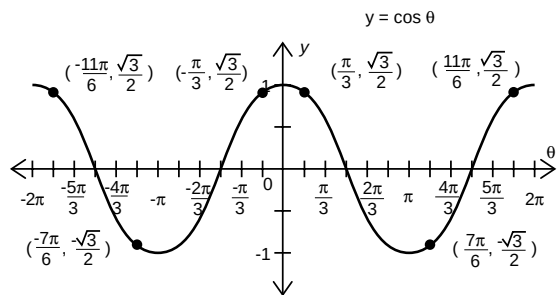


θ	$y = \sin \theta$
-315°	$\frac{1}{\sqrt{2}}$
-225°	$\frac{1}{\sqrt{2}}$
-135°	$-\frac{1}{\sqrt{2}}$
-45°	$-\frac{1}{\sqrt{2}}$
45°	$\frac{1}{\sqrt{2}}$
135°	$\frac{1}{\sqrt{2}}$
225°	$-\frac{1}{\sqrt{2}}$
315°	$-\frac{1}{\sqrt{2}}$

$y = \sin \theta$ is a function because every element of the domain corresponds to only one element in the range. It passes the vertical line test.

14.



θ	$y = \cos \theta$
$-\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$
$-\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$
$-\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$
$-\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$
$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$
$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$

$y = \cos \theta$ is a function because every element of the domain corresponds to only one element in the range. It passes the vertical line test.

15. (a) $AB = 6 \times \sqrt{3} = 6\sqrt{3}$ units, $AD = 6 \times 2 = 12$ units,
 $BC = 6 \times 1 = 6$ units, $CD = 6 \times \sqrt{2} = 6\sqrt{2}$ units,
 $AC = 6\sqrt{3} + 6$ units

(b) $\angle A = 30^\circ$, $\tan \angle A = \frac{1}{\sqrt{3}}$, $\cos \angle A = \frac{\sqrt{3}}{2}$, $\sin \angle A = \frac{1}{2}$,
 $\angle C = 45^\circ$, $\tan \angle C = 1$, $\cos \angle C = \frac{1}{\sqrt{2}}$, $\sin \angle C = \frac{1}{\sqrt{2}}$

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1. (a)

$$\begin{aligned} \text{LS} &= \frac{\cos \theta}{\sin \theta} & \text{RS} &= \frac{1}{\tan \theta} \\ &= \frac{\frac{x}{r}}{\frac{y}{r}} & &= \frac{1}{\frac{y}{x}} \\ &= \frac{x}{r} \times \frac{r}{y} & &= 1 \times \frac{x}{y} \\ &= \frac{x}{y} & &= \frac{x}{y} \end{aligned}$$

$$\therefore \text{LS} = \text{RS}, \therefore \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}.$$

(b)

$$\begin{aligned} \text{LS} &= 1 + \tan^2 \theta & \text{RS} &= \frac{1}{\cos^2 \theta} \\ &= 1 + \left(\frac{y}{x}\right)^2 & &= \frac{1}{\left(\frac{x}{r}\right)^2} \\ &= \frac{x^2}{x^2} + \frac{y^2}{x^2} & &= 1 \times \left(\frac{r}{x}\right)^2 \\ &= \frac{x^2 + y^2}{x^2} & &= \frac{r^2}{x^2} \\ &= \frac{r^2}{x^2} \end{aligned}$$

$$\therefore \text{LS} = \text{RS} \therefore 1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}$$

(c)

$$\begin{aligned} \text{LS} &= 1 - \sin^2 \theta & \text{RS} &= \cos^2 \theta \\ &= 1 - \left(\frac{y}{r}\right)^2 & &= \left(\frac{x}{r}\right)^2 \\ &= \frac{r^2}{r^2} - \frac{y^2}{r^2} & &= \frac{x^2}{r^2} \\ &= \frac{r^2 - y^2}{r^2} \\ &= \frac{x^2}{r^2} \end{aligned}$$

$$\therefore \text{LS} = \text{RS}, \therefore 1 - \sin^2 \theta = \cos^2 \theta$$

