

Chapter 6

Practise, Apply, Solve 6.1, page 509

1. (a)

$$\frac{\sin B}{15.5} = \frac{\sin 105^\circ}{20.5}$$

$$\sin B = \frac{15.5 \sin 105^\circ}{20.5}$$

$$\angle B \doteq \sin^{-1}(0.730\ 334\ 161\ 341)$$

$$\angle B \doteq 47^\circ$$

(b)

$$\frac{\sin E}{9} = \frac{\sin 38^\circ}{5}$$

$$\sin E = \frac{9 \sin 38^\circ}{5}$$

$$\angle E = \sin^{-1}(1.108\ 190\ 655\ 59)$$

$$\angle E = \text{does not exist}$$

(c)

$$\frac{\sin L}{9.1} = \frac{\sin 120^\circ}{12.3}$$

$$\sin L = \frac{9.1 \sin 120^\circ}{12.3}$$

$$\angle L = \sin^{-1}(0.640\ 717\ 981\ 661)$$

$$\angle L \doteq 40^\circ$$

$$\angle N = 180^\circ - (120^\circ + 40^\circ) = 20^\circ$$

(d)

$$\frac{\sin R}{14.2} = \frac{\sin 45^\circ}{10.8}$$

$$\sin R = \frac{14.2 \sin 45^\circ}{10.8}$$

$$\angle R = \sin^{-1}(0.929\ 714\ 471\ 565)$$

$$\angle R \doteq 68^\circ$$

However, $\angle R$ must be obtuse. $\angle R = 180^\circ - 68^\circ = 112^\circ$.

2. (a) $b^2 = 9^2 + 6^2 - 2(9)(6)\cos 110^\circ$

$$b^2 = 153.938\ 175\ 479$$

$$b \doteq 12.4\ \text{cm}$$

(b) $\angle E = 180^\circ - (36^\circ + 44^\circ) = 100^\circ$

$$\frac{e}{\sin 100^\circ} = \frac{8.5}{\sin 44^\circ}$$

$$e = \frac{8.5 \sin 100^\circ}{\sin 44^\circ}$$

OR

$$\frac{e}{\sin 100^\circ} = \frac{7}{\sin 36^\circ}$$

$$e = \frac{7 \sin 100^\circ}{\sin 36^\circ}$$

$$e \doteq 11.7\ \text{cm}$$

(c) $m^2 = 10^2 + 10^2 - 2(10)(10)\cos 130^\circ$

$$m^2 = 328.557\ 521\ 937$$

$$m \doteq 18.1\ \text{cm}$$

(d) $\angle Y = 180^\circ - (115^\circ + 40^\circ) = 25^\circ$

$$\frac{y}{\sin 25^\circ} = \frac{15}{\sin 115^\circ}$$

$$y = \frac{15 \sin 25^\circ}{\sin 115^\circ}$$

$$y \doteq 7.0\ \text{cm}$$

OR

$$\frac{y}{\sin 25^\circ} = \frac{14}{\sin 40^\circ}$$

$$y = \frac{14 \sin 25^\circ}{\sin 40^\circ}$$

$$y \doteq 9.2\ \text{cm}$$

3. (a) $13^2 = 10^2 + 8^2 - 2(10)(8)\cos B$

$$\frac{13^2 - 10^2 - 8^2}{-2(10)(8)} = \cos B$$

$$\cos^{-1}(-0.031\ 25) = \angle B$$

$$92^\circ = \angle B$$

(b) $5^2 = 5.3^2 + 5.5^2 - 2(5.3)(5.5)\cos D$

$$\frac{5^2 - 5.3^2 - 5.5^2}{-2(5.3)(5.5)} = \cos D$$

$$\cos^{-1}(0.571\ 869\ 639\ 794) = \angle D$$

$$55^\circ = \angle D$$

(c) $11.5^2 = 19.2^2 + 21.3^2 - 2(19.2)(21.3)\cos J$

$$\frac{11.5^2 - 19.2^2 - 21.3^2}{-2(19.2)(21.3)} = \cos J$$

$$\cos^{-1}(0.843\ 701\ 095\ 462) = \angle J$$

$$32^\circ = \angle J$$

(d) $15^2 = 16^2 + 13^2 - 2(16)(13)\cos Z$

$$\frac{15^2 - 16^2 - 13^2}{-2(16)(13)} = \cos Z$$

$$\cos^{-1}(0.480\ 769\ 230\ 769) = \angle Z$$

$$61^\circ = \angle Z$$

4. (a)

$$\frac{l}{\sin 80^\circ} = \frac{22.4}{\sin 60^\circ}$$

$$l = \frac{22.4 \sin 80^\circ}{\sin 60^\circ}$$

$$l \doteq 25.5\ \text{cm}$$

(b)

$$\frac{d}{\sin 108^\circ} = \frac{20}{\sin 32^\circ}$$

$$d = \frac{20 \sin 108^\circ}{\sin 32^\circ}$$

$$d \doteq 35.9\ \text{cm}$$

(c)

$$\frac{r}{\sin 49^\circ} = \frac{25.1}{\sin 93^\circ}$$

$$r = \frac{25.1 \sin 49^\circ}{\sin 93^\circ}$$

$$r \doteq 19.0\ \text{cm}$$

(d) $\angle B = 180^\circ - (2 \times 20^\circ) = 140^\circ$

$$b^2 = 8^2 + 8^2 - 2(8)(8)\cos 140^\circ$$

$$b^2 = 226.053\ 688\ 719$$

$$b = 15.0\ \text{cm}$$

5. (a) $h = c \sin B = 9.8(\sin 40^\circ) = 6.299$

$$\therefore h < 6.6 \text{ and } h < 9.8 \Rightarrow 2 \text{ solutions}$$

(b) $h = c \sin B = 11.8(\sin 73^\circ) = 11.28$

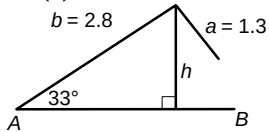
$$\text{But, } b > c \Rightarrow 1 \text{ solution}$$

