

Practise, Apply, Solve 6.6, page 545

1. (a) $5x^2 - 10x = 5x(x - 2)$

(b) $x^2 + 13x + 40 = (x + 8)(x + 5)$

(c) $10x^2 - 11x - 6 = (5x + 2)(2x - 3)$

(d) $x^2 - 81 = (x - 9)(x + 9)$

2. (a) $\sin^2 \theta - \sin \theta = \sin \theta (\sin \theta - 1)$

(b) $\cos^2 \theta - 2 \cos \theta + 1 = (\cos \theta - 1)(\cos \theta - 1)$

(c) $3 \sin^2 \theta - \sin \theta - 2 = (3 \sin \theta + 2)(\sin \theta - 1)$

(d) $4 \cos^2 \theta - 1 = (2 \cos \theta - 1)(2 \cos \theta + 1)$

3. (a)

LS = $2 \sin^2 \theta$ RS = 1

$$= 2 \sin^2 \left(\frac{3\pi}{4} \right)$$

$$= 2 \left(\frac{1}{\sqrt{2}} \right)^2$$

$$= 2 \left(\frac{1}{2} \right)$$

$$= 1$$

$\therefore$ LS = RS, $\therefore \theta = \frac{3\pi}{4}$ is a solution to the equation.

(b)

LS = $4 \cos^2 2\theta$ RS = 3

$$= 4 \cos^2 \left[2 \left(\frac{\pi}{12} \right) \right]$$

$$= 4 \cos^2 \left(\frac{\pi}{6} \right)$$

$$= 4 \left(\frac{\sqrt{3}}{2} \right)^2$$

$$= 4 \left(\frac{3}{4} \right)$$

$$= 3$$

$\therefore$ LS = RS, $\therefore \theta = \frac{\pi}{12}$ is a solution to the equation.

(c)

LS = $2 \sin^2 \theta - \sin \theta$ RS = 1

$$= 2 \sin^2 \left(\frac{\pi}{2} \right) - \sin \left(\frac{\pi}{2} \right)$$

$$= 2(1)^2 - (1)$$

$$= 2 - 1$$

$$= 1$$

$\therefore$ LS = RS, $\therefore \theta = \frac{\pi}{2}$ is a solution to the equation.

4. (a) $\sin x \cos x = 0$

$\therefore \sin x = 0$ and $\cos x = 0$

$x = 0^\circ, 180^\circ, 360^\circ$

$x = 90^\circ, 270^\circ$

$\therefore x = 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$

(b) $\sin x (\cos x - 1) = 0$

$\therefore \sin x = 0$

$x = 0^\circ, 180^\circ, 360^\circ$

and

$\cos x - 1 = 0$

$\cos x = 1$

$x = 0^\circ, 360^\circ$

$\therefore x = 0^\circ, 180^\circ, 360^\circ$

(c) $(\sin x + 1) \cos x = 0$

$\therefore \sin x + 1 = 0$

$\sin x = -1$

$x = 270^\circ$

$\therefore x = 90^\circ, 270^\circ$

and

$\cos x = 0$

$x = 90^\circ, 270^\circ$

(d) $\cos x (2 \sin x - \sqrt{3}) = 0$

$\therefore \cos x = 0$

$x = 90^\circ, 270^\circ$

and

$2 \sin x - \sqrt{3} = 0$

$2 \sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$x = 60^\circ, 120^\circ$