2. (a) $\sin x \left(\frac{1}{\cos x} \right) = \frac{\sin x}{\cos x} = \tan x$

(b) $(\cos x)(\tan x) = (\cos x) \left(\frac{\sin x}{\cos x} \right) = \sin x$

$$\begin{aligned} \text{(c)} \quad 1 - \cos^2 x &= \sin^2 x \\ \text{(d)} \quad 1 - \sin^2 x &= \cos^2 x \\ \text{(e)} \quad \cos^2 x + \sin^2 x &= 1 \\ \text{(f)} \quad (1 - \sin x)(1 + \sin x) &= 1 - \sin^2 x = \cos^2 x \\ \text{(g)} \end{aligned}$$

$$\begin{aligned} \frac{\tan x}{\sin x} &= \frac{\frac{\sin x}{\cos x}}{\sin x} \\ &= \frac{\sin x}{\cos x} \times \frac{1}{\sin x} \\ &= \frac{1}{\cos x} \end{aligned}$$

$$\begin{aligned} \text{(h)} \quad \frac{\frac{\sin x}{\cos x}}{\tan x} &= \frac{\sin x}{\cos x} \times \frac{1}{\tan x} \\ &= \frac{\sin x}{\cos x} \times \frac{1}{\frac{\sin x}{\cos x}} \\ &= \frac{\sin x}{\cos x} \times \frac{\cos x}{\sin x} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{(i)} \quad \left(\frac{1}{\tan x} \right) \sin x &= \left(\frac{1}{\frac{\sin x}{\cos x}} \right) \sin x \\ &= \left(\frac{\cos x}{\sin x} \right) \sin x \\ &= \cos x \end{aligned}$$

$$\begin{aligned} \text{(j)} \quad \frac{1 + \tan^2 x}{\tan^2 x} &= \frac{1 + \frac{\sin^2 x}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}} \\ &= \left(\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} \right) \times \left(\frac{\cos^2 x}{\sin^2 x} \right) \\ &= \left(\frac{1}{\cos^2 x} \right) \left(\frac{\cos^2 x}{\sin^2 x} \right) \\ &= \frac{1}{\sin^2 x} \end{aligned}$$

$$\begin{aligned} \text{(k)} \quad \frac{\sin x \cos x}{1 - \sin^2 x} &= \frac{\sin x \cos x}{\cos^2 x} \\ &= \frac{\sin x}{\cos x} \\ &= \tan x \end{aligned}$$

$$\begin{aligned} \text{(l)} \quad \frac{1 - \cos^2 x}{\sin x \cos x} &= \frac{\sin^2 x}{\sin x \cos x} \\ &= \frac{\sin x}{\cos x} \\ &= \tan x \end{aligned}$$

$$\text{(m)} \quad \frac{1}{\sin x} + \frac{1}{\cos x} = \frac{\cos x + \sin x}{\sin x \cos x}$$

$$\begin{aligned} \text{(n)} \quad \tan x + \frac{1}{\cos x} &= \frac{\sin x}{\cos x} + \frac{1}{\cos x} \\ &= \frac{\sin x + 1}{\cos x} \end{aligned}$$

$$\begin{aligned} \text{(o)} \quad \frac{1}{\tan x} + \sin x &= \frac{1}{\frac{\sin x}{\cos x}} + \sin x \\ &= \frac{\cos x}{\sin x} + \sin x \\ &= \frac{\cos x}{\sin x} + \frac{\sin^2 x}{\sin x} \\ &= \frac{\cos x + \sin^2 x}{\sin x} \end{aligned}$$

$$\begin{aligned} 3. \text{ (a)} \quad 1 - \cos^2 \theta &= (1 - \cos \theta)(1 + \cos \theta) \\ \text{(b)} \quad 1 - \sin^2 \theta &= (1 - \sin \theta)(1 + \sin \theta) \\ \text{(c)} \quad \sin^2 \theta - \cos^2 \theta &= (\sin \theta - \cos \theta)(\sin \theta + \cos \theta) \\ \text{(d)} \quad \sin \theta - \sin^2 \theta &= \sin \theta(1 - \sin \theta) \\ \text{(e)} \quad \cos^2 \theta + 2 \cos \theta + 1 &= (\cos \theta + 1)(\cos \theta + 1) \\ \text{(f)} \quad \sin^2 \theta - 2 \sin \theta + 1 &= (\sin \theta - 1)(\sin \theta - 1) \end{aligned}$$