(c) $h = c \sin B = 5.5(\sin 30^\circ) = 2.75$

$$\therefore 2.5 < h \Rightarrow \text{no solution}$$

(d) $h = b \sin C = 12.2(\sin 34^\circ) = 6.822$
 $\therefore h < 8.2$ and $h < 12.2 \Rightarrow 2$ solutions

6. (a) i.

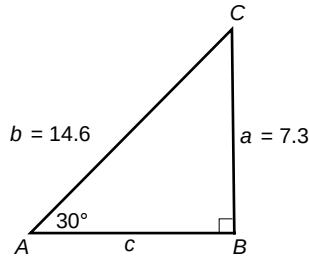


ii. $h = b \sin A = 2.8(\sin 33^\circ) = 1.52$ cm

iii. $1.3 < h \Rightarrow$ no solution

iv. from iii., no solution

(b) i.



ii. $h = b \sin A = 14.6(\sin 30^\circ) = 7.3$ cm

iii. $h = 7.3 = a \Rightarrow$ one solution

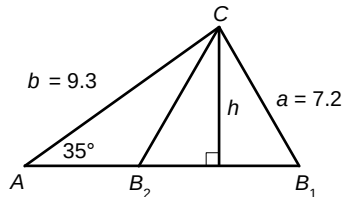
iv.

$$\cos 30^\circ = \frac{c}{14.6}$$

$$14.6(\cos 30^\circ) = c$$

$$12.6 \text{ cm} = c$$

(c) i.



ii. $h = b \sin A = 9.3(\sin 35^\circ) = 5.33$ mm

iii. $h < 7.2$ and $h < 9.3 \Rightarrow$ two solutions

iv.

$$\frac{7.2}{\sin 35^\circ} = \frac{9.3}{\sin B}$$

$$\sin B = \frac{9.3 \sin 35^\circ}{7.2}$$

$$\angle B = \sin^{-1}(0.740869563619)$$

$$\angle B_1 = 47.8^\circ \text{ or } \angle B_2 = 132.2^\circ$$

$$\angle C_1 = 180^\circ - (35^\circ + 132.2^\circ) = 12.8^\circ$$

$$\angle C_2 = 180^\circ - (35^\circ + 47.8^\circ) = 97.2^\circ$$

$$\frac{c_1}{\sin 12.8^\circ} = \frac{7.2}{\sin 35^\circ}$$

$$c_1 = \frac{7.2 \sin 12.8^\circ}{\sin 35^\circ}$$

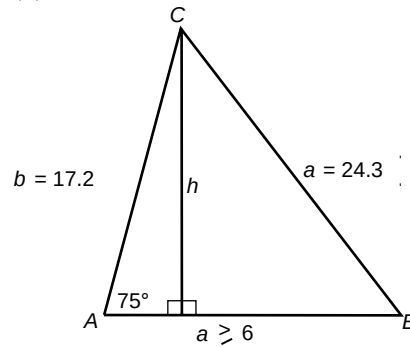
$$c_1 = 2.8 \text{ mm}$$

$$\frac{c_2}{\sin 97.2^\circ} = \frac{7.2}{\sin 35^\circ}$$

$$c_2 = \frac{7.2 \sin 97.2^\circ}{\sin 35^\circ}$$

$$c_2 = 12.5 \text{ mm}$$

(d) i.



ii. $h = b \sin A = 17.2(\sin 75^\circ) = 16.61$ cm

iii. $a > 17.2 \Rightarrow$ one solution

iv.

$$\frac{\sin B}{17.2} = \frac{\sin 75^\circ}{24.3}$$

$$\sin B = \frac{17.2 \sin 75^\circ}{24.3}$$

$$\angle B = \sin^{-1}(0.683700584864)$$

$$\angle B = 43.1^\circ$$

$$\angle C = 180^\circ - (75^\circ + 43.1^\circ) = 61.9^\circ$$

$$\frac{c}{\sin 61.9^\circ} = \frac{24.3}{\sin 75^\circ}$$

$$c = \frac{24.3 \sin 61.9^\circ}{\sin 75^\circ}$$

$$c \doteq 22.2 \text{ cm}$$

7. (a) $m^2 = 11.2^2 + 7.5^2 - 2(11.2)(7.5) \cos 105^\circ$

$$m^2 = 225.1716$$

$$m = 15.0 \text{ cm}$$

$$\frac{15}{\sin 105^\circ} = \frac{11.2}{\sin L}$$

$$\sin L = \frac{11.2 \sin 105^\circ}{15}$$

$$\angle L = \sin^{-1}(0.721224617)$$

$$\angle L \doteq 46^\circ$$

$$\angle N = 180^\circ - (105^\circ + 46^\circ) = 29^\circ$$

(b) $\angle R = 180^\circ - (120^\circ + 28^\circ) = 32^\circ$

$$\frac{t}{\sin 28^\circ} = \frac{25.6}{\sin 120^\circ}$$

$$t = \frac{25.6 \sin 28^\circ}{\sin 120^\circ}$$

$$t \doteq 13.9 \text{ cm}$$

$$\frac{r}{\sin 32^\circ} = \frac{25.6}{\sin 120^\circ}$$

$$r = \frac{25.6 \sin 32^\circ}{\sin 120^\circ}$$

$$r \doteq 15.7 \text{ cm}$$

(c)