$\therefore x = 60^\circ, 90^\circ, 120^\circ, 270^\circ$

(e) $(\sqrt{2} \sin x - 1)(\sqrt{2} \sin x + 1) = 0$

$\therefore \sqrt{2} \sin x - 1 = 0$

and

$\sqrt{2} \sin x + 1 = 0$

$\sin x = \frac{1}{\sqrt{2}}$

$x = 45^\circ, 135^\circ$

$\sin x = \frac{-1}{\sqrt{2}}$

$x = 225^\circ, 315^\circ$

$\therefore x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$

(f) $(\sin x - 1)(\cos x + 1) = 0$

$\therefore \sin x - 1 = 0$

and

$\cos x + 1 = 0$

$\sin x = 1$

$x = 90^\circ$

$\cos x = -1$

$x = 180^\circ$

$\therefore x = 90^\circ, 180^\circ$

5. (a) $(2 \sin x - 1) \cos x = 0$

$\therefore 2 \sin x - 1 = 0$

and

$\cos x = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

(b) $(\sin x + 1)^2 = 0$

$\sin x + 1 = 0$

$\sin x = -1$

$\therefore x = \frac{3\pi}{2}$

(c) $(2 \cos x + \sqrt{3}) \sin x = 0$

$2 \cos x + \sqrt{3} = 0$

and

$\sin x = 0$

$\cos x = \frac{-\sqrt{3}}{2}$

$x = 0, \pi, 2\pi$

$x = \frac{5\pi}{6}, \frac{7\pi}{6}$

$\therefore x = 0, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, 2\pi$

$$(d) (2 \cos x - 1)(2 \sin x + \sqrt{3}) = 0$$

$$2 \cos x - 1 = 0 \quad \text{and} \quad 2 \sin x + \sqrt{3} = 0$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\therefore x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$(e) (\sqrt{2} \cos x - 1)(\sqrt{2} \cos x + 1) = 0$$

$$\sqrt{2} \cos x - 1 = 0 \quad \text{and} \quad \sqrt{2} \cos x + 1 = 0$$

$$\cos x = \frac{1}{\sqrt{2}}$$

$$x = \frac{\pi}{4}, \frac{7\pi}{4}$$

$$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$(f) (\sin x + 1)(\cos x - 1) = 0$$

$$\sin x + 1 = 0 \quad \text{and} \quad \cos x - 1 = 0$$

$$\sin x = -1$$

$$x = \frac{3\pi}{2}$$

$$\therefore x = 0, \frac{3\pi}{2}, 2\pi$$

$$6. (a) \sin^2 \theta = 1 \Rightarrow \sin \theta = \pm 1$$

$$\sin \theta = 1 \quad \text{and} \quad \sin \theta = -1$$

$$\theta = 90^\circ \quad \theta = 270^\circ$$

$$\therefore \theta = 90^\circ, 270^\circ$$

$$(b) \cos^2 \theta = 1 \Rightarrow \cos \theta = \pm 1$$

$$\cos \theta = 1 \quad \text{and} \quad \cos \theta = -1$$

$$\theta = 0^\circ, 360^\circ \quad \theta = 180^\circ$$

$$\therefore \theta = 0^\circ, 180^\circ, 360^\circ$$

$$(c) \tan^2 \theta = 1 \Rightarrow \tan \theta = \pm 1$$

$$\tan \theta = 1 \quad \text{and} \quad \tan \theta = -1$$

$$\theta = 45^\circ, 225^\circ \quad \theta = 135^\circ, 315^\circ$$

$$\therefore \theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

$$(d) 4 \cos^2 \theta = 1 \Rightarrow \cos^2 \theta = \frac{1}{4} \Rightarrow \cos \theta = \pm \frac{1}{2}$$

$$\cos \theta = \frac{1}{2} \quad \text{and} \quad \cos \theta = -\frac{1}{2}$$

$$\theta = 60^\circ, 300^\circ \quad \theta = 120^\circ, 240^\circ$$

$$\therefore \theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$

$$(e) 3 \tan^2 \theta = 1 \Rightarrow \tan^2 \theta = \frac{1}{3} \Rightarrow \tan \theta = \pm \frac{1}{\sqrt{3}}$$

$$\tan \theta = \frac{1}{\sqrt{3}} \quad \text{and} \quad \tan \theta = -\frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ, 210^\circ \quad \theta = 150^\circ, 330^\circ$$

