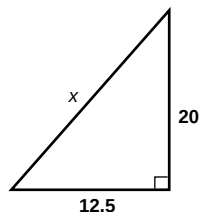
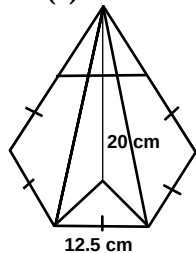


18. (a)



$$\theta = \frac{360^\circ}{6} = 60^\circ \Rightarrow \text{each base triangle is equilateral}$$

$$x^2 = 20^2 + 12.5^2$$

$$x^2 = 556.25$$

$$x = 23.58 \text{ cm}$$

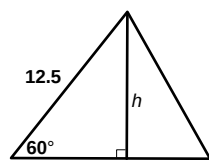
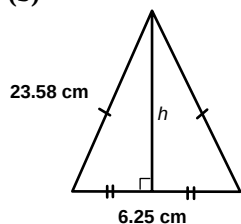
$$12.5^2 = 23.58^2 + 23.58^2 - 2(23.58)(23.58)\cos \alpha$$

$$\cos \alpha = \frac{12.5^2 - 2(23.58)^2}{-2(23.58)}$$

$$\cos \alpha = 0.8594915546$$

$$\alpha = 30.7^\circ$$

(b)



$$23.58^2 = 6.25^2 + h^2$$

$$h^2 = 23.58^2 - 6.25^2$$

$$h^2 = 516.9539$$

$$h = 22.74 \text{ cm}$$

$$\sin 60^\circ = \frac{h}{12.5}$$

$$h = 12.5 (\sin 60^\circ)$$

$$h = 10.83 \text{ cm}$$

$$SA_{\text{pyramid}} = A_{\text{triangle}} + A_{\text{base}}$$

$$SA_{\text{pyramid}} = 6\left(\frac{12.5 \times 22.74}{2}\right) + 6\left(\frac{12.5 \times 10.83}{2}\right)$$

$$SA_{\text{pyramid}} = 852.75 + 406.125$$

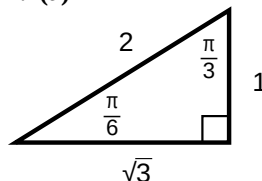
$$SA_{\text{pyramid}} = 1258.875 \text{ cm}^2$$

$$(c) V_{\text{pyramid}} = \frac{A_{\text{base}} \times h}{3} = \frac{406.125 \times 20}{3} = 2707.5 \text{ cm}^3$$

19. The essential thing that you must do in order to solve each trigonometric problem is to draw a diagram. If the triangle is a right triangle, use primary trigonometric ratios. If two angles and any side are known or if two sides and any angle are given, use the sine law. If two sides and a contained angle are given or three sides are given, use the cosine law.

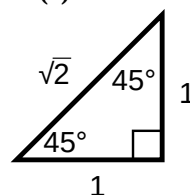
Practise, Apply, Solve 6.3, page 532

1. (a)



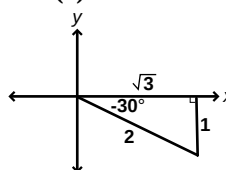
(b) 1 is the shortest side and must lie opposite the smallest angle, which is 30° . 2 is the largest side and must lie opposite the largest angle, which is 90° . $\sqrt{3}$ must then lie opposite the 60° angle.

2. (a)



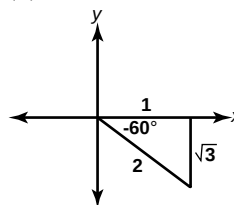
(b) The triangle is isosceles. $\sqrt{2}$ is the largest side and must lie opposite the largest angle, which is 90° . Therefore, the two equal sides lie opposite the 45° angles.

3. (a)



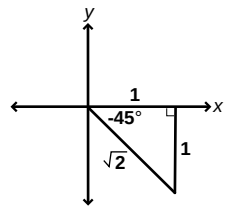
$$\cos \theta = \frac{x}{r}, \cos (-30^\circ) = \frac{\sqrt{3}}{2}$$

(b)



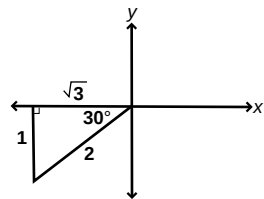
$$\tan \theta = \frac{y}{x}, \tan (-60^\circ) = \frac{-\sqrt{3}}{1} = -\sqrt{3}$$

(c)