$$10^2 = 8^2 + 5^2 - 2(5)(8) \cos A$$

$$\frac{10^2 - 8^2 - 5^2}{-2(8)(5)} = \cos A$$

$$-0.1375 = \cos A$$

$$\angle A = \cos^{-1}(-0.1375)$$

$$\angle A \doteq 98^\circ$$

$$5^2 = 10^2 + 8^2 - 2(10)(8) \cos B$$

$$\frac{5^2 - 10^2 - 8^2}{-2(10)(8)} = \cos B$$

$$0.86875 = \cos B$$

$$\angle B = \cos^{-1}(0.86875)$$

$$\angle B \doteq 30^\circ$$

$$\angle C = 180^\circ - (98^\circ + 30^\circ) = 52^\circ$$

$$(d) \angle X = 180^\circ - (21^\circ + 35^\circ) = 124^\circ$$

$$\frac{z}{\sin 35^\circ} = \frac{18.7}{\sin 124^\circ}$$

$$z = \frac{18.7 \sin 35^\circ}{\sin 124^\circ}$$

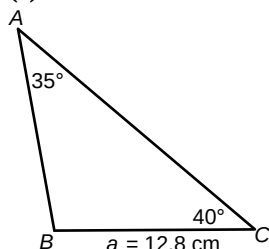
$$z \doteq 12.9 \text{ cm}$$

$$\frac{y}{\sin 21^\circ} = \frac{18.7}{\sin 124^\circ}$$

$$y = \frac{18.7 \sin 21^\circ}{\sin 124^\circ}$$

$$y \doteq 8.1 \text{ cm}$$

(e)



$$\angle B = 180^\circ - (35^\circ + 40^\circ) = 105^\circ$$

$$\frac{b}{\sin 105^\circ} = \frac{12.8}{\sin 35^\circ}$$

$$b = \frac{12.8 \sin 105^\circ}{\sin 35^\circ}$$

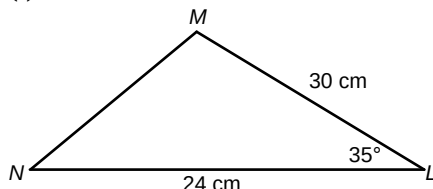
$$b \doteq 21.6 \text{ cm}$$

$$\frac{c}{\sin 40^\circ} = \frac{12.8}{\sin 35^\circ}$$

$$c = \frac{12.8 \sin 40^\circ}{\sin 35^\circ}$$

$$c \doteq 14.3 \text{ cm}$$

(f)



$$l^2 = 30^2 + 24^2 - 2(30)(24) \cos 35^\circ$$

$$l^2 = 296.4211$$

$$l = 17.2 \text{ cm}$$

$$\frac{\sin M}{24} = \frac{\sin 35^\circ}{17.2}$$

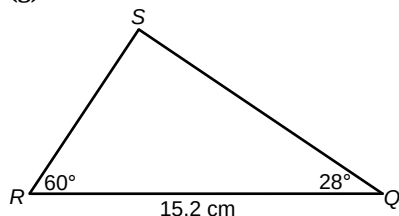
$$\sin M = \frac{24 \sin 35^\circ}{17.2}$$

$$\angle M = \sin^{-1}(0.8003392135)$$

$$\angle M \doteq 53^\circ$$

$$\angle N = 180^\circ - (53^\circ + 35^\circ) = 92^\circ$$

(g)



$$\angle S = 180^\circ - (60^\circ + 28^\circ) = 92^\circ$$

$$\frac{q}{\sin 28^\circ} = \frac{15.2}{\sin 92^\circ}$$

$$q = \frac{15.2 \sin 28^\circ}{\sin 92^\circ}$$

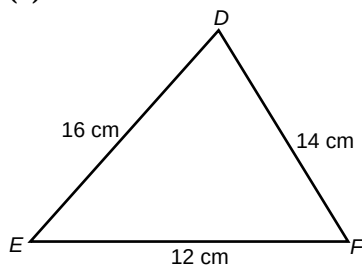
$$q \doteq 7.1 \text{ cm}$$

$$\frac{r}{\sin 60^\circ} = \frac{15.2}{\sin 92^\circ}$$

$$r = \frac{15.2 \sin 60^\circ}{\sin 92^\circ}$$

$$r \doteq 13.2 \text{ cm}$$

(h)



$$16^2 = 14^2 + 12^2 - 2(14)(12) \cos F$$

$$\cos F = \frac{16^2 - 14^2 - 12^2}{-2(14)(12)}$$

$$\angle F = \cos^{-1}(0.25)$$

$$\angle F \doteq 76^\circ$$

$$14^2 = 16^2 + 12^2 - 2(16)(12) \cos E$$

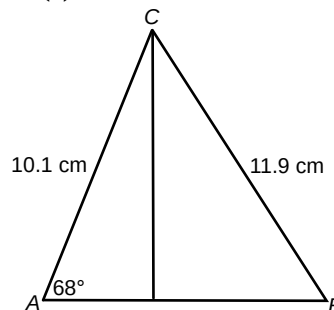
$$\cos E = \frac{14^2 - 16^2 - 12^2}{-2(16)(12)}$$

$$\angle E = \cos^{-1}(0.53125)$$

$$\angle E \doteq 58^\circ$$

$$\angle D = 180^\circ - (58^\circ + 76^\circ) = 46^\circ$$

8. (a)