$$\begin{aligned} 4. \text{ (a)} \quad \frac{\cos \alpha}{\tan \alpha} &= \frac{\cos \alpha}{\frac{\sin \alpha}{\cos \alpha}} \\ &= \cos \alpha \left(\frac{\cos \alpha}{\sin \alpha} \right) \\ &= \frac{\cos^2 \alpha}{\sin \alpha} \\ &= \frac{1 - \sin^2 \alpha}{\sin \alpha} \\ &= \frac{1}{\sin \alpha} - \sin \alpha \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \text{LS} &= \frac{\cos \alpha}{\tan \alpha} & \text{RS} &= \frac{1}{\sin \alpha} - \sin \alpha \\ &= \frac{\cos \alpha}{\frac{\sin \alpha}{\cos \alpha}} & &= \frac{1}{\sin \alpha} - \frac{\sin^2 \alpha}{\sin \alpha} \\ &= \cos \alpha \left(\frac{\cos \alpha}{\sin \alpha} \right) & &= \frac{1 - \sin^2 \alpha}{\sin \alpha} \\ &= \frac{\cos^2 \alpha}{\sin \alpha} & &= \frac{\cos^2 \alpha}{\sin \alpha} \\ \therefore \text{LS} &= \text{RS}, \therefore \frac{\cos \alpha}{\tan \alpha} = \frac{1}{\sin \alpha} - \sin \alpha. \end{aligned}$$

$$\begin{aligned} 5. \text{ (a)} \quad \frac{\cos^2 x}{1 - \sin x} &= \frac{1 - \sin^2 x}{1 - \sin x} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{1 - \sin^2 x}{1 - \sin x} &= \frac{(1 - \sin x)(1 + \sin x)}{1 - \sin x} \end{aligned}$$

(c)

$$\begin{aligned}
\text{LS} &= \frac{\cos^2 x}{1 - \sin x} & \text{RS} &= 1 + \sin x \\
&= \frac{1 - \sin^2 x}{1 - \sin x} \\
&= \frac{(1 - \sin x)(1 + \sin x)}{1 - \sin x} \\
&= 1 + \sin x \\
\therefore \text{LS} &= \text{RS}, \therefore \frac{\cos^2 x}{1 - \sin x} = 1 + \sin x
\end{aligned}$$

6. (a)

$$\begin{aligned}
\text{LS} &= \frac{\sin x}{\tan x} & \text{RS} &= \cos x \\
&= \frac{\sin x}{\frac{\sin x}{\cos x}} \\
&= \sin x \left(\frac{\cos x}{\sin x} \right) \\
&= \cos x \\
\therefore \text{LS} &= \text{RS}, \therefore \frac{\sin x}{\tan x} = \cos x
\end{aligned}$$

(b)

$$\begin{aligned}
\text{LS} &= \frac{\tan \theta}{\cos \theta} & \text{RS} &= \frac{\sin \theta}{1 - \sin^2 \theta} \\
&= \frac{\frac{\sin \theta}{\cos \theta}}{\cos \theta} & &= \frac{\sin \theta}{\cos^2 \theta} \\
&= \left(\frac{\sin \theta}{\cos \theta} \right) \left(\frac{1}{\cos \theta} \right) \\
&= \frac{\sin \theta}{\cos^2 \theta} \\
\therefore \text{LS} &= \text{RS}, \therefore \frac{\tan \theta}{\cos \theta} = \frac{\sin \theta}{1 - \sin^2 \theta}.
\end{aligned}$$

(c)

$$\begin{aligned}
\text{LS} &= \frac{1}{\cos \alpha} + \tan \alpha & \text{RS} &= \frac{1 + \sin \alpha}{\cos \alpha} \\
&= \frac{1}{\cos \alpha} + \frac{\sin \alpha}{\cos \alpha} \\
&= \frac{1 + \sin \alpha}{\cos \alpha} \\
\therefore \text{LS} &= \text{RS}, \therefore \frac{1}{\cos \alpha} + \tan \alpha = \frac{1 + \sin \alpha}{\cos \alpha}.
\end{aligned}$$