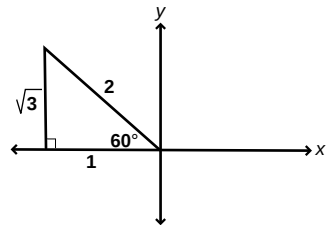
$$\sin \theta = \frac{y}{r}, \sin (-45^\circ) = \frac{-1}{\sqrt{2}}$$

4. (a)



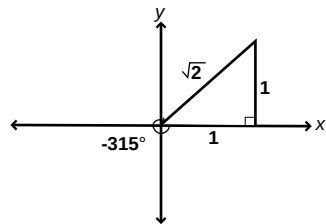
$$\tan \theta = \frac{y}{x}, \tan (-150^\circ) = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$$

(b)



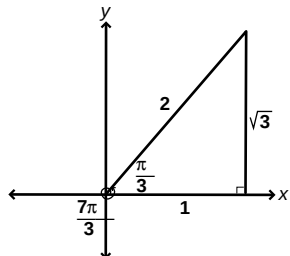
$$\sin \theta = \frac{y}{r}, \sin (240^\circ) = \frac{\sqrt{3}}{2}$$

(c)



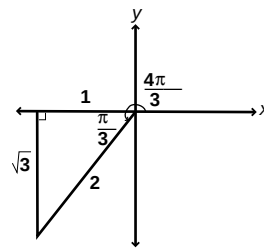
$$\cos \theta = \frac{x}{r}, \cos (-315^\circ) = \frac{1}{\sqrt{2}}$$

5. (a)



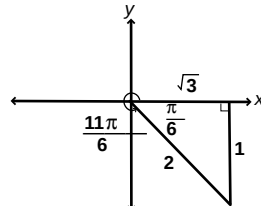
$$\sin \theta = \frac{y}{r}, \sin \left(\frac{3\pi}{4} \right) = \frac{1}{\sqrt{2}}$$

(b)



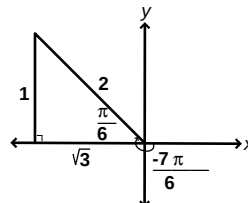
$$\tan \theta = \frac{y}{x}, \tan \left(\frac{4\pi}{3} \right) = \frac{-\sqrt{3}}{-1} = \sqrt{3}$$

(c)



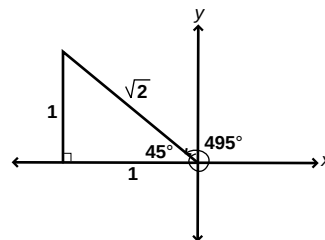
$$\cos \theta = \frac{x}{r}, \cos \left(\frac{11\pi}{6} \right) = \frac{\sqrt{3}}{2}$$

(d)



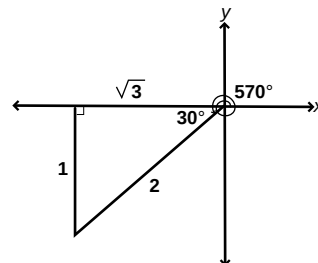
$$\tan \theta = \frac{y}{x}, \tan \left(\frac{-7\pi}{6} \right) = \frac{1}{-\sqrt{3}}$$

6. (a)



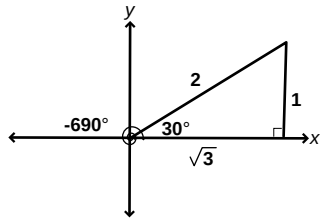
$$\sin \theta = \frac{y}{r}, \sin (495^\circ) = \frac{1}{\sqrt{2}}$$

(b)



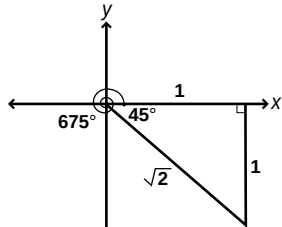
$$\cos \theta = \frac{x}{r}, \cos (570^\circ) = \frac{-\sqrt{3}}{2}$$

(c)



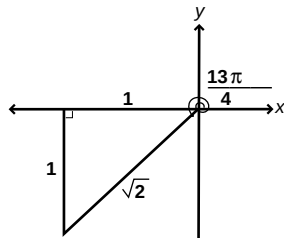
$$\tan \theta = \frac{y}{x}, \tan (-690^\circ) = \frac{1}{\sqrt{3}}$$

(d)