$$\therefore \theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

$$(f) 2 \sin^2 \theta = 1 \Rightarrow \sin^2 \theta = \frac{1}{2} \Rightarrow \sin \theta = \pm \frac{1}{\sqrt{2}}$$

$$\sin \theta = \frac{1}{\sqrt{2}} \quad \text{and} \quad \sin \theta = -\frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ, 135^\circ \quad \theta = 225^\circ, 315^\circ$$

$$\therefore \theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

$$7. (a) 2 \sin^2 x - \sin x - 1 = (2 \sin x + 1)(\sin x - 1)$$

$$(b) 2 \sin x + 1 = 0 \quad \text{and} \quad \sin x - 1 = 0$$

$$\sin x = -\frac{1}{2}$$

$$\sin x = 1$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{\pi}{2}$$

$$\therefore x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$8. (a) 2 \cos^2 x + \cos x - 1 = (2 \cos x - 1)(\cos x + 1)$$

$$(b) 2 \cos x - 1 = 0 \quad \text{and} \quad \cos x + 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$\cos x = -1$$

$$x = 60^\circ, 300^\circ$$

$$x = 180^\circ$$

$$\therefore x = 60^\circ, 180^\circ, 300^\circ$$

$$9. (a) 2 \sin^2 x - \sin x = 0$$

$$\sin x(2 \sin x - 1) = 0$$

$$\sin x = 0 \quad \text{and} \quad 2 \sin x - 1 = 0$$

$$x = 0^\circ, 180^\circ, 360^\circ$$

$$\sin x = \frac{1}{2}$$

$$x = 30^\circ, 150^\circ$$

$$\therefore x = 0^\circ, 30^\circ, 150^\circ, 180^\circ, 360^\circ$$

$$(b) \cos^2 x = \cos x$$

$$\cos^2 x - \cos x = 0$$

$$\cos x(\cos x - 1) = 0$$

$$\cos x = 0$$

$$x = 90^\circ, 270^\circ$$

$$\cos x - 1 = 0$$

$$\cos x = 1$$

$$x = 0^\circ, 360^\circ$$

$$\therefore x = 0^\circ, 90^\circ, 270^\circ, 360^\circ$$

$$(c) 2 \tan^2 x + \tan x - 3 = 0$$

$$(2 \tan x + 3)(\tan x - 1) = 0$$

$$2 \tan x + 3 = 0$$

$$\text{and} \quad \tan x - 1 = 0$$

$$\tan x = -\frac{3}{2}$$

$$\tan x = 1$$

$$x = -56.3^\circ$$

$$x = 45^\circ, 225^\circ$$

$$x = 124^\circ, 304^\circ$$

$$\therefore x = 45^\circ, 124^\circ, 225^\circ, 304^\circ$$

$$(d) 6 \sin^2 x - \sin x = 1$$

$$6 \sin^2 x - \sin x - 1 = 0$$

$$(3 \sin x + 1)(2 \sin x - 1) = 0$$

$$3 \sin x + 1 = 0$$

$$\text{and} \quad 2 \sin x - 1 = 0$$

$$\sin x = -\frac{1}{3}$$

$$\sin x = \frac{1}{2}$$

$$x = -19.5^\circ, 200^\circ, 341^\circ$$

$$x = 30^\circ, 150^\circ$$

$$\therefore x = 30^\circ, 150^\circ, 200^\circ, 341^\circ$$

$$(e) \cos^2 x - 6 \cos x + 5 = 0$$

$$(\cos x - 1)(\cos x - 5) = 0$$

$$\cos x - 1 = 0$$

$$\text{and} \quad \cos x - 5 = 0$$

$$\cos x = 1$$

$$\cos x = 5$$

$$x = 0^\circ, 360^\circ$$

$$x \text{ has no solutions}$$

$$\therefore x = 0^\circ, 360^\circ$$

$$(f) 4 \sin^2 x - 3 = -\sin x$$

$$4 \sin^2 x + \sin x - 3 = 0$$

$$(4 \sin x - 3)(\sin x + 1) = 0$$

$$4 \sin x - 3 = 0$$

$$\text{and} \quad \sin x + 1 = 0$$