(d)

$$\begin{aligned}
\text{LS} &= 1 - \cos^2 \theta & \text{RS} &= \tan \theta \cos \theta \sin \theta \\
&= \sin^2 \theta & &= \left(\frac{\sin \theta}{\cos \theta} \right) (\cos \theta \sin \theta) \\
& & &= \sin^2 \theta \\
\therefore \text{LS} &= \text{RS}, \therefore 1 - \cos^2 \theta = \tan \theta \cos \theta \sin \theta.
\end{aligned}$$

7. (a)

$$\begin{aligned}
\text{LS} &= \frac{\sin^2 x}{1 - \cos x} & \text{RS} &= 1 + \cos x \\
&= \frac{1 - \cos^2 x}{1 - \cos x} \\
&= \frac{(1 - \cos x)(1 + \cos x)}{1 - \cos x} \\
&= 1 + \cos x \\
\therefore \text{LS} &= \text{RS}, \therefore \frac{\sin^2 x}{1 - \cos x} = 1 + \cos x.
\end{aligned}$$

(b)

$$\begin{aligned}
\text{LS} &= \frac{\tan^2 \alpha}{1 + \tan^2 \alpha} & \text{RS} &= \sin^2 \alpha \\
&= \frac{\frac{\sin^2 \alpha}{\cos^2 \alpha}}{1 + \frac{\sin^2 \alpha}{\cos^2 \alpha}} \\
&= \frac{\frac{\sin^2 \alpha}{\cos^2 \alpha}}{\frac{\cos^2 \alpha + \sin^2 \alpha}{\cos^2 \alpha}} \\
&= \frac{\sin^2 \alpha}{\cos^2 \alpha} \cdot \frac{\cos^2 \alpha}{1} \\
&= \sin^2 \alpha \\
\therefore \text{LS} &= \text{RS}, \therefore \frac{\tan^2 \alpha}{1 + \tan^2 \alpha} = \sin^2 \alpha.
\end{aligned}$$

(c)

$$\begin{aligned}
\text{LS} &= \cos^2 x & \text{RS} &= (1 - \sin x)(1 + \sin x) \\
& & &= 1 - \sin^2 x \\
& & &= \cos^2 x \\
\therefore \text{LS} &= \text{RS}, \therefore \cos^2 x = (1 - \sin x)(1 + \sin x).
\end{aligned}$$

(d)

$$\begin{aligned}
\text{LS} &= \sin^2 \theta + 2 \cos^2 \theta - 1 & \text{RS} &= \cos^2 \theta \\
&= (1 - \cos^2 \theta) + 2 \cos^2 \theta - 1 \\
&= 1 - \cos^2 \theta + 2 \cos^2 \theta - 1 \\
&= \cos^2 \theta \\
\therefore \text{LS} &= \text{RS}, \therefore \sin^2 \theta + 2 \cos^2 \theta - 1 = \cos^2 \theta.
\end{aligned}$$

8. (a)

$$\begin{aligned}
\text{LS} &= (\sin x + \cos x)^2 & \text{RS} &= 1 + 2 \sin x \cos x \\
&= \sin^2 x + 2 \sin x \cos x + \cos^2 x \\
&= (\sin^2 x + \cos^2 x) + 2 \sin x \cos x \\
&= 1 + 2 \sin x \cos x \\
\therefore \text{LS} &= \text{RS}, \therefore (\sin x + \cos x)^2 = 1 + 2 \sin x \cos x
\end{aligned}$$

(b)

$$\begin{aligned}
 \text{LS} &= \frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} \\
 &= \frac{\cos^2 \theta}{\sin^2 \theta \cos^2 \theta} + \frac{\sin^2 \theta}{\sin^2 \theta \cos^2 \theta} \\
 &= \frac{\cos^2 \theta + \sin^2 \theta}{\sin^2 \theta \cos^2 \theta} \\
 &= \frac{1}{\sin^2 \theta \cos^2 \theta} \\
 \therefore \text{LS} &= \text{RS}, \therefore \frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} = \frac{1}{\sin^2 \theta - \sin^4 \theta}.
 \end{aligned}$$

(c)

$$\begin{aligned}
 \text{LS} &= \cos x + \sin x \tan x \\
 &= \cos x + \sin x \left(\frac{\sin x}{\cos x} \right) \\
 &= \cos x + \frac{\sin^2 x}{\cos x} \\
 &= \frac{\cos^2 x + \sin^2 x}{\cos x} \\
 &= \frac{1}{\cos x} \\
 \therefore \text{LS} &= \text{RS}, \therefore \cos x + \sin x \tan x = \frac{1}{\cos x}.
 \end{aligned}$$