SSA and $a \geq b \Rightarrow$ one solution

$$\frac{\sin B}{10.1} = \frac{\sin 68^\circ}{11.9}$$

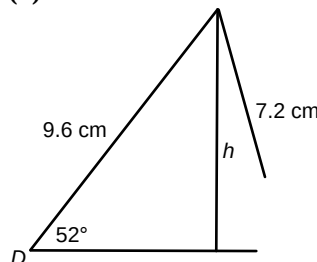
$$\sin B = \frac{10.1 \sin 68^\circ}{11.9}$$

$$\angle B = \sin^{-1}(0.7869375572)$$

$$\angle B \doteq 52^\circ$$

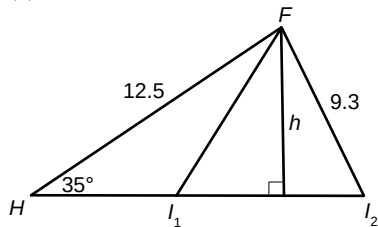
$$\therefore \angle C = 180^\circ - 68^\circ - 52^\circ = 60^\circ$$

(b)



$$h = 9.6(\sin 52^\circ) = 7.56, \therefore 7.2 < h \Rightarrow \text{no solution}$$

(c)



$$h = 12.5(\sin 35^\circ) = 7.16$$

$\therefore$ SSA and $h < 9.3$ and $h < 12.5 \Rightarrow$ two solutions

$$\frac{9.3}{\sin 35^\circ} = \frac{12.5}{\sin I}$$

$$\sin I = \frac{12.5 \sin 35^\circ}{9.3}$$

$$\angle I = \sin^{-1}(0.770\ 936\ 070\ 4)$$

$$\angle I = 50^\circ$$

$$\therefore \angle I_2 = 50^\circ \Rightarrow \angle F_2 = 180^\circ - (35^\circ + 50^\circ) = 95^\circ$$

$$\therefore \angle I_1 = 180^\circ - 50^\circ = 130^\circ \Rightarrow \angle F_1 = 15^\circ$$

$$\frac{f_1}{\sin 15^\circ} = \frac{9.3}{\sin 35^\circ}$$

$$f_1 = \frac{9.3 \sin 15^\circ}{\sin 35^\circ}$$

$$f_1 = 4.2 \text{ cm}$$

$$\frac{f_2}{\sin 95^\circ} = \frac{9.3}{\sin 35^\circ}$$

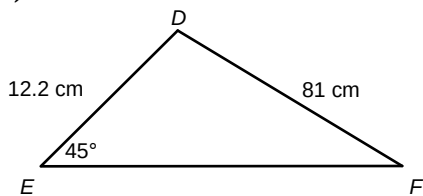
$$f_2 = \frac{9.3 \sin 95^\circ}{\sin 35^\circ}$$

$$f_2 = 16.2 \text{ cm}$$

$\therefore$ one triangle: $\angle I = 50^\circ$, $\angle F = 95^\circ$, $f = 16.2 \text{ cm}$

OR $\angle I = 130^\circ$, $\angle F = 15^\circ$, $f = 4.2 \text{ cm}$

(d)



SSA and $e \geq f \Rightarrow$ one solution

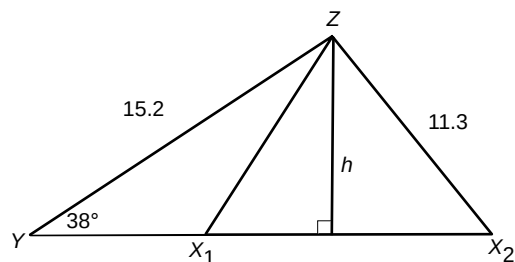
$$\angle D = 180^\circ - (45^\circ + 6^\circ) = 129^\circ$$

$$\frac{d}{\sin 129^\circ} = \frac{81}{\sin 45^\circ}$$

$$d = \frac{81 \sin 129^\circ}{\sin 45^\circ}$$

$$d \doteq 89.0 \text{ cm}$$

(e)



$$h = 15.2(\sin 38^\circ) = 9.36$$

SSA and $h < 11.3$ and $h < 15.2 \Rightarrow$ two solutions

$$\frac{\sin X}{15.2} = \frac{\sin 38^\circ}{11.3}$$

$$\sin X = \frac{15.2 \sin 38^\circ}{11.3}$$

$$\angle X = \sin^{-1}(0.828\ 146\ 409)$$

$$\angle X \doteq 56^\circ$$

$$\therefore \angle X_2 = 56^\circ \Rightarrow \angle Z_2 = 180^\circ - (38^\circ + 56^\circ) = 86^\circ$$

$$\therefore \angle X_1 = 180^\circ - 56^\circ = 124^\circ \Rightarrow \angle Z_1 = 18^\circ$$

$$\frac{z_2}{\sin 86^\circ} = \frac{11.3}{\sin 38^\circ}$$

$$z_2 = \frac{11.3 \sin 86^\circ}{\sin 38^\circ}$$

$$z_2 = 18.3 \text{ cm}$$

$$\frac{z_1}{\sin 18^\circ} = \frac{11.3}{\sin 38^\circ}$$

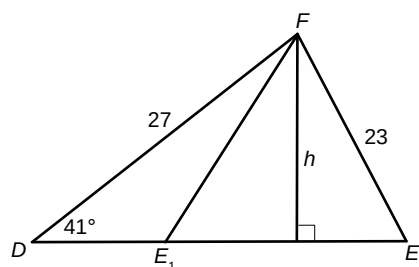
$$z_1 = \frac{11.3 \sin 18^\circ}{\sin 38^\circ}$$

$$z_1 = 5.7 \text{ cm}$$

$\therefore$ one triangle: $\angle X = 56^\circ$, $\angle Z = 86^\circ$, $z = 18.3 \text{ cm}$