$$\sin x = \frac{3}{4}$$

$$\sin x = -1$$

$$x = 49^\circ, 131^\circ$$

$$x = 270^\circ$$

$$\therefore x = 49^\circ, 131^\circ, 270^\circ$$

$$10. (a) 2 \cos^2 \theta + \cos \theta - 1 = 0$$

$$(2 \cos \theta - 1)(\cos \theta + 1) = 0$$

$$2 \cos \theta - 1 = 0 \quad \text{and} \quad \cos \theta + 1 = 0$$

$$\cos \theta = \frac{1}{2} \quad \cos \theta = -1$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3} \quad \theta = \pi$$

$$\therefore \theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

$$(b) 2 \sin^2 \theta = 1 - \sin \theta$$

$$2 \sin^2 \theta + \sin \theta - 1 = 0$$

$$(2 \sin \theta - 1)(\sin \theta + 1) = 0$$

$$2 \sin \theta - 1 = 0 \quad \text{and} \quad \sin \theta + 1 = 0$$

$$\sin \theta = \frac{1}{2} \quad \sin \theta = -1$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad \theta = \frac{3\pi}{2}$$

$$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$(c) \cos^2 \theta = 2 + \cos \theta$$

$$\cos^2 \theta - \cos \theta - 2 = 0$$

$$(\cos \theta + 1)(\cos \theta - 2) = 0$$

$$\cos \theta + 1 = 0 \quad \text{and} \quad \cos \theta - 2 = 0$$

$$\cos \theta = -1 \quad \cos \theta = 2$$

$$\theta = \pi \quad \theta \text{ has no solutions}$$

$$\therefore \theta = \pi$$

$$(d) 2 \sin^2 \theta + 5 \sin \theta - 3 = 0$$

$$(2 \sin \theta - 1)(\sin \theta + 3) = 0$$

$$2 \sin \theta - 1 = 0 \quad \text{and} \quad \sin \theta + 3 = 0$$

$$\sin \theta = \frac{1}{2} \quad \sin \theta = -3$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad \theta \text{ has no solutions}$$

$$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$(e) 3 \tan^2 \theta - 2 \tan \theta = 1$$

$$3 \tan^2 \theta - 2 \tan \theta - 1 = 0$$

$$(3 \tan \theta + 1)(\tan \theta - 1) = 0$$

$$3 \tan \theta + 1 = 0 \quad \text{and} \quad \tan \theta - 1 = 0$$

$$\tan \theta = -\frac{1}{3} \quad \tan \theta = 1$$

$$\theta = -0.32 \quad \theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$\theta = 2.82, 5.96$$

$$\therefore \theta = \frac{\pi}{4}, 2.82, \frac{5\pi}{4}, 5.96$$

$$(f) 12 \sin^2 \theta + \sin \theta - 6 = 0$$

$$(3 \sin \theta - 2)(4 \sin \theta + 3) = 0$$

$$3 \sin \theta - 2 = 0 \quad \text{and} \quad 4 \sin \theta + 3 = 0$$

$$\sin \theta = \frac{2}{3} \quad \sin \theta = -\frac{3}{4}$$

$$\theta = 0.73, 2.41 \quad \theta = -0.85$$

$$\therefore \theta = 0.73, 2.41, 3.99, 5.44$$

$$11. (a) \cos^2 2x = -\cos 2x$$

$$\cos^2 2x + \cos 2x = 0$$

$$\cos 2x(\cos 2x + 1) = 0$$

$$\cos 2x = 0 \quad \text{and} \quad \cos 2x + 1 = 0$$

$$2x = 90^\circ, 270^\circ \quad \cos 2x = -1$$

$$x = 45^\circ, 135^\circ \quad 2x = 180^\circ$$

$$\therefore x = 45^\circ, 90^\circ, 135^\circ, 225^\circ, 315^\circ$$

(b)