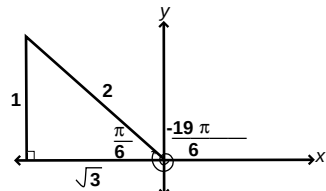
$$\sin \theta = \frac{y}{r}, \sin (675^\circ) = \frac{-1}{\sqrt{2}}$$

7. (a)



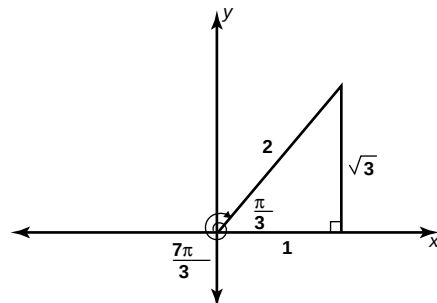
$$\cos \theta = \frac{x}{r}, \cos \left(\frac{13\pi}{4} \right) = \frac{-1}{\sqrt{2}}$$

(b)



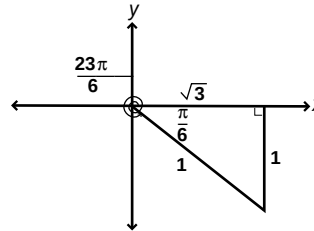
$$\cos \theta = \frac{x}{r}, \cos \left(\frac{-19\pi}{6} \right) = \frac{-\sqrt{3}}{2}$$

(c)



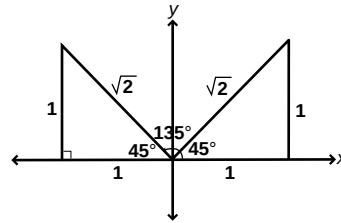
$$\tan \theta = \frac{y}{x}, \tan \left(\frac{7\pi}{3} \right) = \frac{\sqrt{3}}{1} = \sqrt{3}$$

(d)



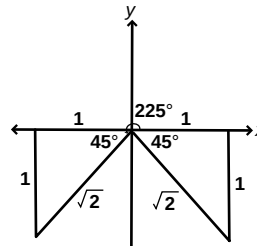
$$\cos \theta = \frac{x}{r}, \cos \left(\frac{23\pi}{6} \right) = \frac{\sqrt{3}}{2}$$

8. (a)



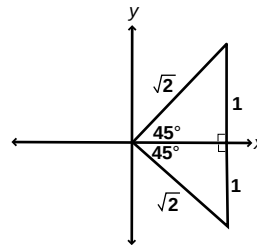
$$\sin 45^\circ = \frac{1}{\sqrt{2}}, \sin 135^\circ = \frac{1}{\sqrt{2}}, \therefore \sin 45^\circ = \sin 135^\circ$$

(b)



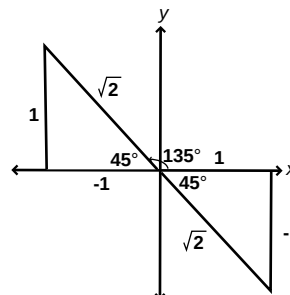
$$\sin 225^\circ = \frac{-1}{\sqrt{2}}, \sin (-45^\circ) = \frac{-1}{\sqrt{2}}, \therefore \sin 225^\circ = \sin (-45^\circ)$$

(c)



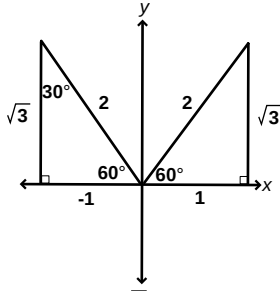
$$\cos (-45^\circ) = \frac{1}{\sqrt{2}}, \cos 45^\circ = \frac{1}{\sqrt{2}}, \therefore \cos (-45^\circ) = \cos 45^\circ$$

(d)



$$\tan 135^\circ = \frac{1}{-1} = -1, \tan (-45^\circ) = \frac{-1}{1} = -1, \therefore \tan 135^\circ = \tan (-45^\circ)$$

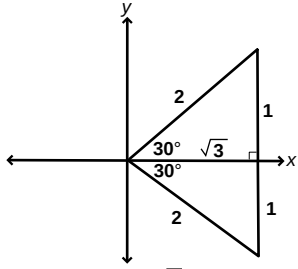
9. (a)



$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \sin(-240^\circ) = \frac{\sqrt{3}}{2},$$

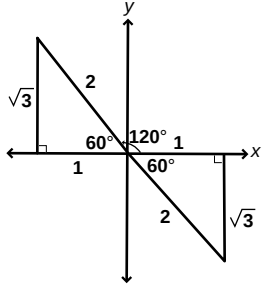
$$\therefore \sin 60^\circ = \sin(-240^\circ)$$