OR $\angle X = 124^\circ$, $\angle Z = 18^\circ$, $z = 5.7 \text{ cm}$

9.



$$h = 27(\sin 41^\circ) = 17.71$$

$\therefore$ SSA, $h < 23$ and $h < 27 \Rightarrow$ two solutions

$$\frac{\sin E}{27} = \frac{\sin 41^\circ}{23}$$

$$\sin E = \frac{27 \sin 41^\circ}{23}$$

$$\angle E = \sin^{-1}(0.770\ 156\ 251\ 4)$$

$$\angle E \doteq 50^\circ$$

$$\therefore \angle E_2 = 50^\circ \Rightarrow \angle F_2 = 180^\circ - (50^\circ + 41^\circ) = 89^\circ$$

$$\text{OR } \angle E_1 = 180^\circ - 50^\circ = 130^\circ \Rightarrow \angle F_1 = 9^\circ$$

$$\frac{f_1}{\sin 9^\circ} = \frac{23}{\sin 41^\circ}$$

$$f_1 = \frac{23 \sin 9^\circ}{\sin 41^\circ}$$

$$f_1 = 5.5 \text{ cm}$$

$$\frac{f_2}{\sin 89^\circ} = \frac{23}{\sin 41^\circ}$$

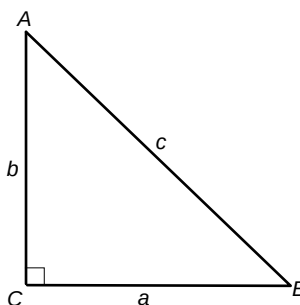
$$f_2 = \frac{23 \sin 89^\circ}{\sin 41^\circ}$$

$$f_2 = 35.1 \text{ cm}$$

$\therefore$ one triangle: $\angle E = 50^\circ$, $\angle F = 89^\circ$, $f = 35.1 \text{ cm}$

OR $\angle E = 130^\circ$, $\angle F = 9^\circ$, $f = 5.5 \text{ cm}$

10.



$$c^2 = a^2 + b^2 - 2ab \cos C$$

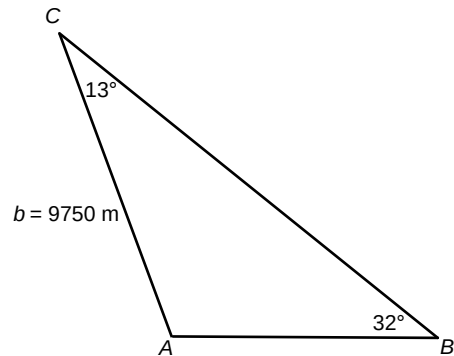
Let $\angle C = 90^\circ$.

$$\therefore c^2 = a^2 + b^2 - 2ab \cos 90^\circ$$

$$c^2 = a^2 + b^2 - 2ab(0)$$

$$c^2 = a^2 + b^2$$

11.



$$\angle C = 45^\circ - 32^\circ = 13^\circ, \therefore \angle CBA = 32^\circ$$

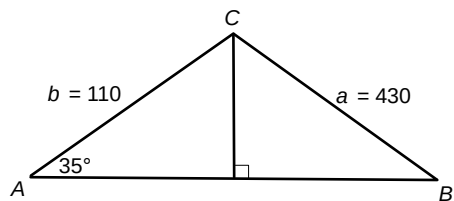
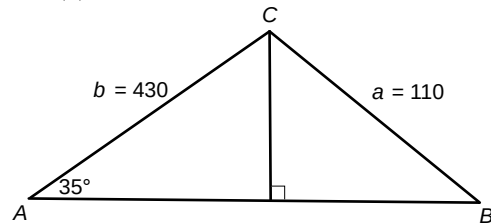
$$\frac{AB}{\sin 13^\circ} = \frac{9750}{\sin 32^\circ}$$

$$AB = \frac{9750 \sin 13^\circ}{\sin 32^\circ}$$

$$AB \doteq 4139 \text{ m}$$

$\therefore$ the distance across the lake is 4139 m.

12. (a)



There are two possible triangles that Mrs. Mardle is describing.

$$h = 430(\sin 35^\circ) = 246.64, \therefore 110 < h \Rightarrow \text{not possible}$$

$$h = 110(\sin 35^\circ) = 63.09, a \geq b \Rightarrow \text{one triangle possible}$$

$\therefore$ this is a true statement. The second triangle justifies the statement.