$$2 \sin^2 \left(\frac{x}{2} \right) = 1$$

$$\sin^2 \left(\frac{x}{2} \right) = \frac{1}{2}$$

$$\sin \left(\frac{x}{2} \right) = \pm \frac{1}{\sqrt{2}}$$

$$\sin \left(\frac{x}{2} \right) = \frac{1}{\sqrt{2}} \quad \text{and} \quad \sin \left(\frac{x}{2} \right) = \frac{-1}{\sqrt{2}}$$

$$\frac{x}{2} = 45^\circ, 135^\circ \quad \frac{x}{2} = 225^\circ, 315^\circ$$

$$x = 90^\circ, 270^\circ \quad x = 450^\circ, 630^\circ$$

$$\therefore x = 90^\circ, 270^\circ$$

$$12. (a) \cos^2 \theta - \sin^2 \theta = 1$$

$$(1 - \sin^2 \theta) - \sin^2 \theta = 1$$

$$1 - 2 \sin^2 \theta = 1$$

$$-2 \sin^2 \theta = 0$$

$$\sin^2 \theta = 0$$

$$\therefore \sin \theta = 0$$

$$\therefore \theta = 0, \pi, 2\pi$$

$$(b) \sin \theta - \cos^2 \theta - 1 = 0$$

$$\sin \theta - (1 - \sin^2 \theta) - 1 = 0$$

$$\sin \theta - 1 + \sin^2 \theta - 1 = 0$$

$$\sin^2 \theta + \sin \theta - 2 = 0$$

$$(\sin \theta + 2)(\sin \theta - 1) = 0$$

$$\sin \theta + 2 = 0 \quad \text{and} \quad \sin \theta - 1 = 0$$

$$\sin \theta = -2 \quad \sin \theta = 1$$

$$\theta \text{ has no solutions} \quad \theta = \frac{\pi}{2}$$

$$\therefore \theta = \frac{\pi}{2}$$

$$(c) 2 \sin \theta - \cos^2 \theta = 2$$

$$2 \sin \theta - (1 - \sin^2 \theta) = 2$$

$$2 \sin \theta - 1 + \sin^2 \theta = 2$$

$$\sin^2 \theta + 2 \sin \theta - 3 = 0$$

$$(\sin \theta + 3)(\sin \theta - 1) = 0$$

$$\sin \theta + 3 = 0 \quad \text{and} \quad \sin \theta - 1 = 0$$

$$\sin \theta = -3 \quad \sin \theta = 1$$

$$\theta \text{ has no solutions} \quad \theta = \frac{\pi}{2}$$

$$\therefore \theta = \frac{\pi}{2}$$

$$(d) 13 - 15 \sin^2 \theta + \cos \theta = 0$$

$$13 - 15(1 - \cos^2 \theta) + \cos \theta = 0$$

$$13 - 15 + 15 \cos^2 \theta + \cos \theta = 0$$

$$15 \cos^2 \theta + \cos \theta - 2 = 0$$

$$(3 \cos \theta - 1)(5 \cos \theta + 2) = 0$$

$$3 \cos \theta - 1 = 0 \quad \text{and}$$

$$\cos \theta = \frac{1}{3}$$

$$\theta = 1.23, 5.05$$

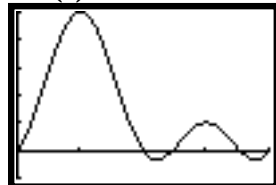
$$\therefore \theta = 1.23, 1.98, 4.30, 5.05$$

$$5 \cos \theta + 2 = 0$$

$$\cos \theta = \frac{-2}{5}$$

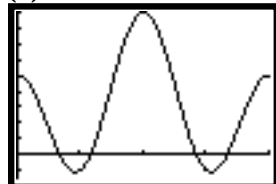
$$\theta = 1.98, 4.30$$

13. (a)