(b)



$$\cos(-30^\circ) = \frac{\sqrt{3}}{2}, \cos 30^\circ = \frac{\sqrt{3}}{2}, \therefore \cos(-30^\circ) = \cos 30^\circ$$

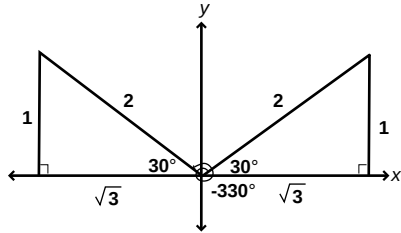
(c)



$$\tan 120^\circ = \frac{-\sqrt{3}}{1}, \tan(-60^\circ) = \frac{-\sqrt{3}}{1},$$

$$\therefore \tan 120^\circ = \tan(-60^\circ)$$

(d)



$$\sin 150^\circ = \frac{1}{2}, \sin(-330^\circ) = \frac{1}{2}, \therefore \sin 150^\circ = \sin(-330^\circ)$$

10. (a) Show $\sin^2(45^\circ) + \cos^2(45^\circ) = 1$.

$$\text{LS} = \sin^2(45^\circ) + \cos^2(45^\circ) \quad \text{RS} = 1$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

$$\therefore \text{LS} = \text{RS}, \therefore \sin^2(45^\circ) + \cos^2(45^\circ) = 1.$$

(b) Show $\sin^2\left(\frac{\pi}{3}\right) + \cos^2\left(\frac{\pi}{3}\right) = 1$.

$$\text{LS} = \sin^2\left(\frac{\pi}{3}\right) + \cos^2\left(\frac{\pi}{3}\right) \quad \text{RS} = 1$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= 1$$

$$\therefore \text{LS} = \text{RS}, \therefore \sin^2\left(\frac{\pi}{3}\right) + \cos^2\left(\frac{\pi}{3}\right) = 1.$$

(c) Show $\sin^2(60^\circ) + \cos^2(60^\circ) = 1$.

$$\text{LS} = \sin^2(60^\circ) + \cos^2(60^\circ) \quad \text{RS} = 1$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= 1$$

$$\therefore \text{LS} = \text{RS}, \therefore \sin^2(60^\circ) + \cos^2(60^\circ) = 1.$$

(d) Show $\sin^2\left(\frac{7\pi}{4}\right) + \cos^2\left(\frac{7\pi}{4}\right) = 1$.

$$\text{LS} = \sin^2\left(\frac{7\pi}{4}\right) + \cos^2\left(\frac{7\pi}{4}\right) \quad \text{RS} = 1$$

$$= \left(\frac{-1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

$$\therefore \text{LS} = \text{RS}, \therefore \sin^2\left(\frac{7\pi}{4}\right) + \cos^2\left(\frac{7\pi}{4}\right) = 1.$$

11. (a) Show that $\tan 120^\circ = \frac{\sin 120^\circ}{\cos 120^\circ}$.

$$\text{LS} = \tan 120^\circ \quad \text{RS} = \frac{\sin 120^\circ}{\cos 120^\circ}$$

$$= -\sqrt{3}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}}$$

$$= \frac{\sqrt{3}}{2} \times \frac{2}{-1}$$

$$= -\sqrt{3}$$

$$\therefore \text{LS} = \text{RS}, \therefore \tan 120^\circ = \frac{\sin 120^\circ}{\cos 120^\circ}.$$

(b) Show that $\tan(-150^\circ) = \frac{\sin(-150^\circ)}{\cos(-150^\circ)}$.

$$\text{LS} = \tan(-150^\circ) \quad \text{RS} = \frac{\sin(-150^\circ)}{\cos(-150^\circ)}$$

$$= \frac{1}{\sqrt{3}} \quad = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$$

$$= \frac{-1}{2} \times \frac{2}{-\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}$$

$$\therefore \text{LS} = \text{RS}, \therefore \tan(-150^\circ) = \frac{\sin(-150^\circ)}{\cos(-150^\circ)}.$$

(c) Show that $\tan\left(\frac{11\pi}{3}\right) = \frac{\sin\left(\frac{11\pi}{3}\right)}{\cos\left(\frac{11\pi}{3}\right)}$.