(b) Consider $\triangle ABC$, with $a = 430 \text{ m}$, $\angle A = 35^\circ$, $b = 110 \text{ m}$.

$$\frac{\sin B}{110} = \frac{\sin 35^\circ}{430}$$

$$\sin B = \frac{110 \sin 35^\circ}{430}$$

$$\angle B = \sin^{-1}(0.1467288558)$$

$$\angle B = 8.4^\circ$$

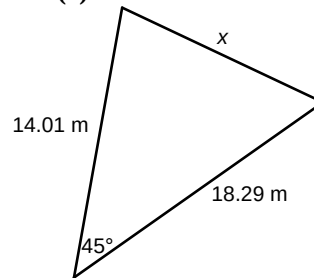
$$\angle C = 180^\circ - (35^\circ + 8.4^\circ) = 136.6^\circ$$

$$\frac{c}{\sin 136.6^\circ} = \frac{430}{\sin 35^\circ}$$

$$c = \frac{430 \sin 136.6^\circ}{\sin 35^\circ}$$

$$c = 515.1 \text{ m}$$

13. (a)

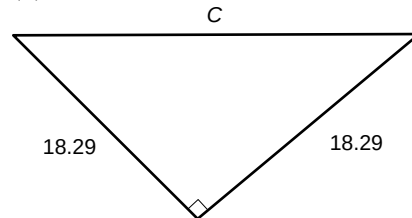


$$x^2 = 14.01^2 + 18.29^2 - 2(14.01)(18.29)\cos 45^\circ$$

$$x^2 = 168.4220156$$

$$x \doteq 12.98 \text{ m}$$

(b)



$$c^2 = 18.29^2 + 18.29^2$$

$$c^2 = 669.0482$$

$$c \doteq 25.87 \text{ m}$$

$$(c) d = 25.87 - 14.01 = 11.86 \text{ m}$$

$$14. \angle ACB = 180^\circ - (80^\circ + 35^\circ) = 65^\circ$$

$$\frac{AC}{\sin 80^\circ} = \frac{10}{\sin 65^\circ}$$

$$AC = \frac{10 \sin 80^\circ}{\sin 65^\circ}$$

$$AC = 10.87 \text{ cm}$$

$$CD^2 = 8^2 + 10.87^2 - 2(8)(10.87)\cos 75^\circ$$

$$CD^2 = 64 + 118.1569 - 45.01$$

$$CD = 11.7 \text{ cm}$$

15.

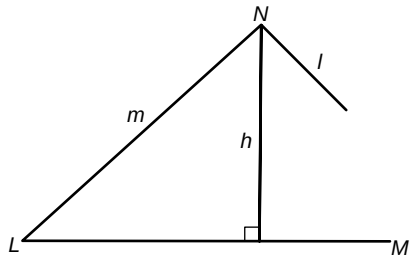
$$d^2 = 10^2 + 10^2 - 2(10)(10)\cos 28.5^\circ$$

$$d^2 = 24.23657747$$

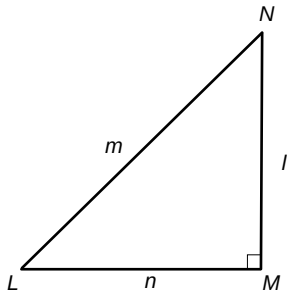
$$d \doteq 4.92 \text{ cm}$$

$\therefore$ the diameter of the cup is 4.92 cm.

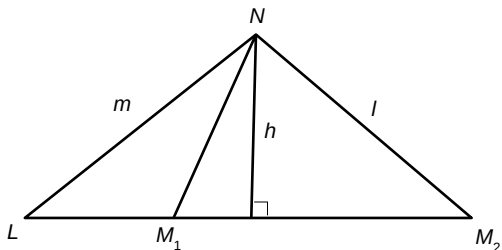
16. (a) For no triangles to be possible, $l < h \Rightarrow l < m \sin L$



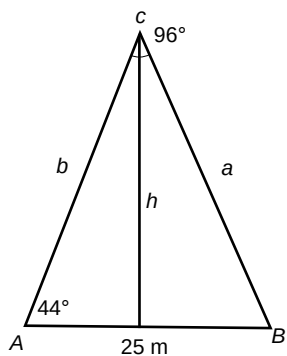
(b) For one triangle to be possible, $l = h \Rightarrow l = m \sin L$



(c) For two triangles to be possible, $h < l$ and $h < m$ so that $m \sin L < l < m$



17. (a)



$$\angle B = 180^\circ - (96^\circ + 44^\circ) = 40^\circ$$

$$\frac{a}{\sin 44^\circ} = \frac{25}{\sin 96^\circ}$$

$$a = \frac{25 \sin 44^\circ}{\sin 96^\circ}$$

$$a = 17.5 \text{ m}$$

$\therefore$ one wire is 17.5 m and the other wire is 16.2 m long.

$$\frac{b}{\sin 40^\circ} = \frac{25}{\sin 96^\circ}$$

$$b = \frac{25 \sin 40^\circ}{\sin 96^\circ}$$

$$b \doteq 16.2 \text{ m}$$

(b)

$$\sin 44^\circ = \frac{h}{b}$$

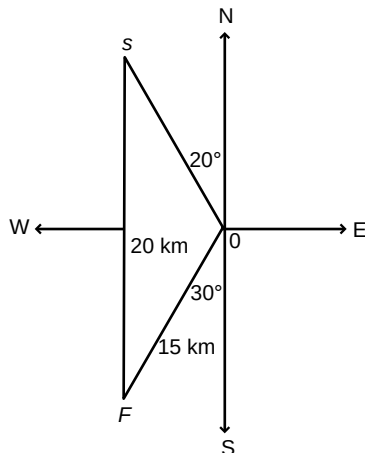
$$\sin 44^\circ = \frac{h}{16.2}$$

$$h = 16.2(\sin 44^\circ)$$

$$h = 11.3 \text{ m}$$

$\therefore$ the tower is 11.3 m high.