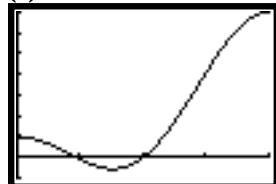
$$\theta = 0^\circ, 180^\circ, 222^\circ, 318^\circ, 360^\circ$$

(b)



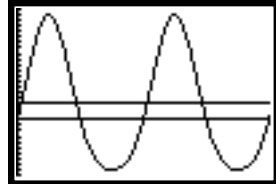
$$\theta = 60^\circ, 104^\circ, 256^\circ, 300^\circ$$

(c)



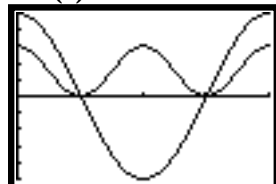
$$\theta = 83^\circ, 180^\circ$$

(d)



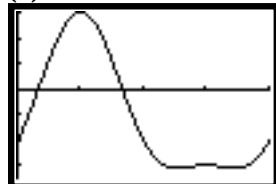
$$\theta = 6^\circ, 84^\circ, 186^\circ, 264^\circ$$

14. (a)



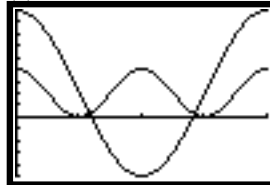
$$x = 1.57, 4.71$$

(b)



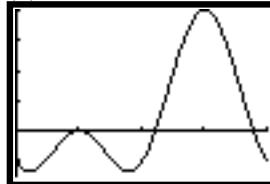
$$x = 0.52, 2.62$$

(c)



$$x = 1.82, 4.46$$

(d)



$$x = 1.57, 3.48, 5.94$$

15. (a) Example: $2 \cos^2 x = \cos x$ OR $\sin^2 x - 4 \sin x + 4 = 0$

(b) No, it is not possible to factor all quadratic trigonometric equations. Some equations have irrational roots.

(c) The two factors that play a role when determining the number of solutions of a quadratic trigonometric equation are period and domain.

16. (a) $3 \tan^2 2x = 1$

$$\tan^2 2x = \frac{1}{3}$$

$$\tan 2x = \pm \frac{1}{\sqrt{3}}$$

$$\tan 2x = \frac{1}{\sqrt{3}} \quad \text{and} \quad \tan 2x = \frac{-1}{\sqrt{3}}$$

$$2x = 30^\circ \quad 2x = 150^\circ$$

$$x = 15^\circ, 105^\circ, 195^\circ, 285^\circ \quad x = 75^\circ, 165^\circ, 255^\circ, 345^\circ$$

$$\therefore x = 15^\circ, 75^\circ, 105^\circ, 165^\circ, 195^\circ, 255^\circ, 285^\circ, 345^\circ$$

$$(b) (\sqrt{2} \sin \theta)^2 = (\sqrt{3} - \cos \theta)^2$$

$$2 \sin^2 \theta = 3 - 2\sqrt{3} \cos \theta + \cos^2 \theta$$

$$2(1 - \cos^2 \theta) = 3 - 2\sqrt{3} \cos \theta + \cos^2 \theta$$

$$2 - 2 \cos^2 \theta = 3 - 2\sqrt{3} \cos \theta + \cos^2 \theta$$

$$3 \cos^2 \theta - 2\sqrt{3} \cos \theta + 1 = 0$$

$$\cos \theta = \frac{\sqrt{3}}{3}$$

$$\theta = 0.96$$

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1.

Given Information	What Can Be Found	Law Required
two angles and any side (AAS or ASA)	side	Sine Law
two sides and the contained angle (SAS)	side	Cosine Law
three sides (SSS)	angle	Cosine Law
two sides and an angle opposite one of them (SSA)	angle	Sine Law