$$\text{LS} = \tan\left(\frac{11\pi}{3}\right) \quad \text{RS} = \frac{\sin\left(\frac{11\pi}{3}\right)}{\cos\left(\frac{11\pi}{3}\right)}$$

$$= -\sqrt{3} \quad = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \frac{-\sqrt{3}}{2} \times \frac{2}{1}$$

$$= -\sqrt{3}$$

$$\therefore \text{LS} = \text{RS}, \therefore \tan\left(\frac{11\pi}{3}\right) = \frac{\sin\left(\frac{11\pi}{3}\right)}{\cos\left(\frac{11\pi}{3}\right)}.$$

(d) Show that $\tan\left(\frac{13\pi}{4}\right) = \frac{\sin\left(\frac{13\pi}{4}\right)}{\cos\left(\frac{13\pi}{4}\right)}$.

$$\text{LS} = \tan\left(\frac{13\pi}{4}\right) \quad \text{RS} = \frac{\sin\left(\frac{13\pi}{4}\right)}{\cos\left(\frac{13\pi}{4}\right)}$$

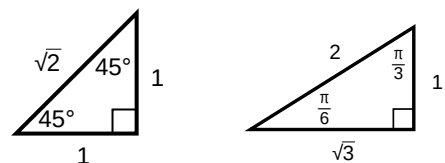
$$= 1 \quad = \frac{\frac{-1}{\sqrt{2}}}{\frac{-1}{\sqrt{2}}}$$

$$= \frac{-1}{\sqrt{2}} \times \frac{\sqrt{2}}{-1}$$

$$= 1$$

$$\therefore \text{LS} = \text{RS}, \therefore \tan\left(\frac{13\pi}{4}\right) = \frac{\sin\left(\frac{13\pi}{4}\right)}{\cos\left(\frac{13\pi}{4}\right)}.$$

12.



(a) $\tan \theta = -1, \therefore \theta = 135^\circ, 315^\circ$

(b) $\sin \theta = \frac{1}{\sqrt{2}}, \therefore \theta = 45^\circ, 135^\circ$

(c) $\cos \theta = \frac{-\sqrt{3}}{2}, \therefore \theta = 150^\circ, 210^\circ$

(d) $\sin \theta = \frac{\sqrt{3}}{2}, \therefore \theta = 60^\circ, 120^\circ$

Part 3: Trigonometric Functions

(e) $\cos \theta = \frac{1}{\sqrt{2}}, \therefore \theta = 45^\circ, 315^\circ$

(f) $\tan \theta = -\sqrt{3}, \therefore \theta = 120^\circ, 300^\circ$

(g) $\sin \theta = \frac{1}{2}, \therefore \theta = 30^\circ, 150^\circ$

(h) $\tan \theta = \frac{-1}{\sqrt{3}}, \therefore \theta = 150^\circ, 330^\circ$

(i) $\sin 2x = \frac{1}{\sqrt{2}}$

$$2x = 45^\circ$$

$$x = 22.5^\circ$$

$$2x = 135^\circ$$

$$x = 67.5^\circ$$

$$\therefore x = 22.5^\circ, 67.5^\circ, 202.5^\circ, 247.5^\circ$$

(j) $\cos 2x = \frac{1}{2}$

$$2x = 60^\circ$$

$$x = 30^\circ$$

$$2x = 300^\circ$$

$$x = 150^\circ$$

$$\therefore x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

(k) $5 \cos x - \sqrt{3} = 3 \cos x$

$$2 \cos x = \sqrt{3}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$\therefore x = 30^\circ, 330^\circ$$

(l) $\sin\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

$$\frac{x}{2} = 60^\circ$$

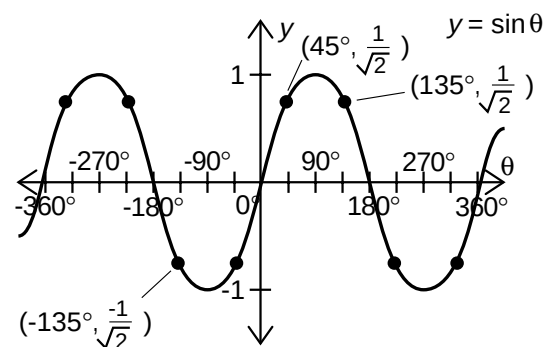
$$x = 120^\circ$$

$$\frac{x}{2} = 120^\circ$$

$$x = 240^\circ$$

$$\therefore x = 120^\circ, 240^\circ$$

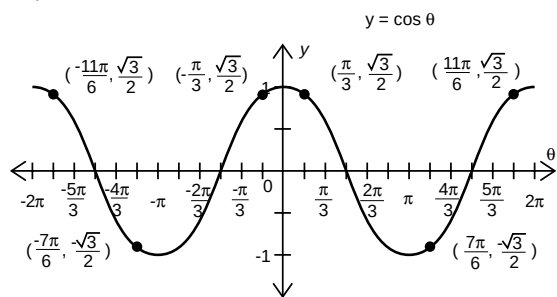
13.