18.



$$\angle SOF = 180^\circ - (20^\circ + 30^\circ) = 130^\circ$$

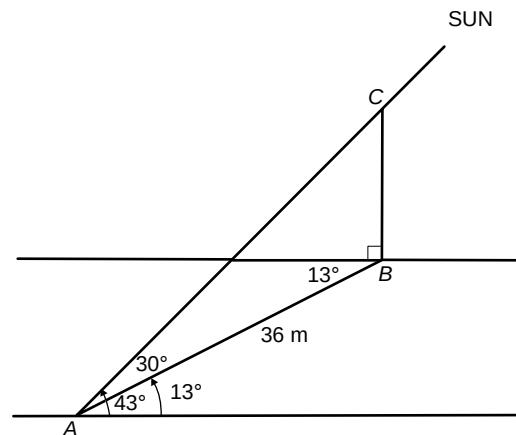
$$o^2 = 20^2 + 15^2 - 2(20)(15)\cos 130^\circ$$

$$o^2 = 1010.672 \ 566$$

$$o \doteq 32 \text{ km}$$

$\therefore$ at that point, the two boats are 32 km apart.

19.



$$\angle A = 43^\circ - 13^\circ = 30^\circ, \angle B = 90^\circ + 13^\circ = 103^\circ,$$

$$\angle C = 180^\circ - (30^\circ + 103^\circ) = 47^\circ$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

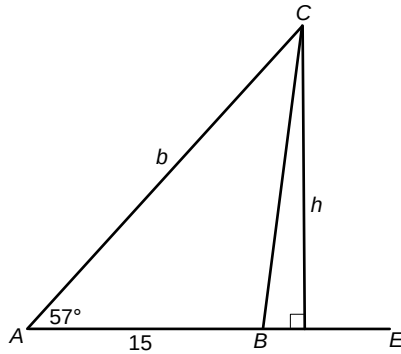
$$\frac{a}{\sin 30^\circ} = \frac{36}{\sin 47^\circ}$$

$$a = \frac{36 \sin 30^\circ}{\sin 47^\circ}$$

$$a = 24.6 \text{ m}$$

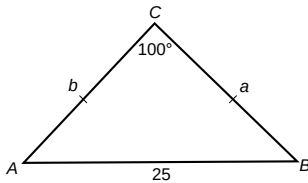
$\therefore$ the tower is 24.6 m high.

20.



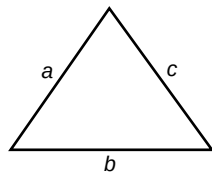
$$\begin{aligned}\angle ABC &= 180^\circ - 83^\circ = 97^\circ \\ \angle ACB &= 180^\circ - (57^\circ + 97^\circ) = 26^\circ \\ \frac{b}{\sin 97^\circ} &= \frac{15}{\sin 26^\circ} & \sin 57^\circ &= \frac{h}{33.96} \\ b &= \frac{15 \sin 97^\circ}{\sin 26^\circ} & h &= 33.96 (\sin 57^\circ) \\ b &\doteq 33.96 & h &= 28.5 \text{ km} \\ \therefore \text{the balloon is at an altitude of 28.5 km.}\end{aligned}$$

21.

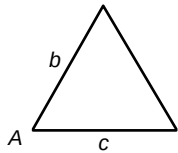


$$\begin{aligned}\angle CAB &= \angle CBA = \frac{180^\circ - 100^\circ}{2} = 40^\circ, a = b \\ \frac{b}{\sin 40^\circ} &= \frac{25}{\sin 100^\circ} \\ b &= \frac{25 \sin 40^\circ}{\sin 100^\circ} \\ b &\doteq 16.32 \text{ cm} \\ P &= 16.32 + 16.32 + 25 = 57.6 \text{ cm} \\ \therefore \text{the perimeter of the triangle is 57.6 cm.}\end{aligned}$$

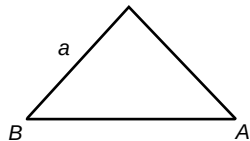
22.



SSS – you can find all 3 angles using the cosine law:
 $a^2 = b^2 + c^2 - 2bc \cos A$



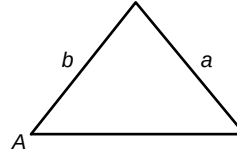
SAS – you can find the side opposite the given angle using the cosine law: $a^2 = b^2 + c^2 - 2bc \cos A$



Part 3: Trigonometric Functions

ASA – you can find the side opposite a given angle using the

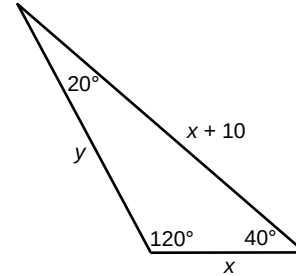
sine law: $\frac{a}{\sin A} = \frac{b}{\sin B}$



SSA – you can find the angle(s) opposite the given side using

the sine law: $\frac{a}{\sin A} = \frac{b}{\sin B}$

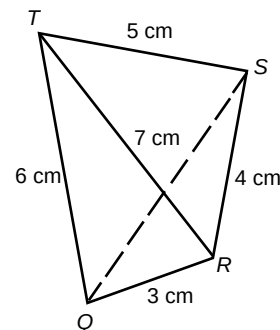
23.



$$\begin{aligned}\frac{x}{\sin 20^\circ} &= \frac{x+10}{\sin 120^\circ} \\ \frac{x}{0.3420} &= \frac{x+10}{0.8660} \\ 0.866x &= 0.342(x+10) \\ 0.866x &= 0.342x + 3.42 \\ 0.524x &= 3.42 \\ x &= 6.53\end{aligned}$$