(d)

$$\begin{aligned}
 \text{LS} &= \frac{\sin^2 x}{\cos^2 x} \times \frac{1}{\tan x} \\
 &= \tan^2 x \times \frac{1}{\tan x} \\
 &= \tan x \\
 \therefore \text{LS} &= \text{RS}, \therefore \frac{\sin^2 x}{\cos^2 x} \times \frac{1}{\tan x} = \tan x.
 \end{aligned}$$

9. (a)

$$\begin{aligned}
 \text{LS} &= \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin \theta \cos \theta} \\
 &= \frac{(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)}{\cos \theta (\cos \theta + \sin \theta)} \\
 &= \frac{\cos \theta - \sin \theta}{\cos \theta} \\
 \therefore \text{LS} &= \text{RS}, \therefore \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin \theta \cos \theta} = 1 - \tan \theta.
 \end{aligned}$$

$$\text{RS} = 1 - \tan \theta$$

$$= 1 - \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\cos \theta - \sin \theta}{\cos \theta}$$

(b)

$$\begin{aligned}
 \text{LS} &= \tan^2 x - \sin^2 x \\
 &= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x \\
 &= \frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x \cos^2 x}{\cos^2 x} \\
 &= \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x} \\
 &= \frac{\sin^2 x (\sin^2 x)}{\cos^2 x} \\
 &= \frac{\sin^4 x}{\cos^2 x} \\
 \therefore \text{LS} &= \text{RS}, \therefore \tan^2 x - \sin^2 x = \sin^2 x \tan^2 x.
 \end{aligned}$$

$$\text{RS} = \sin^2 x \tan^2 x$$

$$= \sin^2 x \left(\frac{\sin^2 x}{\cos^2 x} \right)$$

$$= \frac{\sin^4 x}{\cos^2 x}$$

(c)

$$\begin{aligned}
 \text{LS} &= \tan^2 x - \cos^2 x \\
 &= \frac{\sin^2 x}{\cos^2 x} - \cos^2 x \\
 &= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^4 x}{\cos^2 x} \\
 &= \frac{\sin^2 x - \cos^4 x}{\cos^2 x} \\
 \therefore \text{LS} &= \text{RS}, \therefore \tan^2 x - \cos^2 x = \frac{1}{\cos^2 x} - 1 - \cos^2 x.
 \end{aligned}$$

$$\text{RS} = \frac{1}{\cos^2 x} - 1 - \cos^2 x$$

$$= \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} - \frac{\cos^4 x}{\cos^2 x}$$

$$= \frac{(1 - \cos^2 x) - \cos^4 x}{\cos^2 x}$$

$$= \frac{\sin^2 x - \cos^4 x}{\cos^2 x}$$

(d)

$$\begin{aligned}
 \text{LS} &= \frac{1}{1 + \cos \theta} + \frac{1}{1 - \cos \theta} \\
 &= \frac{1 - \cos \theta + 1 + \cos \theta}{(1 + \cos \theta)(1 - \cos \theta)} \\
 &= \frac{2}{1 - \cos^2 \theta} \\
 &= \frac{2}{\sin^2 \theta} \\
 \therefore \text{LS} &= \text{RS}, \therefore \frac{1}{1 + \cos \theta} + \frac{1}{1 - \cos \theta} = \frac{2}{\sin^2 \theta}
 \end{aligned}$$

$$\text{RS} = \frac{2}{\sin^2 \theta}$$

10. (a)

$$\begin{aligned}
 \text{LS} &= \frac{1 + 2 \sin \beta \cos \beta}{\cos \beta + \sin \beta} \\
 &= \frac{\sin^2 \beta + \cos^2 \beta + 2 \sin \beta \cos \beta}{\cos \beta + \sin \beta} \\
 &= \frac{(\sin \beta + \cos \beta)(\sin \beta + \cos \beta)}{\cos \beta + \sin \beta} \\
 &= \sin \beta + \cos \beta \\
 \therefore \text{LS} &= \text{RS}, \therefore \text{the equation is an identity.}
 \end{aligned}$$

