \frac{(64 - 0.5)9.8}{28^2}$$

$$2\theta = \sin^{-1} \left(\frac{(64 - 0.5)9.8}{28^2} \right)$$

$$\sin^{-1} \left(\frac{(64 - 0.5)9.8}{28^2} \right) = 52.537^\circ$$

The angles are 52.537° and $180^\circ - 52.537^\circ = 127.463^\circ$

$$\therefore 2\theta = 52.537^\circ \text{ and } 2\theta = 127.463^\circ$$

$$\text{So, } \theta \doteq 26^\circ \text{ and } \theta \doteq 64^\circ$$

Note: The other two angles $26^\circ + 180^\circ = 206^\circ$

and $64^\circ + 180^\circ = 244^\circ$ do not fit the context of this question.

So, the fly ball is struck at an angle of 64° . The line drive is struck at an angle of 26.3° .

12.

Since $k \geq 1$ and $k \in \mathbb{I}$, the graphs will be of the form:

$\sin \theta$, $\sin 2\theta$, $\sin 3\theta$ etc.

$$\text{For } \sin \theta = \frac{1}{2}:$$

$$\theta = \sin^{-1} \left(\frac{1}{2} \right)$$

$$\theta = \frac{\pi}{6}, \text{ (Quadrant I) and } \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}, \text{ (Quadrant II)}$$

Since the period is 2π , $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ are the only two solutions

for $0 \leq \theta \leq 2\pi$. $\therefore$ for $k=1$, there are $2(1) = 2$ solutions.

$$\text{For } \sin 2\theta = \frac{1}{2}:$$

$$2\theta = \sin^{-1} \left(\frac{1}{2} \right)$$

$$2\theta = \frac{\pi}{6}, \text{ (Quadrant I) and } 2\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}, \text{ (Quadrant II)}$$

$$\therefore \theta = \frac{\pi}{12} \text{ and } \theta = \frac{5\pi}{12}$$

Since the period is π , $\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \pi + \frac{\pi}{12} = \frac{13\pi}{12}, \pi + \frac{5\pi}{12} = \frac{17\pi}{12}$

for $0 \leq \theta \leq 2\pi$. $\therefore$ for $k=2$, there are $2(2) = 2$ solutions.

Similarly, this can be shown for $k=3, 4, 5, \dots$ i.e for $k \in \mathbb{I}$.

Thus, $2k$ solutions exist for $\sin k\theta = \frac{1}{2}$, where $\{k \in \mathbb{I} \mid k \geq 1\}$

and $0 \leq \theta \leq 2\pi$.