θ	$y = \sin \theta$
-315°	$\frac{1}{\sqrt{2}}$
-225°	$\frac{1}{\sqrt{2}}$
-135°	$\frac{-1}{\sqrt{2}}$
-45°	$\frac{-1}{\sqrt{2}}$
45°	$\frac{1}{\sqrt{2}}$
135°	$\frac{1}{\sqrt{2}}$
225°	$\frac{-1}{\sqrt{2}}$
315°	$\frac{-1}{\sqrt{2}}$

$y = \sin \theta$ is a function because every element of the domain corresponds to only one element in the range. It passes the vertical line test.

14.



θ	$y = \cos \theta$
$\frac{-11\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{-7\pi}{6}$	$\frac{-\sqrt{3}}{2}$
$\frac{-5\pi}{6}$	$\frac{-\sqrt{3}}{2}$
$\frac{-\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{5\pi}{6}$	$\frac{-\sqrt{3}}{2}$
$\frac{7\pi}{6}$	$\frac{-\sqrt{3}}{2}$
$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$

$y = \cos \theta$ is a function because every element of the domain corresponds to only one element in the range. It passes the vertical line test.

15. (a) $AB = 6 \times \sqrt{3} = 6\sqrt{3}$ units, $AD = 6 \times 2 = 12$ units,
 $BC = 6 \times 1 = 6$ units, $CD = 6 \times \sqrt{2} = 6\sqrt{2}$ units,
 $AC = 6\sqrt{3} + 6$ units

(b) $\angle A = 30^\circ$, $\tan \angle A = \frac{1}{\sqrt{3}}$, $\cos \angle A = \frac{\sqrt{3}}{2}$, $\sin \angle A = \frac{1}{2}$,
 $\angle C = 45^\circ$, $\tan \angle C = 1$, $\cos \angle C = \frac{1}{\sqrt{2}}$, $\sin \angle C = \frac{1}{\sqrt{2}}$

Practise, Apply, Solve 6.5, page 538

1. (a)

$$\begin{aligned} \text{LS} &= \frac{\cos \theta}{\sin \theta} & \text{RS} &= \frac{1}{\tan \theta} \\ &= \frac{\frac{x}{r}}{\frac{y}{r}} & &= \frac{1}{\frac{y}{x}} \\ &= \frac{x}{r} \times \frac{r}{y} & &= 1 \times \frac{x}{y} \\ &= \frac{x}{y} & &= \frac{x}{y} \end{aligned}$$

$$\therefore \text{LS} = \text{RS}, \therefore \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}.$$

(b)

$$\begin{aligned} \text{LS} &= 1 + \tan^2 \theta & \text{RS} &= \frac{1}{\cos^2 \theta} \\ &= 1 + \left(\frac{y}{x}\right)^2 & &= \frac{1}{\left(\frac{x}{r}\right)^2} \\ &= \frac{x^2}{x^2} + \frac{y^2}{x^2} & &= 1 \times \left(\frac{r}{x}\right)^2 \\ &= \frac{x^2 + y^2}{x^2} & &= \frac{r^2}{x^2} \\ &= \frac{r^2}{x^2} \end{aligned}$$

$$\therefore \text{LS} = \text{RS} \therefore 1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}$$

(c)

$$\begin{aligned} \text{LS} &= 1 - \sin^2 \theta & \text{RS} &= \cos^2 \theta \\ &= 1 - \left(\frac{y}{r}\right)^2 & &= \left(\frac{x}{r}\right)^2 \\ &= \frac{r^2}{r^2} - \frac{y^2}{r^2} & &= \frac{x^2}{r^2} \\ &= \frac{r^2 - y^2}{r^2} \\ &= \frac{x^2}{r^2} \end{aligned}$$

$$\therefore \text{LS} = \text{RS}, \therefore 1 - \sin^2 \theta = \cos^2 \theta$$

2. (a) $\sin x \left(\frac{1}{\cos x} \right) = \frac{\sin x}{\cos x} = \tan x$

(b) $(\cos x)(\tan x) = (\cos x) \left(\frac{\sin x}{\cos x} \right) = \sin x$