$$\text{RS} = \sin \beta + \cos \beta$$

(b)

$$\begin{aligned}
\text{LS} &= \frac{\tan x \sin x}{\tan x + \sin x} \\
&= \frac{\left(\frac{\sin x}{\cos x}\right) \sin x}{\left(\frac{\sin x}{\cos x}\right) + \sin x} \\
&= \frac{\frac{\sin^2 x}{\cos x}}{\frac{\sin x + \sin x \cos x}{\cos x}} \\
&= \left(\frac{\sin^2 x}{\cos x}\right) \left(\frac{\cos x}{\sin x + \sin x \cos x}\right) \\
&= \frac{\sin^2 x}{\sin x + \sin x \cos x} \\
&= \frac{\sin^2 x}{\sin x(1 + \cos x)} \\
&= \frac{\sin x}{1 + \cos x}
\end{aligned}$$

$\therefore \text{LS} \neq \text{RS}$, $\therefore$ the equation is not an identity.

(c)

$$\begin{aligned}
\text{LS} &= (1 - \cos^2 \theta)(1 - \tan^2 \theta) & \text{RS} &= \frac{\sin^2 \theta - 2 \sin^4 \theta}{1 - \sin^2 \theta} \\
&= \sin^2 \theta (1 - \tan^2 \theta) \\
&= \sin^2 \theta - \sin^2 \theta \tan^2 \theta \\
&= \sin^2 \theta - \sin^2 \theta \left(\frac{\sin^2 \theta}{\cos^2 \theta}\right) \\
&= \sin^2 \theta - \frac{\sin^4 \theta}{\cos^2 \theta} \\
&= \frac{\sin^2 \theta \cos^2 \theta - \sin^4 \theta}{1 - \sin^2 \theta} \\
&= \frac{\sin^2 \theta (1 - \sin^2 \theta) - \sin^4 \theta}{1 - \sin^2 \theta} \\
&= \frac{\sin^2 \theta - 2 \sin^4 \theta}{1 - \sin^2 \theta}
\end{aligned}$$

$\therefore \text{LS} = \text{RS}$, $\therefore$ the equation is an identity.

(d)

$$\begin{aligned}
\text{LS} &= 1 - 2 \cos^2 x & \text{RS} &= \sin^4 x - \cos^4 x \\
& & &= (1 - \cos^2 x)^2 - \cos^4 x \\
& & &= 1 - 2 \cos^2 x + \cos^4 x - \cos^4 x \\
& & &= 1 - 2 \cos^2 x
\end{aligned}$$

$\therefore \text{LS} = \text{RS}$, $\therefore$ the equation is an identity.

11. (a)

$$\begin{aligned}
\text{LS} &= \cos x \tan^3 x & \text{RS} &= \sin x \tan^2 x \\
&= \cos x \left(\frac{\sin^3 x}{\cos^3 x}\right) & &= \sin x \left(\frac{\sin^2 x}{\cos^2 x}\right) \\
&= \frac{\sin^3 x}{\cos^2 x} & &= \frac{\sin^3 x}{\cos^2 x}
\end{aligned}$$

$\therefore \text{LS} = \text{RS}$, $\therefore \cos x \tan^3 x = \sin x \tan^2 x$.

(b)

$$\begin{aligned}
\text{LS} &= \sin^2 \theta + \cos^4 \theta & \text{RS} &= \cos^2 \theta + \sin^4 \theta \\
&= \sin^2 \theta + (1 - \sin^2 \theta)^2 & &= 1 - \sin^2 \theta + \sin^4 \theta \\
&= \sin^2 \theta + 1 - 2 \sin^2 \theta + \sin^4 \theta \\
&= 1 - \sin^2 \theta + \sin^4 \theta \\
\therefore \text{LS} &= \text{RS}, \therefore \sin^2 \theta + \cos^4 \theta = \cos^2 \theta + \sin^4 \theta.
\end{aligned}$$