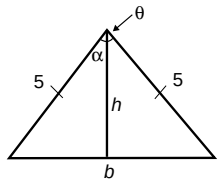
$$\begin{aligned}\frac{y}{\sin 40^\circ} &= \frac{6.53}{\sin 20^\circ} \\ y &= \frac{6.53 \sin 40^\circ}{\sin 20^\circ} \\ y &= 12.27 \\ \text{Thus, } P &= x + y + x + 10 \\ P &= 6.53 + 12.27 + 16.53 \\ P &= 35.33 \text{ cm}\end{aligned}$$

24.



$$\begin{aligned}5^2 &= 7^2 + 4^2 - 2(7)(4) \cos \angle TRS \\ \frac{5^2 - 7^2 - 4^2}{-2(7)(4)} &= \cos \angle TRS \\ 44.4^\circ &= \angle TRS \\ 6^2 &= 7^2 + 3^2 - 2(7)(3) \cos \angle TRQ \\ \cos \angle TRQ &= \frac{6^2 - 7^2 - 3^2}{-2(7)(3)} \\ 58.4^\circ &= \angle TRQ \\ \therefore \angle QRS &= 44.4^\circ + 58.4^\circ = 102.8^\circ \\ \therefore SQ^2 &= 3^2 + 4^2 - 2(3)(4) \cos 102.8^\circ \\ SQ^2 &= 30.317 \text{ 163 94} \\ SQ &= 5.5 \text{ cm}\end{aligned}$$

25.



$$\theta = \frac{360^\circ}{5} = 72^\circ, \alpha = \frac{72^\circ}{2} = 36^\circ$$

$$\cos 36^\circ = \frac{h}{5}$$

$$h = 5 (\cos 36^\circ)$$

$$h = 4.05 \text{ cm}$$

$$b^2 = 5^2 + 5^2 - 2(5)(5)\cos 72^\circ$$

$$b^2 = 34.549 150 28$$

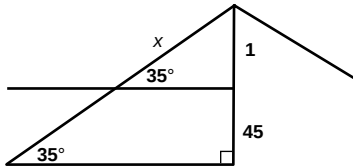
$$b = 5.88$$

$$A_{\text{triangle}} = \frac{1}{2}bh = (5.88 \times 4.05) \div 2 = 11.907 \text{ cm}^2$$

$$A_{\text{pentagon}} = 5 \times 11.907 = 59.5 \text{ cm}^2$$

Practise, Apply, Solve 6.2, page 522

1.



$$\sin 35^\circ = \frac{46}{x}$$

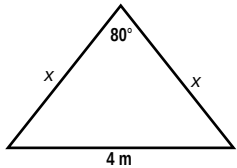
$$x \sin 35^\circ = 46$$

$$x = \frac{46}{\sin 35^\circ}$$

$$x = 80.2 \text{ m}$$

$\therefore$ Morana must let out 80.2 m of line.

2.



$$4^2 = x^2 + x^2 - 2(x)(x) \cos 80^\circ$$

$$16 = 2x^2 - 0.347 296 355 3x^2$$

$$16 = 1.652 703 645x^2$$

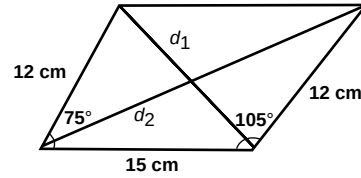
$$x^2 = 9.681 106 502$$

$$x = 3.11$$

$$\therefore 3.1 + 0.5 = 3.6 \text{ m}$$

$\therefore$ each rafter must be 3.6 m long.

3. (a)



$$d_1^2 = 12^2 + 15^2 - 2(12)(15) \cos 75^\circ$$

$$d_1^2 = 275.825 143 8$$

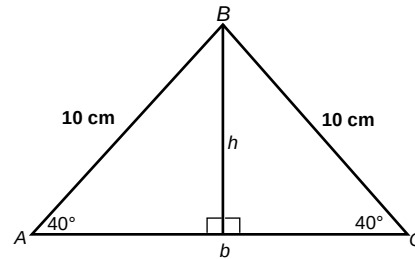
$$d_1 = 16.6 \text{ cm}$$

(b) $d_2^2 = 15^2 + 12^2 - 2(15)(12) \cos 105^\circ$

$$d_2^2 = 462.174 856 2$$

$$d_2 = 21.5 \text{ cm}$$

4.



$$\sin 40^\circ = \frac{h}{10}$$

$$10 (\sin 40^\circ) = h$$

$$6.4 \text{ cm} = h$$

$$\angle ABC = 180^\circ - (40^\circ + 40^\circ) = 100^\circ$$

$$b^2 = 10^2 + 10^2 - 2(10)(10) \cos 100^\circ$$

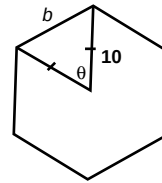
$$b^2 = 234.729 635 5$$

$$b = 15.3 \text{ cm}$$

$$A_{\text{triangle}} = \frac{1}{2}bh = (15.3 \times 6.4) \div 2 = 49.0 \text{ cm}^2$$

$\therefore$ the height of the triangle is 6.4 cm and the area of the triangle is 49.0 cm^2 .

5.

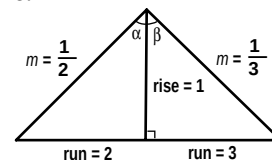


$$\theta = \frac{360^\circ}{6} = 60^\circ$$

$\therefore$ each triangle is an equilateral triangle $\Rightarrow b = 10 \text{ cm}$

$$P = 6 \times 10 = 60 \text{ cm}$$

6.



$$m = \frac{1}{2}$$

$$m = \frac{1}{3}$$

$$\text{run} = 2$$

$$\text{run} = 3$$