(c)

$$\begin{aligned}
\text{LS} &= (\sin x + \cos x) \left(\frac{\tan^2 x + 1}{\tan x}\right) & \text{RS} &= \frac{1}{\cos x} + \frac{1}{\sin x} \\
&= (\sin x + \cos x) \left(\frac{\frac{\sin^2 x}{\cos^2 x} + 1}{\frac{\sin x}{\cos x}}\right) & &= \frac{\sin x + \cos x}{\cos x \sin x} \\
&= (\sin x + \cos x) \left(\frac{\cos x}{\sin x}\right) \left(\frac{\sin^2 x}{\cos^2 x} + 1\right) \\
&= (\sin x + \cos x) \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) \\
&= (\sin x + \cos x) \left(\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right) \\
&= (\sin x + \cos x) \left(\frac{1}{\cos x \sin x}\right) \\
&= \frac{\sin x + \cos x}{\cos x \sin x}
\end{aligned}$$

$$\therefore \text{LS} = \text{RS}, \therefore (\sin x + \cos x) \left(\frac{\tan^2 x + 1}{\tan x}\right) = \frac{1}{\cos x} + \frac{1}{\sin x}.$$

(d)

$$\begin{aligned}
\text{LS} &= \tan^2 \beta + \cos^2 \beta + \sin^2 \beta & \text{RS} &= \frac{1}{\cos^2 \beta} \\
&= \tan^2 \beta + 1 \\
&= \frac{\sin^2 \beta}{\cos^2 \beta} + 1 \\
&= \frac{\sin^2 \beta}{\cos^2 \beta} + \frac{\cos^2 \beta}{\cos^2 \beta} \\
&= \frac{\sin^2 \beta + \cos^2 \beta}{\cos^2 \beta} \\
&= \frac{1}{\cos^2 \beta}
\end{aligned}$$

$$\therefore \text{LS} = \text{RS}, \therefore \tan^2 \beta + \cos^2 \beta + \sin^2 \beta = \frac{1}{\cos^2 \beta}.$$

12.

$$\begin{aligned}
 \text{LS} &= \frac{\frac{1-\cos x}{\cos x}}{\tan x} & \text{RS} &= \frac{\frac{\tan x}{1+\cos x}}{\cos x} \\
 &= \frac{\frac{1-\cos x}{\cos x}}{\frac{\sin x}{\cos x}} & &= \frac{\frac{\sin x}{\cos x}}{\frac{1+\cos x}{\cos x}} \\
 &= \left(\frac{1-\cos x}{\cos x} \right) \left(\frac{\cos x}{\sin x} \right) & &= \left(\frac{\sin x}{\cos x} \right) \left(\frac{\cos x}{1+\cos x} \right) \\
 &= \frac{1-\cos x}{\sin x} & &= \frac{\sin x}{1+\cos x} \\
 & & &= \left(\frac{\sin x}{1+\cos x} \right) \left(\frac{1-\cos x}{1-\cos x} \right) \\
 & & &= \frac{\sin x(1-\cos x)}{1-\cos^2 x} \\
 & & &= \frac{\sin x(1-\cos x)}{\sin^2 x} \\
 & & &= \frac{1-\cos x}{\sin x} \\
 \therefore \text{LS} &= \text{RS}, \therefore \frac{\frac{1-\cos x}{\cos x}}{\tan x} = \frac{\tan x}{1+\cos x}.
 \end{aligned}$$

13.

$$\begin{aligned}
 \text{LS} &= \sin^2 x \left(1 + \frac{1}{\tan^2 x} \right) & \text{RS} &= 1 \\
 &= \sin^2 x + \frac{\sin^2 x}{\tan^2 x} \\
 &= \sin^2 x + \frac{\sin^2 x}{\frac{\sin^2 x}{\cos^2 x}} \\
 &= \sin^2 x + \sin^2 x \left(\frac{\cos^2 x}{\sin^2 x} \right) \\
 &= \sin^2 x + \cos^2 x \\
 &= 1 \\
 \therefore \text{LS} &= \text{RS}, \therefore \sin^2 x \left(1 + \frac{1}{\tan^2 x} \right) = 1.
 \end{aligned}$$

14. (a)

$$\begin{aligned}
 \text{LS} &= \frac{1-\cos \beta}{\sin \beta} & \text{RS} &= \frac{\sin \beta}{1+\cos \beta} \\
 &= \left(\frac{1-\cos \beta}{\sin \beta} \right) \left(\frac{1+\cos \beta}{1+\cos \beta} \right) \\
 &= \frac{1-\cos^2 \beta}{\sin \beta(1+\cos \beta)} \\
 &= \frac{\sin^2 \beta}{\sin \beta(1+\cos \beta)} \\
 &= \frac{\sin \beta}{1+\cos \beta} \\
 \therefore \text{LS} &= \text{RS}, \therefore \frac{1-\cos \beta}{\sin \beta} = \frac{\sin \beta}{1+\cos \beta}.
 \end{aligned}$$

(b)

$$\begin{aligned}
 \text{LS} &= \frac{\sin x}{1+\cos x} & \text{RS} &= \frac{1}{\sin x} - \frac{1}{\tan x} \\
 &= \frac{\sin x}{1+\cos x} \left(\frac{1-\cos x}{1-\cos x} \right) & &= \frac{1}{\sin x} - \frac{1}{\frac{\sin x}{\cos x}} \\
 &= \frac{\sin x(1-\cos x)}{1-\cos^2 x} & &= \frac{1}{\sin x} - \frac{\cos x}{\sin x} \\
 &= \frac{\sin x(1-\cos x)}{\sin^2 x} & &= \frac{1-\cos x}{\sin x} \\
 &= \frac{1-\cos x}{\sin x} \\
 \therefore \text{LS} &= \text{RS}, \therefore \frac{\sin x}{1+\cos x} = \frac{1}{\sin x} - \frac{1}{\tan x}.
 \end{aligned}$$

15.

$$\begin{aligned}
 \text{LS} &= \frac{\sin^2 \phi + 2 \cos \phi - 1}{\sin^2 \phi + 3 \cos \phi - 3} & \text{RS} &= \frac{\cos^2 \phi + \cos \phi}{-\sin^2 \phi} \\
 &= \frac{(1-\cos^2 \phi) + 2 \cos \phi - 1}{(1-\cos^2 \phi) + 3 \cos \phi - 3} & &= \frac{\cos^2 \phi + \cos \phi}{-(1-\cos^2 \phi)} \\
 &= \frac{-\cos^2 \phi + 2 \cos \phi}{-\cos^2 \phi + 3 \cos \phi - 2} & &= \frac{\cos \phi(\cos \phi + 1)}{-(1-\cos \phi)(1+\cos \phi)} \\
 &= \frac{-\cos \phi(\cos \phi - 2)}{-(\cos \phi - 1)(\cos \phi - 2)} & &= \frac{\cos \phi}{\cos \phi - 1} \\
 &= \frac{\cos \phi}{\cos \phi - 1} \\
 \therefore \text{LS} &= \text{RS}, \therefore \frac{\sin^2 \phi + 2 \cos \phi - 1}{\sin^2 \phi + 3 \cos \phi - 3} = \frac{\cos^2 \phi + \cos \phi}{-\sin^2 \phi}.
 \end{aligned}$$

16.

$$\begin{aligned}
 \text{LS} &= \sin^2 \phi - \cos^2 \phi - \tan^2 \phi & \text{RS} &= \frac{2 \sin^2 \phi - 2 \sin^4 \phi - 1}{1 - \sin^2 \phi} \\
 &= \sin^2 \phi - (1 - \sin^2 \phi) - \frac{\sin^2 \phi}{\cos^2 \phi} \\
 &= -1 + 2 \sin^2 \phi - \frac{\sin^2 \phi}{\cos^2 \phi} \\
 &= \frac{-\cos^2 \phi + 2 \sin^2 \phi \cos^2 \phi - \sin^2 \phi}{\cos^2 \phi} \\
 &= \frac{-(1 - \sin^2 \phi) + 2 \sin^2 \phi(1 - \sin^2 \phi) - \sin^2 \phi}{\cos^2 \phi} \\
 &= \frac{-1 + \sin^2 \phi + 2 \sin^2 \phi - 2 \sin^4 \phi - \sin^2 \phi}{\cos^2 \phi} \\
 &= \frac{2 \sin^2 \phi - 2 \sin^4 \phi - 1}{1 - \sin^2 \phi} \\
 \therefore \text{LS} &= \text{RS}, \\
 \therefore \sin^2 \phi - \cos^2 \phi - \tan^2 \phi &= \frac{2 \sin^2 \phi - 2 \sin^4 \phi - 1}{1 - \sin^2 \phi}.
 \end{aligned}$$